Chapter 16
Superposition and Standing Waves

Topics:
- Superposition
- Constructive and destructive interference
- Standing waves
- Resonant modes of systems
- Beats
Principle of superposition  When two or more waves are simultaneously present at a single point in space, the displacement of the medium at that point is the sum of the displacements due to each individual wave.
An example of superposition

Two waves approach each other.

The net displacement is the point-by-point summation of the individual waves.

Both waves emerge unchanged.
An example of destructive interference

Two waves approach each other.

The leading edges of the waves meet, and the displacements offset each other at this point.

At this moment, the net displacement of the medium is zero.

Both waves emerge unchanged.
Checking Understanding

Two waves on a string are moving toward each other. A picture at $t = 0$ s appears as follows:

How does the string appear at $t = 2$ s?
Answer

Two waves on a string are moving toward each other. A picture at $t = 0$ s appears as follows:

How does the string appear at $t = 2$ s?
Standing waves
(a) This wave is traveling along the string to the right.

This wave is traveling along the string to the left.

(b) The blue wave is the superposition of the red and green waves.

At different times:
- $t = 0$
- $t = \frac{1}{8} T$
- $t = \frac{2}{8} T$
- $t = \frac{3}{8} T$
- $t = \frac{4}{8} T$
- $t = \frac{5}{8} T$
- $t = \frac{6}{8} T$
- $t = \frac{7}{8} T$
- $t = T$

(c) The superposition of the two waves is a standing wave with the same wavelength as the original waves.

At different times:
- $t = 0$
- $t = \frac{1}{8} T$
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- $t = \frac{3}{8} T$
- $t = \frac{4}{8} T$
- $t = \frac{5}{8} T$
- $t = \frac{6}{8} T$
- $t = \frac{7}{8} T$
- $t = T$
The intensity is maximum at the antinodes.

The intensity is zero at the nodes.
Standing waves on a string

Before:

The reflected pulse is inverted and its amplitude is unchanged.

After:

Wiggle the string in the middle.

Two waves move away from the middle.

The reflected waves travel through each other. This creates a standing wave.
Standing Wave Modes on a string: wavelengths

In this case we have one anti-node (max amplitude), and two nodes. The wavelength is 
\[ = 2L \]

Here we have two-antinodes (and three nodes). The wavelength is 
\[ L (= 2L/2) \]

And here three anti-nodes (and four nodes). The wavelength is 
\[ 2L/3 \]

In general, if we have \( m \) anti-nodes, the wavelength is 
\[ 2L/m. \]
• The case $m=1$ is called the fundamental mode, or first harmonic (with fundamental or primary frequency and wavelength).

• The case $m=2$ is called the second harmonic.

• And so on.

• ‘$m$’ is called the mode number.

\[ \lambda_m = \frac{2L}{m}, \quad m = 1, 2, 3, 4, \ldots \quad (16.1) \]

Wavelengths of standing wave modes of a string of length $L$
Imagine that the string where we have a standing wave is that one of a guitar or violin.

The standing wave produces vibrations in the air surrounding the string, creating a sound wave.

Does the pitch of the tone become higher or lower as we go to higher harmonics?

\[ f_m = \frac{v}{\lambda_m} = \frac{v}{2L/m} = m \left( \frac{v}{2L} \right) \quad m = 1, 2, 3, 4, \ldots \]  

(Frequencies of standing wave modes of a string of length \( L \))

Notice that here \( v \) is the speed of propagation of waves on the string, not the speed of sound in air. Why?
String instruments

\[ f_m = \frac{v}{\lambda_m} = \frac{v}{2L/m} = m\left(\frac{v}{2L}\right) \quad m = 1, 2, 3, 4, \ldots \]  

Frequencies of standing wave modes of a string of length \( L \)

Recall that the velocity of sound in a string is given by:

\[ v_{\text{string}} = \sqrt{\frac{T_s}{m}} \]

For a fixed speed, the frequency (pitch) gets higher with shorter strings.

For a fixed length, the higher the velocity the higher the frequency (pitch). In that case, the pitch changes with:

1) Mass density \( m \): The heavier the string the lower the note/pitch.
2) Tension \( T \): the tighter the string, the higher the note/pitch. For a fixed length, the higher the velocity the higher the frequency (pitch).
Standing Sound Waves

\[
\begin{align*}
\lambda_m &= \frac{2L}{m} \\
f_m &= m\left(\frac{v}{2L}\right) = mf_1 \\
m &= 1, 2, 3, 4, \ldots
\end{align*}
\]

Wavelengths and frequencies of standing sound wave modes in an open-open or closed-closed tube

\[
\begin{align*}
\lambda_m &= \frac{4L}{m} \\
f_m &= m\left(\frac{v}{4L}\right) = mf_1 \\
m &= 1, 3, 5, 7, \ldots
\end{align*}
\]

Wavelengths and frequencies of standing sound wave modes in an open-closed tube
Physics of Speech and Hearing

(a) Frequencies from the vocal cords

(b) Actual spoken frequencies (Vowel sound “ee”)

First formant

Second formant

$\Delta p$

3.81 ms

Interference of Spherical Waves

TACTICS BOX 16.1 Identifying constructive and destructive interference

1. Identify the path length from each source to the point of interest. Compute the path-length difference \( \Delta r = |r_2 - r_1| \).
2. Find the wavelength, if it is not specified.
3. If the path-length difference is a whole number of wavelengths (\( \lambda, 2\lambda, 3\lambda \ldots \)), crests are aligned with crests and there is constructive interference.
4. If the path-length difference is a whole number of wavelengths plus a half wavelength (\( 1\frac{1}{2}\lambda, 2\frac{1}{2}\lambda, 3\frac{1}{2}\lambda \ldots \)), crests are aligned with troughs and there is destructive interference.
Beats

\[ f_{\text{beat}} = |f_1 - f_2| \]