Self-Force for Comparable Mass Binaries

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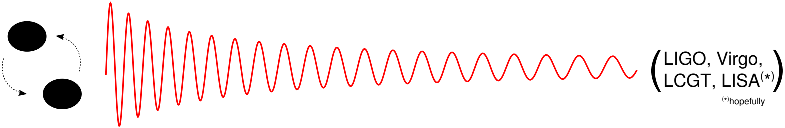
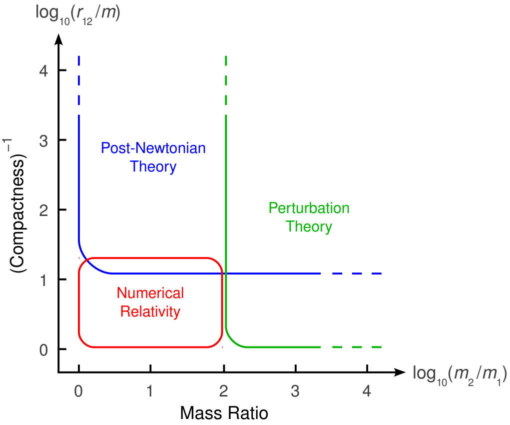


Based on collaborations with:
L. Barack, E. Barausse, L. Blanchet, A. Buonanno, A. H. Mroué,
H. P. Pfeiffer, N. Sago, A. Taracchini, and B. F. Whiting

Methods to compute GW templates for compact binaries



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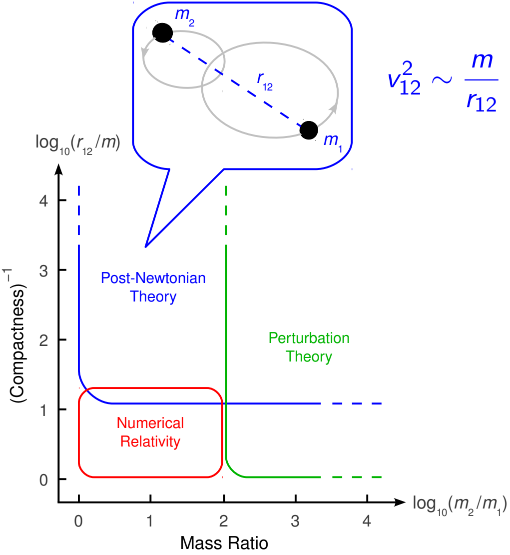
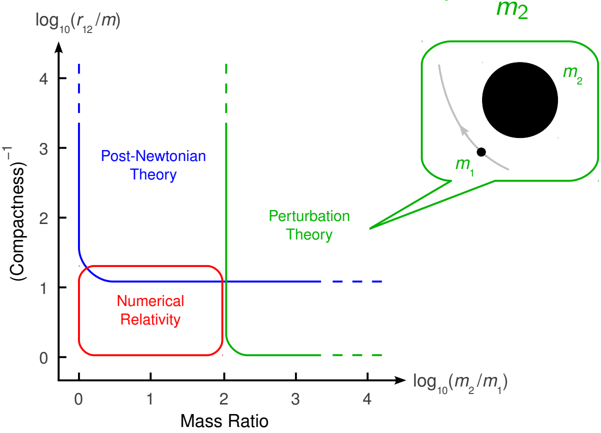


$$v_{12}^2 \sim \frac{m}{r_{12}} \ll 1$$

Methods to compute GW templates for compact binaries

$$q \equiv \frac{m_1}{m_2} \ll 1$$



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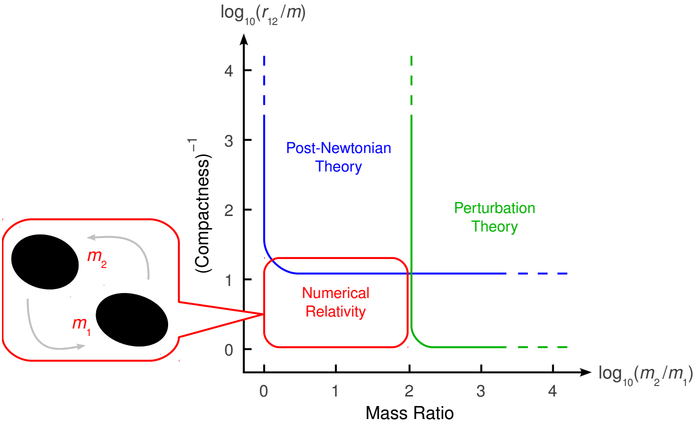

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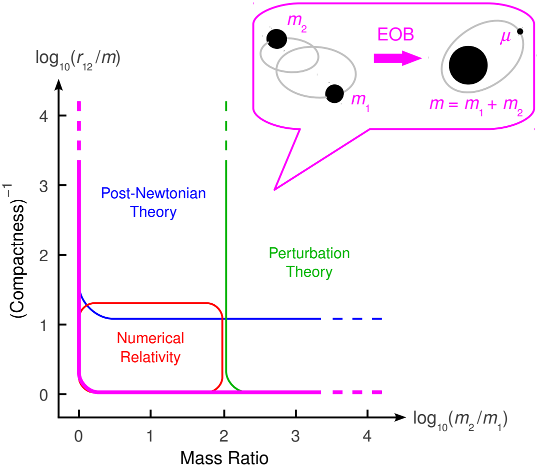
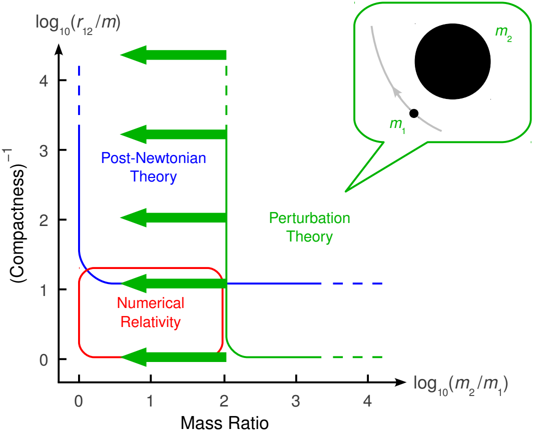


Methods to compute GW templates for compact binaries



Beware of confusing mass conventions

	SF	PN/NR
mass of the "particle"	μ	m_1
mass of the "black hole"	M	m_2
total mass	$\mu + M \simeq M$	$m = m_1 + m_2$
reduced mass	$\frac{\mu M}{\mu+M} \simeq \mu$	$\mu = \frac{m_1 m_2}{m}$
symmetric mass ratio	$\frac{\mu M}{(\mu+M)^2} \simeq \frac{\mu}{M}$	$\nu = \frac{m_1 m_2}{m^2}$
(asymmetric) mass ratio	$\frac{\mu}{M} \ll 1$	$q = \frac{m_1}{m_2}$

We shall use the PN/NR mass conventions

Outline

1. Gravitational waveforms
2. Periastron advance in black hole binaries
3. First law of binary black hole mechanics
4. Binding energy and angular momentum

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Head-on collision of two black holes

[Smarr (1979); Detweiler (1979)]

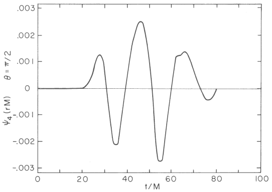


Figure 3. The curvature $\psi_4 \cdot rM$ in the equatorial plane crossing the 2-sphere at r = 25M as a function of time. This is for the two black hole collision Run II.

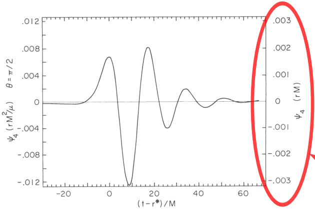
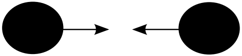


Figure 4. The same quantity as in Figure 3 except from the perturbation calculation of a particle of mass μ falling into a black hole of mass M. The abscissa is retarded time. The vertical scales are explained in the text. Only the quadrupole contribution is shown here.

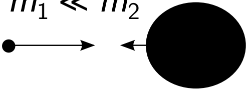
Numerical Relativity

$m_1 = m_2$



Perturbation Theory

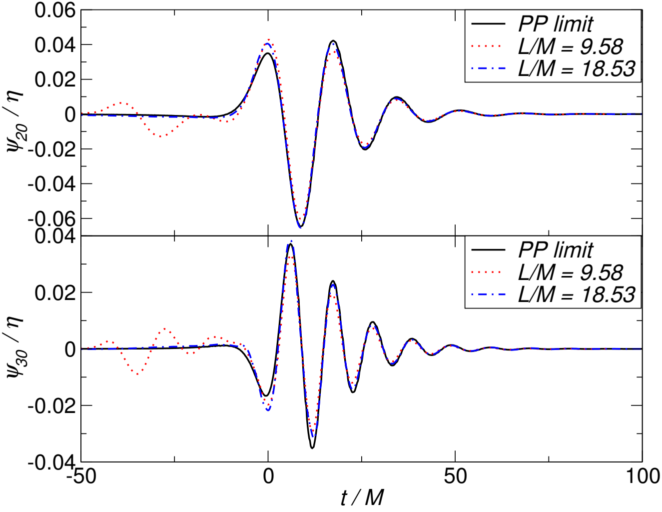
$m_1 \ll m_2$



Rescaling $m_1 \to \mu$, $m_2 \to m$

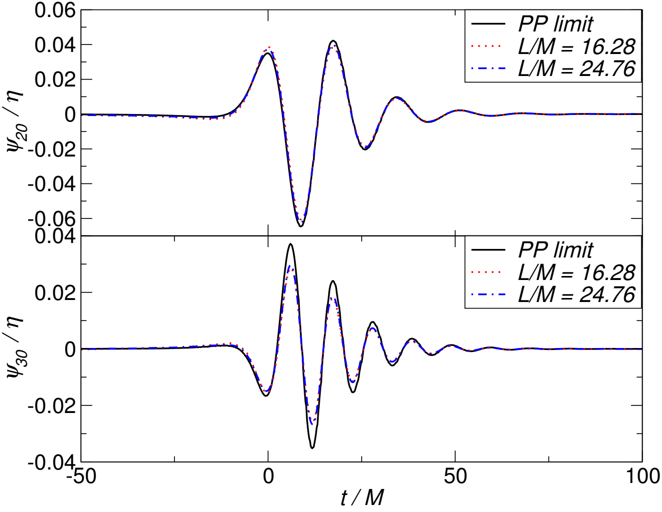
Head-on collision for a mass ratio 1:100

[Sperhake, Cardoso *et al.* (2011)]



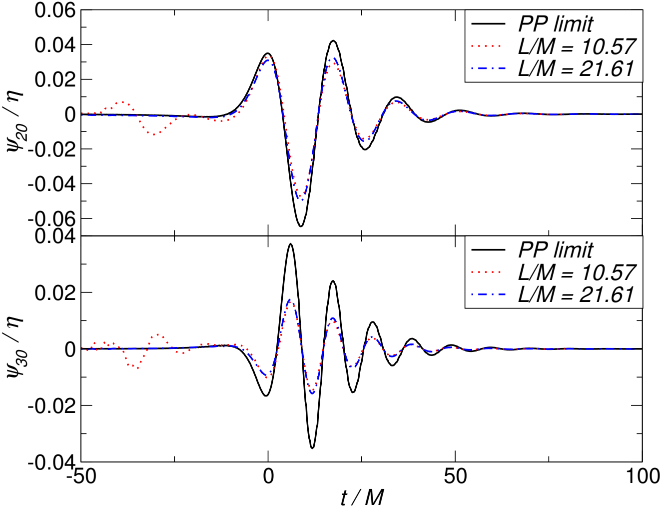
Head-on collision for a mass ratio 1:10

[Sperhake, Cardoso *et al.* (2011)]



Head-on collision for a mass ratio 1:4

[Sperhake, Cardoso *et al.* (2011)]



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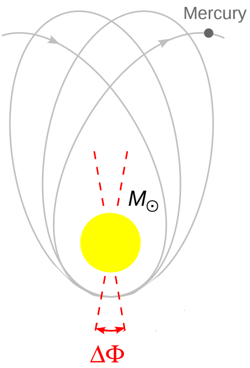
Relativistic perihelion advance of Mercury

- Observed anomalous precession of Mercury's perihelion of $\sim$ 43" /cent.
- Accounted for by the leading-order relativisic angular advance per orbit

$$\Delta\Phi_{\mathrm{GR}} = \frac{6\pi G M_\odot}{c^2 a\,(1-e^2)}$$

- One of the first successes of Einstein's general theory of relativity
- Relativisic periastron advance of $\sim$ °/yr now measured in binary pulsars



Periastron advance in black hole binaries

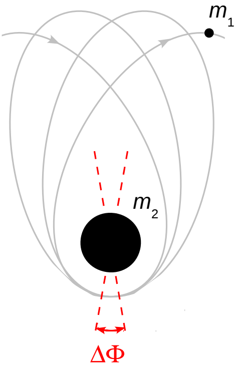
- Conservative part of the dynamics only
- Generic non-circular orbit parametrized by the two frequencies

$$\Omega_r = \frac{2\pi}{P}\,, \quad \Omega_\varphi = \frac{1}{P}\int_0^P \dot{\varphi}(t)\,\mathrm{d}t$$

- Periastron advance per radial period

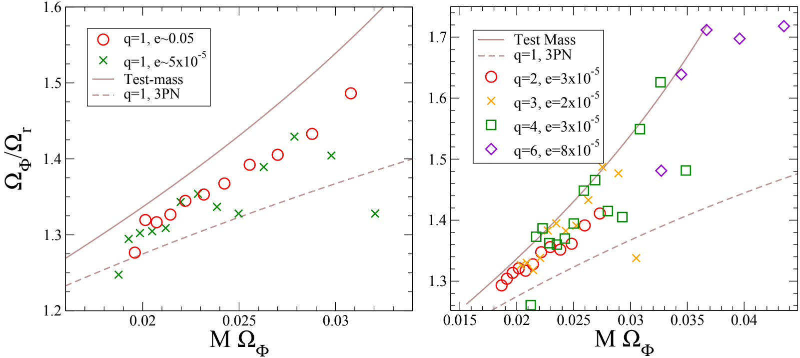
$$K \equiv \frac{\Omega_\varphi}{\Omega_r} = 1 + \frac{\Delta\Phi}{2\pi}$$

- In the circular orbit limit $e \to 0$, the relation $K(\Omega_\varphi)$ is coordinate invariant



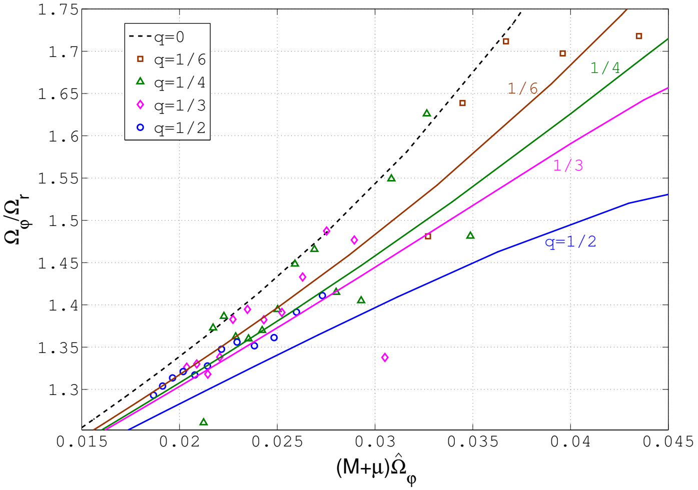
Early results in numerical relativity

[Mroué, Pfeiffer, Kidder & Teukolsky (2010)]



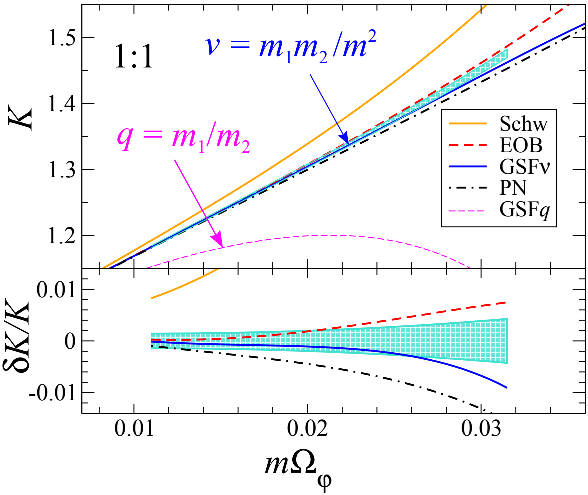
Tentative comparison with self-force results

[Barack & Sago (2011)]



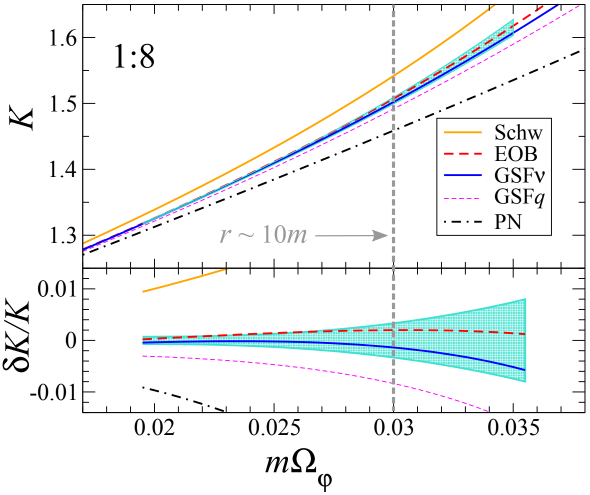
Extensive comparison for a mass ratio 1:1

[Le Tiec, Mroué *et al.* (2011)]



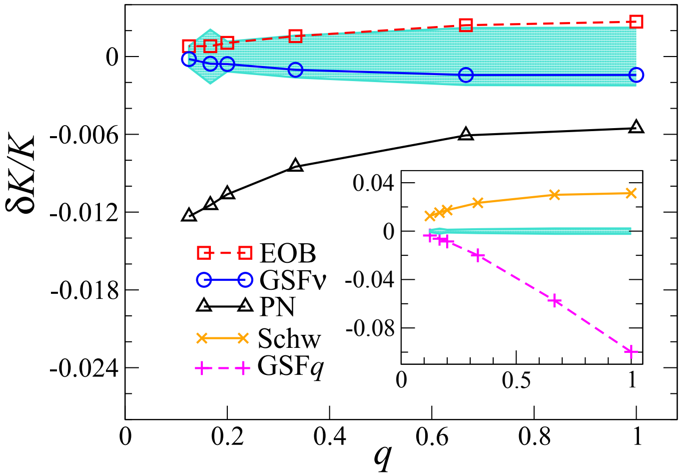
Extensive comparison for a mass ratio 1:8

[Le Tiec, Mroué *et al.* (2011)]



Variation with respect to the mass ratio

[Le Tiec, Mroué *et al.* (2011)]



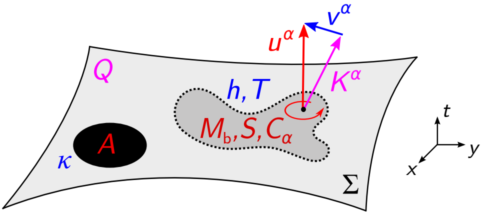
Outline

Generalized first law of mechanics

[Friedman, Uryū & Shibata (2002)]

- Spacetimes with black holes + perfect fluid matter sources
- One-parameter family of solutions $\{g_{\alpha\beta}(\lambda), u^\alpha(\lambda), \rho(\lambda), s(\lambda)\}$
- Globally defined Killing vector field $K^\alpha \to$ conserved charge Q

$$\delta Q = \sum_i \frac{\kappa_i}{8\pi}\,\delta A_i + \int_\Sigma \left[\,\bar{h}\,\Delta(\mathrm{d}M_\mathrm{b}) + \bar{T}\,\Delta(\mathrm{d}S) + v^\alpha \Delta(\mathrm{d}C_\alpha)\right]$$



Application to compact binaries on circular orbits

- For circular orbits, the geometry has a helical Killing vector

$$K^\alpha \to (\partial_t)^\alpha + \Omega\,(\partial_\varphi)^\alpha \quad \text{(when } r \to +\infty)$$

- For asymptotically flat spacetimes [Friedman *et al.* (2002)]

$$\delta Q = \delta M - \Omega\,\delta J$$

- In the exact theory, helically symmetric spacetimes are not asymptotically flat [Gibbons & Stewart (1983); Klein (2004)]
- Asymptotic flatness can be recovered if gravitational radiation can be "turned off", *e.g.*
  - Conformal Flatness Condition
  - Post-Newtonian theory

Application to compact binaries on circular orbits

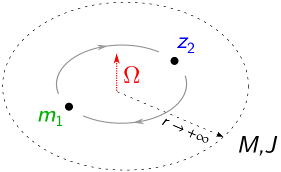
[Le Tiec, Blanchet & Whiting (2012)]

- Conservative dynamics only → no gravitational radiation
- Non-spinning compact objects modeled as point masses m_A:

$$T^{\alpha\beta} = \sum_{A=1}^{2} m_A \, z_A \, u_A^\alpha u_A^\beta \, \frac{\delta(\mathbf{x} - \mathbf{y}_A)}{\sqrt{-g}}$$

- For two point masses on a circular orbit, the first law becomes

$$\boxed{\delta M - \Omega \, \delta J = z_1 \, \delta m_1 + z_2 \, \delta m_2}$$



First integral associated with the variational law

[Le Tiec, Blanchet & Whiting (2012)]

- Variational first law: $\delta M - \Omega\,\delta J = z_1\,\delta m_1 + z_2\,\delta m_2$
- Since $\{M, J, z_A\}$ are all functions of $\{\Omega, m_A\}$, we have

$$\frac{\partial M}{\partial \Omega} = \Omega \frac{\partial J}{\partial \Omega} \quad \text{and} \quad z_A = \frac{\partial (M - \Omega J)}{\partial m_A}$$

- After a few algebraic manipulations, we obtain

$$\boxed{M - 2\Omega J = m_1 z_1 + m_2 z_2}$$

- Alternative derivations based on:
  - Euler's theorem applied to the function $M(J^{1/2}, m_1, m_2)$
  - The combination $M_\mathrm{K} - 2\Omega J_\mathrm{K}$ of the Komar quantities

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Binding energy beyond the test-mass approximation

[Le Tiec, Barausse & Buonanno (2012)]

- The binding energy $E \equiv M - m$ is a function of $x \equiv (m\Omega)^{2/3}$
- In the "small" mass ratio limit $\nu \to 0$:

$$z_1 = \sqrt{1-3x} + \nu\, z_{\text{GSF}}(x) + \mathcal{O}(\nu^2)$$
$$\frac{E}{\mu} = \left(\frac{1-2x}{\sqrt{1-3x}} - 1\right) + \nu\, E_{\text{GSF}}(x) + \mathcal{O}(\nu^2)$$

- The self-force contribution $z_{\text{GSF}}(x)$ is known numerically
  [Detweiler (2008); Sago, Barack & Detweiler (2008); Shah *et al.* (2011)]
- The first law provides a relationship $E \leftrightarrow z_1$, which implies

$$E_{\text{GSF}}(x) = \frac{1}{2}\, z_{\text{GSF}}(x) - \frac{x}{3}\, z'_{\text{GSF}}(x) + f(x)$$

GSF correction to the Schwarzschild ISCO frequency

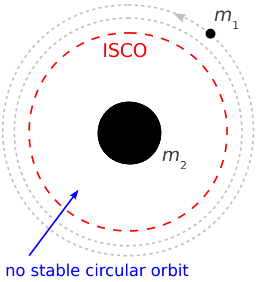
- The orbital frequency of the Schwarzschild ISCO is shifted under the effect of the conservative self-force:

$$m\Omega_{\mathrm{ISCO}} = \underbrace{6^{-3/2}}_{\text{Schwarz. result}} \left\{ 1 + \underbrace{\nu \, C_\Omega}_{\text{conservative GSF effect}} + \mathcal{O}(\nu^2) \right\}$$

- A stability analysis of slightly eccentric orbits near the ISCO yields [Barack & Sago (2009)]

$$C_\Omega^{\mathrm{BS}} = 1.2512(4)$$

- Strong-field benchmark used for comparison with PN/NR/EOB



GSF correction to the Schwarzschild ISCO frequency

- The angular frequency of the minimum energy circular orbit (MECO) is solution of

$$\frac{\partial E}{\partial \Omega}\bigg|_{\Omega_{\text{MECO}}} = 0$$

- Hamiltonian system: ISCO $\Leftrightarrow$ MECO [Buonanno *et al.* (2003)]
- Our result for the energy $E_{\text{GSF}}(x)$ yields [Le Tiec *et al.* (2012)]

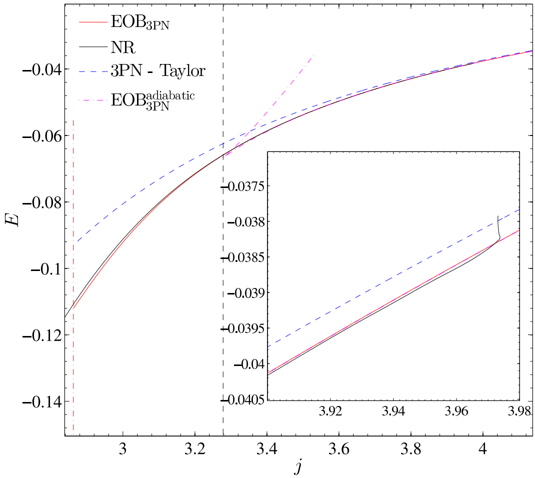
$$C_\Omega = \frac{1}{2} + \frac{1}{4\sqrt{2}}\left\{\frac{1}{3}\, z''_{\text{GSF}}(1/6) - z'_{\text{GSF}}(1/6)\right\}$$

- Using accurate numerical self-force data for $z_{\text{GSF}}(x)$, we find

$$C_\Omega = 1.2510(2) \qquad \left[C_\Omega^{\text{BS}} = 1.2512(4)\right]$$

NR/EOB comparison for an equal mass binary

[Damour, Nagar, Pollney & Reisswig (2012)]



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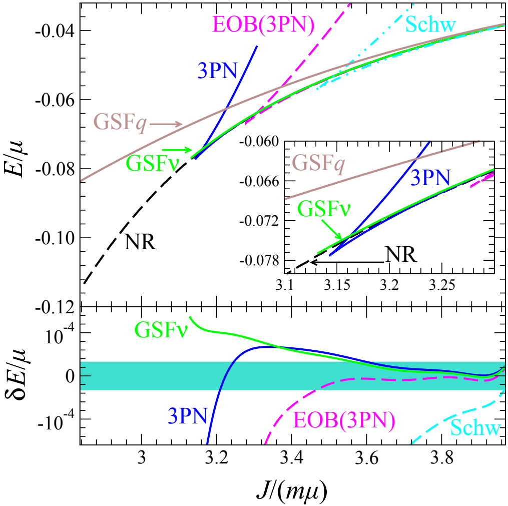


Why do the GSFν results perform so well?

- In perturbation theory, one traditionally expands as

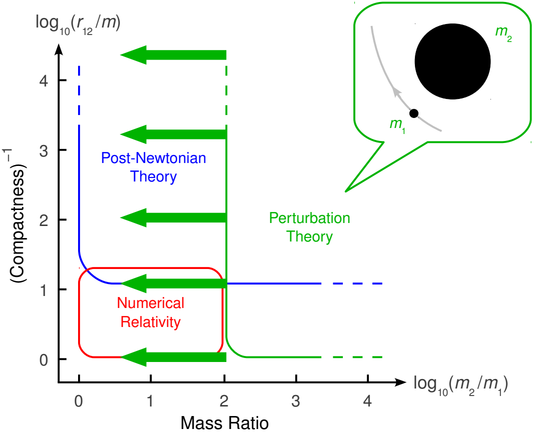
$$\text{GSF}q: \quad \sum_{n=0}^{n_{\max}} A_n(m_2\Omega)\, q^n \quad \text{where} \quad q \equiv m_1/m_2 \in [0,1]$$

- However, the relations $K(\Omega; m_A)$, $E(\Omega; m_A)$, and $J(\Omega; m_A)$ must be symmetric under exchange $m_1 \longleftrightarrow m_2$
- Hence, a better-motivated expansion is

$$\text{GSF}\nu: \quad \sum_{n=0}^{n_{\max}} B_n(m\Omega)\, \nu^n \quad \text{where} \quad \nu \equiv m_1 m_2/m^2 \in [0,1/4]$$

- In a PN expansion, we have $B_n = \mathcal{O}\big(1/c^{2n}\big) = n\text{PN} + \cdots$

Perturbation theory for comparable mass binaries



How about spins?

- Calculation of $z_{\text{GSF}}(\Omega; S)$ for a particle on a circular equatorial orbit in a Kerr background [Shah, Friedman & Keidl (in progress)]
- Generalization of the first law to spinning point particles [Blanchet, Buonanno & Le Tiec (in progress)]

$$\delta M - \Omega\,\delta L = \sum_{A=1}^{2} \left(z_A\,\delta m_A + \Omega_A\,\delta S_A \right)$$

- Exact spin effects at linear order in ν in binding energy E and total angular momentum J
- Shift of the Kerr ISCO under the effect of the conservative SF
- Spin-orbit and spin-spin contributions to EOB potentials

How about orbital evolution?

- Consider a binary on a quasicircular orbit with frequency $\Omega(t)$
- Binding energy $E[\Omega(t)]$ known to $\mathcal{O}(\nu)$ [Le Tiec *et al.* (2012)]
- Compute the second order metric perturbation at $\mathscr{I}^+$:

$$\mathcal{O}(\nu) \text{ corrections in } h_+[\Omega(t)], h_\times[\Omega(t)], \mathcal{F}[\Omega(t)]$$

- Apply energy balance in the adiabatic approximation:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \mathcal{F} \quad \Longrightarrow \quad \Omega(t) \text{ accurate to } \mathcal{O}(\nu)$$

- The resulting templates should model the adiabatic inspiral and GW emission from EMRIs and IMRIs accurately

Summary and prospects

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  - Non-quasicircular orbits?