The Magnetic Reconnection Code: Framework and Application

K. Geraschewski, A. Bhattacharjee (University of Iowa)
T. Linde, R. Rosner, A. Siegel (University of Chicago)
D. Keyes, F. Dobrian (Old Dominion University)

1 Adaptive Mesh Refinement

1.1 Example: 2D ideal MHD (previous work)

1.2 Quad/Oct-Tree vs. arbitrary patches

1.3 Tree constructed refinement and load balancing

1.4 Time substepping

2 Elliptic solvers

2.1 Additive Schwarz Iteration

Decomposition into 3 × 3 grids, 4 points overlap

Adaptivity in space only, same timestep on all levels

Adaptivity in time (Galerkin-Offen time-stepping)

Adaptive mesh refinement

Decomposition into 3 × 3 grids, 4 points overlap

Elliptic problem

Hyperbolic problem

Elliptic problem

5.4 Time substepping

5.5 Case π/4

5.6 Growth rate vs. α, β/aspect ratio

6 Implicit solvers

The example of rectangular in two-dimensional incompressible Hall-MHD is used to evaluate the trade-offs between explicit and implicit time stepping. Being incompressible, the fast sound waves have already been filtered out of the problem, so that neither the explicit nor the implicit scheme need to handle them, for the explicit scheme this comes at the expense of solving elliptic problems at each time step. However, the explicit scheme is still limited by the Courant-Friedrichs-Lewy stability criterion, maintaining small time steps to avoid spatial over-resolution. These time steps are still much smaller than necessary for the desired accuracy, since the reconnection phenomena takes place at a slower time scale. On the other hand, the implicit steps are reached much faster than explicit steps at all low magnitudes, making the implicit code the preferred approach. Since the implicit method is not constrained to time step limitations as solution increases and can be implemented to scale as \( \mathcal{O}(N^2) \) for large problems using Newton-Krylov-Schwarz methods, we expect a break-even point to exist at which the implicit solver proves favorable to the explicit time stepping.

The set of equations that are solved by the explicit solver is

\[
\Delta \mathbf{B} + \frac{\partial \mathbf{U}}{\partial t} = \nabla \times \mathbf{E} + \frac{\partial \mathbf{E}}{\partial t}
\]

where \( \mathbf{E} = -\nabla \psi \), \( \mathbf{B} = \nabla \times \mathbf{U} \).

To compare these two fundamentally different algorithms, we are using the PETSc library, which is being optimized for the given problem in collaboration with David Keyes / the TOPS group.