

## Eitan TADMOR – Career Narrative

**Eitan Tadmor** is a Distinguished University Professor at the University of Maryland, College Park (UMd). Tadmor began his scientific career in 1980 as a Bateman Research Instructor in CalTech. He later chaired the Department of Applied Mathematics in his alma mater, Tel-Aviv U. 1991–1993. In 1995 he was recruited by the UCLA Department of Mathematics in 1995 where he was the Founding Co-Director of the NSF Institute for Pure and Applied Mathematics ([IPAM](#)), 1999–2001. He moved to UMd in 2002, and served as the Director of the University Center for Scientific Computation and Mathematical Modeling ([CSCAMM](#)), 2002–2016.

Synergetic activities include co-chairing International Conferences on Hyperbolic Problems, hosted at CalTech (Hyp2002) and at UMd (Hyp2008). Tadmor was the Principal Investigator (PI) for an NSF Focus Research Group on “Kinetic Description of Multiscale Phenomena” (2008–2012). He was on the Scientific Committee of Abel Symposium on “Nonlinear PDEs” held in Oslo, 2010. In 2012 he was the PI awarded the NSF Research network “[Kinetic Description of Emerging Challenges in Natural Sciences](#)” (Ki-Net) and is currently serving as the Ki-Net Director.

Tadmor is currently a senior fellow at the Institute for Theoretical Studies (ITS), ETH-Zurich, 2016-2017. He held numerous visiting positions, including longer visits at the universities of Michigan, Paris VI, Brown, at the Courant Institute and at the Weizmann Institute. He serves/served on the editorial boards of more than a dozen leading journals including European Math Surveys in Math. Sciences (2014–), Acta Numerica (2009–), SIAM J. Math. Analysis (2004–), J. Foundations of Computational Math. (2004–) and SIAM J. on Numerical Analysis (1990–2013).

Invited addresses include an invited lecture at the 2002 ICM (Beijing), plenary addresses in the international conferences on hyperbolic problems (Zürich 1990 and Beijing 1998), the 2008 Foundations of Computational Math. meeting in Hong-Kong, and the SIAM invited address at the 2014 Joint Mathematical meeting in Baltimore.

Tadmor was listed on the 2003 ISI most cited researchers in Mathematics. In 2012 he was in the inaugural class of Fellows of the American Mathematical Society. In 2015 he was awarded the SIAM-ETH Henrici prize for “*original, broad and fundamental contributions to the applied and numerical analysis of nonlinear differential equations and their applications ...*”.

The signature of Tadmor’s work is the interplay between analytical theories and computational algorithms with diverse applications to shock waves, kinetic transport, incompressible flows, image processing, and self-organized collective dynamics. In particular, he has made a series of fundamental contributions to the development of high-resolution methods for nonlinear conservation laws, introducing the classes of *central schemes*, *entropy conservative/stable schemes* and *spectral viscosity methods*. He has carried out ground-breaking work on the rigorous derivation of transport models and their relation to *kinetic theories*, co-authored with P.-L. Lions, B. Perthame (1994) and T. Tao (2002), and a separate line of work on *critical thresholds phenomena* in such models. He introduced novel ideas of multi-scale *hierarchical decompositions* of images and solutions of PDEs in critical regularity spaces. He leads an interdisciplinary research program in modeling and analysis of *social (hydro-)dynamics* with applications to flocking and opinion dynamics.

## Eitan TADMOR – 10 principal Publications

### 1 Convergence of spectral methods for nonlinear conservation laws

*SIAM J. Numerical Analysis* 26 (1989) 30–44

This paper introduced the Spectral Viscosity method — the first systematic method to treat shock discontinuities with spectral calculations. A follow-up a large body of related works.

### 2 Non-oscillatory central differencing for hyperbolic conservation laws

(with H. Nessyahu), *J. of Computational Physics* 87 (1990) 408–463

This paper introduced the Nessyahu-Tadmor scheme — the forerunner for the class of high-resolution "central schemes", and led to a large number of publications on related black-box solvers for a wide variety of problems governed by multi-dimensional systems of non-linear conservation laws and related PDEs.

### 3 A kinetic formulation of multidimensional scalar conservation laws and related equations

(with P.-L. Lions & B. Perthame) *J. Amer. Math. Soc.* 7 (1994) 169–191

and a follow-up work

### Velocity averaging, kinetic formulations and regularizing effects in quasi-linear PDEs

(with Terence Tao) *Communications Pure & Applied Math.* 60 (2007), 1488–1521

These papers provide the systematic treatment of kinetic formulation of entropic solutions for nonlinear conservation laws and related convection-diffusion equations and derivation of novel regularizing results.

### 4 Local error estimates for discontinuous solutions of nonlinear hyperbolic equations

*SIAM J. Numerical Anal.* 28 (1991) 891–906

This paper introduced a novel convergence rate theory for nonlinear conservation laws and related Hamilton-Jacobi equations that led to optimal  $L^1$ -convergence rates.

### 5 High order time discretization methods with the strong stability property

(with S. Gottlieb and C.-W. Shu), *SIAM Review* 43 (2001) 89–112

This is the standard reference for Strong Stability Preserving (SSP) numerical solvers of ODEs.

### 6 Spectral dynamics of the velocity gradient field in restricted flows

(with H. Liu), *Comm. Math. Physics* 228 (2002), 435–466

A systematic study of global regularity vs. finite time blow-up using spectral dynamics of velocity gradient matrix.

### 7 Entropy stability theory for difference approximations of nonlinear conservation laws and related time dependent problems, *Acta Numerica* 12 (2003), 451–512

Here we introduced a novel family of entropy conservative schemes and provides a general framework for studying entropy stability of difference approximations for nonlinear systems of conservation laws by comparison.

### 8 A multiscale image representation using hierarchical (BV, $L^2$ ) decompositions

(with S. Nezzar and L. Vese) *Multiscale Modeling & Simulation* 2 (2004) 554–579

We introduce a novel hierarchical decomposition of images and solutions of equations in critical regularity spaces into multi-scale components.

### 9 A new model for self-organized dynamics and its flocking behavior

(with S. Motsch), *J. Stat. Physics* 144(5) (2011) 923–947

Introduced a new model for far from equilibrium self-organized dynamics based on relative distances.

### 10 ENO reconstruction and ENO interpolation are stable

(with U. Fjordholm and S. Mishra) *Foundations Computational Math.* 13(2) (2012), 139–159

First stability proof for the ENO reconstruction, indicating a remarkable rigidity for ENO procedure of arbitrary order of accuracy and on non-uniform meshes.

## Eitan TADMOR – significant contributions<sup>1</sup>

### ★ High-resolution approximations for nonlinear conservation laws

#### 1 Non-oscillatory central differencing for hyperbolic conservation laws

H. Nessyahu and E. Tadmor, *J. of Computational Physics* 87 (1990) 408–463.

This paper introduced the Nessyahu-Tadmor scheme — the forerunner for the class of high-resolution ”central schemes”, which offer black-box solvers for a wide variety of problems governed by multi-dimensional systems of non-linear conservation laws and related PDEs, consult [CentPack](#). Related follow-up work can be found in the 1990 JCP work with A. Kurganov on [New high resolution central schemes for nonlinear conservation laws and convection-diffusion equations](#).

#### 2 Local error estimates for discontinuous solutions of nonlinear hyperbolic equations

E. Tadmor, *SIAM J. Numerical Analysis* 28 (1991) 891–906.

This paper introduced a novel  $L^1$ -convergence rate theory for nonlinear conservation laws and related Hamilton-Jacobi equations. The theory, based on one-sided stability estimates, provides an alternative to the standard Krushkov-Kuznetsov theory and led to optimal  $L^1$ -convergence rates. A large body of related works, including the 2001 work with C.-T Lin on [L<sup>1</sup>-stability and error estimates for approximate Hamilton-Jacobi solutions](#) can be found in [www2.cscamm.umd.edu/tadmor/convergence\\_rate/](http://www2.cscamm.umd.edu/tadmor/convergence_rate/).

#### 3 ENO reconstruction and ENO interpolation are stable (+errata)

U. Fjordholm, S. Mishra and E. Tadmor, *Foundations of Computational Mathematics* 13(2) (2012), 139–159.

The ENO reconstruction procedure was introduced in 1987 by Harten et. al. in the context of accurate simulations for piecewise smooth solutions of nonlinear conservation laws. Despite the extensive literature on the construction and implementation of ENO method and its variants, the question of its stability remained open during the last 25 years. Here we prove that the ENO reconstruction and ENO interpolation procedures are stable in the sense that the size of the jumps after ENO reconstruction relative to the jump of the underlying cell averages is bounded. These estimates, which are shown to hold for ENO reconstruction and interpolation of arbitrary order of accuracy and on non-uniform meshes, indicate a remarkable rigidity of the piecewise-polynomial ENO procedure.

### ★ Kinetic formulation and regularizing effects in nonlinear PDEs

#### 4 A kinetic formulation of multidimensional scalar conservation laws and related equations

P.-L. Lions, B. Perthame and E. Tadmor, *J. American Math. Society* 7 (1994) 169–191.

This paper provides a systematic treatment of kinetic formulation of entropic solutions for nonlinear conservation laws and related convection-diffusion equations and the first derivation of regularizing effects using the averaging lemma. It was followed by a large body of works which utilized the kinetic formulation and their new regularizing effects for conservation laws, related degenerate equations and their numerical approximations. In particular, a follow-up 1994 CMP work with P.-L. Lions and B. Perthame on [Kinetic formulation of the isentropic gas dynamics and p-systems](#).

<sup>1</sup>The electronic version of this pdf document includes hyperlinks to works available at <http://www.cscamm.umd.edu/tadmor/pub/significant.htm#narrative>

## 5 Velocity averaging, kinetic formulations and regularizing effects in quasi-linear PDEs

E. Tadmor and Terence Tao, *Communications Pure & Applied Mathematics* 60 (2007), 1488–1521.

The present paper provides the first quantitative velocity averaging result for second order equations, thus paving the way for a full family of new results for regularizing effects in second-order degenerate nonlinear parabolic equations, in particular in the anisotropic cases where relatively little was known prior to this contribution. The method of proof is based on a delicate multipliers techniques, dyadic decomposition and refined estimates on Littlewood-Paley blocks to verify the so-called "truncation property".

## ★ The spectral viscosity method—computation of Gibbs phenomenon

### 6 Recovering pointwise values of discontinuous data within spectral accuracy

D. Gottlieb and E. Tadmor, in "Progress and Supercomputing in Computational Fluid Dynamics", Proc. 1984 U.S.-Israel Workshop on Progress in Scientific Computing, Vol. 6 (E. M. Murman and S. S. Abarbanel, eds.), Birkhauser, Boston (1985) 357–375 [[SIAM Rev 28\(4\) 1986](#)].

Here we show how the pointvalues of a piecewise smooth function can be recovered from its spectral content, so that the accuracy depend solely on the local smoothness. This paper was the forerunner for large body of work which followed on the 90s and 00s, on computation of the Gibbs phenomenon (Gottlieb, Shu, Gelb, Tanner and others). In particular, these Gottlieb–Tadmor mollifiers were subsequently improved to [\(root-\) exponential accuracy](#) (with J. Tanner) and motivated the development of [effective spectral edge detectors](#) (with A. Gelb).

### 7 Convergence of spectral methods for nonlinear conservation laws

E. Tadmor, *SIAM J. Numerical Analysis* 26 (1989) 30–44.

The Spectral Viscosity (SV) method was developed in 1989 as a systematic approach for treating shock discontinuities in spectral calculations, by adding a spectrally small amount of high-frequencies diffusion. The resulting SV-approximation is stable without sacrificing spectral accuracy, recovering a spectrally accurate approximation of (the projection of the) entropy solution. Subsequently, the SV method was implemented by many practitioners in highly accurate spectral computations of nonlinear equations; consult [www2.cscamm.umd.edu/tadmor/spectral\\_viscosity/](http://www2.cscamm.umd.edu/tadmor/spectral_viscosity/).

### 8 Filters, mollifiers and the computation of the Gibbs phenomenon

E. Tadmor, *Acta Numerica* 16 (2007) 305-378.

This 2007 *Acta Numerica* review contains a summary of the developments during 1985-2007 on detection of edges in piecewise smooth spectral data and the high resolution reconstruction of the data between those edges.

## ★ Critical thresholds in Eulerian dynamics

### 9 Spectral dynamics of the velocity gradient field in restricted flows

H. Liu and E. Tadmor, *Communications in Mathematical Physics* 228 (2002), 435–466.

In this paper we initiate a systematic study of global regularity vs. finite time blow-up gradients of the fundamental Eulerian equation,  $\mathbf{u}_t + \mathbf{u} \cdot \nabla_{\mathbf{x}} \mathbf{u} = \mathbf{F}$ , which shows up in different contexts dictated by the different modeling of  $\mathbf{F}$ 's. The analysis is based on the spectral dynamics tracing the eigenvalues of the velocity gradient which determine the boundaries of the critical threshold surfaces in configuration space. It led to a large body of work which demonstrated *a generic scenario of critical threshold phenomena*, where global regularity depends on the initial configurations of density, velocity divergence and the spectral gap of the  $2 \times 2$  velocity gradient. This includes showing that rotational forcing prolongs the life-span of sub-critical 2D shallow-water solutions, global regularity results for sub-critical 2D restricted Euler-Poisson and for 3D radial Euler-Poisson equations, and a surprising global existence result for a large set of sub-critical initial data in the 4D restricted Euler eqs. Additional references can be found in [www2.cscamm.umd.edu/tadmor/critical\\_thresholds/](http://www2.cscamm.umd.edu/tadmor/critical_thresholds/).

## ★ Entropy stability — difference approximations of nonlinear conservation laws

### 10 Entropy stability theory for difference approximations of nonlinear conservation laws and related time dependent problems

E. Tadmor, *Acta Numerica* 12 (2003), 451-512.

Entropy stability plays an important role in the dynamics of nonlinear systems of conservation laws and related convection-diffusion equations. The paper provides a state-of-the-art summary for the a body of works during 1987-2007 on the topic of entropy stability (beginning with the 1987 Math. Comp. work on [The numerical viscosity of entropy stable schemes for systems of conservation laws. I](#)). Here, we developed a general theory of entropy stability for difference approximations for nonlinear equations, which provides precise characterization of entropy stability using comparison principles. In particular, we construct a new family of entropy stable schemes which retain the precise entropy decay of the Navier-Stokes equations. They contain no artificial numerical viscosity. The theory can be found in the follow-up developments [www2.cscamm.umd.edu/tadmor/entropy\\_stability/](http://www2.cscamm.umd.edu/tadmor/entropy_stability/).

### 11 Construction of approximate entropy measure valued solutions for hyperbolic systems of conservation laws

U. Fjordholm, R. Kappeli S. Mishra and E. Tadmor, *Foundations of Computational Mathematics* (2016) DOI [10.1007/s10208-015-9299-z](https://doi.org/10.1007/s10208-015-9299-z).

Entropy solutions have been widely accepted as the suitable solution framework for systems of conservation laws in several space dimensions. However, recent results of De Lellis and Székelyhidi have demonstrated that entropy solutions may not be unique. In this paper, we present numerical evidence that demonstrates that state of the art numerical schemes may not necessarily converge to an entropy solution of systems of conservation laws as the mesh is refined. Combining these two facts, we argue that entropy solutions may not be suitable as a solution framework for systems of conservation laws, particularly in several space dimensions. We advocate a more general notion due to DiPerna — that of entropy measure valued solutions, as an appropriate solution paradigm for systems of conservation laws. To this end, we present the first detailed numerical procedure which constructs stable approximations to entropy measure valued solutions and provide sufficient conditions that guarantee these approximations converge to an entropy measure valued solution as the mesh is refined, thus providing a viable numerical framework for systems of conservation laws in several space dimensions. A large number of numerical experiments that illustrate the proposed schemes are presented and are utilized to examine several interesting properties of the computed entropy measure valued solutions.

## ★ Translatory boundary conditions. SSP time-discretization

### 12 Scheme-independent stability criteria for difference approximations of hyperbolic initial-boundary value problems. II

M. Goldberg and E. Tadmor, *Mathematics of Computation* 36 (1981) 603–626.

The development of easily checkable stability criteria for finite-difference approximations of initial boundary value systems with translatory boundary conditions. This led a series of works in the eighties, and it became a standard tool in the stability theory for approximations of initial-boundary value problem of hyperbolic type.

### 13 High order time discretization methods with the strong stability property

S. Gottlieb, C.-W. Shu and E. Tadmor, *SIAM Review* 43 (2001) 89–112.

We construct, analyze and implement the class of strong stability-preserving (SSP) high-order time discretizations for semi-discrete method of lines approximations of PDEs. These high-order methods preserve the strong stability properties of first-order Euler time stepping and have proved very useful, especially in solving hyperbolic PDEs. Since its publication in 2001, this work has become a standard reference on SSP solvers.

## ★ Hierarchical decompositions. Applications in image processing, PDEs

### 14 A multiscale image representation using hierarchical (BV, $L^2$ ) decompositions

E. Tadmor, S. Nezzar and L. Vese, *Multiscale Modeling & Simulation* 2 (2004) 554–579.

The paper introduces a novel hierarchical decomposition of images, the forerunner of recently developed iteration methods in image processing. Questions of convergence, energy decomposition, localization and adaptivity are discussed. Subsequently, our approach was used in applications to synthetic and real images. The approach developed here was pursued in a series of works, including the 2011 SIAM J. Imaging Sciences paper with P. Athavale on [Integro-differential equations based on \(BV,  \$L^1\$ \) image decomposition](#).

Additional references can be found [www2.cscamm.umd.edu/tadmor/hierarchical\\_decompositions/](http://www2.cscamm.umd.edu/tadmor/hierarchical_decompositions/).

## ★ Self-organized dynamics — flocking and opinion dynamics

### 15 Heterophilious dynamics enhances consensus

S. Motsch and E. Tadmor, *SIAM Review* 56(4) (2014) 577–621 (with [Introduction](#) by D. J. Higham).

Nature and human societies offer many examples of self-organized behavior. Ants form colonies, birds fly in flocks, mobile networks coordinate a rendezvous, and human opinions evolve into parties. These are simple examples of collective dynamics that tend to self-organize into large-scale clusters of colonies, flocks, parties, etc. We review a general class of models for self-organized dynamics based on alignment. Prototypical examples are the local Hegselmann-Krause model for opinion dynamics, and the Vicsek model for flocking, the global Cucker-Smale model for flocking, and its local version version advocated by us in the 2011 *J. Stat. Physics* paper “[A new model for self-organized dynamics and its flocking behavior](#)” (with S. Motsch). A natural question which arises in this context is to ask when and how clusters emerge through the self-alignment of agents, and what types of rules of engagement influence the formation of such clusters. Of particular interest to us are cases in which the self-organized behavior tends to concentrate into one cluster, reflecting a consensus of opinions. Standard models for self-organized dynamics assume that the intensity of alignment increases as agents get closer, reflecting a common tendency to align with those who think or act alike. Similarity breeds connection reflects our intuition that increasing the intensity of alignment as the difference of positions decreases is more likely to lead to a consensus. We argue here that the converse is true: when the dynamics is driven by local interactions, it is more likely to approach a consensus when the interactions among agents increase as a function of their difference in position. Heterophily — the tendency to bond more with those who are different rather than with those who are similar, plays a decisive role in the process of clustering. We point out that the number of clusters in heterophilious dynamics decreases as the heterophily dependence among agents increases. In particular, sufficiently strong heterophilious interactions enhance consensus. What is the qualitative behavior of self-organized dynamics for very large groups ( $N \rightarrow \infty$ )? Agent-based models lend themselves to standard kinetic and hydrodynamics descriptions originated in the 2008 KRM paper “[From particle to kinetic and hydrodynamic descriptions of flocking](#)” (with S.-Y. Ha). The latter govern “social hydrodynamics” and their critical threshold phenomena are studied the 2016 M3AS paper “[Critical thresholds in 1D Euler equations with nonlocal forces](#)” (with J. A. Carrillo, Y.-P. Choi and C. Tan) and the recent series works on [Eulerian dynamics with a commutator forcing](#) (with Shvydkoy and He).