This is a major paper concerning scalar conservation laws. Although the results hold independently on the space dimension $N$, they are new even if $N = 1$: a quantitative version of L. Tartar’s compactness result [in Nonlinear analysis and mechanics: Heriot-Watt Symposium, Vol. IV, 136–212, Pitman, Boston, MA, 1979; MR0584398 (81m:35014)] is given.

Thanks to a forthcoming paper of two of the authors, the equation $\partial_t u + \text{div} A(u) = 0, \ t > 0$, is reformulated as a kinetic equation. Its well-posedness, regarding the Cauchy problem, is proved.

Let us assume that $\text{meas}\{-R < v < R; \tau + A'(v) \cdot \xi = 0\} = 0$ for any $R$. Then any bounded sequence of solutions, bounded in mass, is actually relatively compact in $L^1_{\text{loc}}$. If $N = 1$, this is Tartar’s result, otherwise it is new.

More precisely, one may assume that

$$\text{meas}\{-R < v < R; |\tau + A'(v) \cdot \xi|^2 \leq \delta^2(\tau^2 + |\xi|^2)\} \leq C(R)\delta^\alpha,$$

uniformly in $(\tau, \xi)$. Then such a sequence of solutions is actually bounded in $W^{s,p}_{\text{loc}}$, uniformly in time, for $s < \alpha/(\alpha + 2)$ and $p = (\alpha + 4)(\alpha + 2)$. Examples indicate that the bound for $s$ is not accurate; $\alpha$ would be expected.

Reviewed by Denis Serre

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