Lagrangian Data Assimilation and Its Application to Geophysical Fluid Flows

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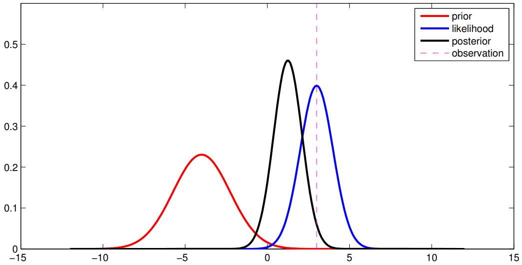
June 11, 2013

Data Assimilation

Setup:

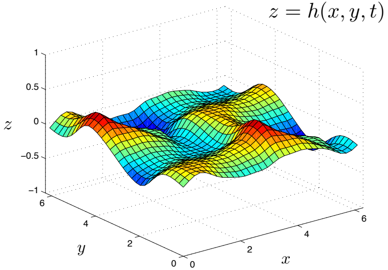
- Given dynamical system $\dot{\mathbf{x}} = f(\mathbf{x})$ (deterministic or stochastic)
- Uncertainty in initial conditions $x(0)$
- Would like to estimate state at specific time: $x(t_k)$
- Observations $y = h(x) + \textit{noise}$

Application of Bayes' Rule: $p(x|y) \propto p(x)p(y|x)$

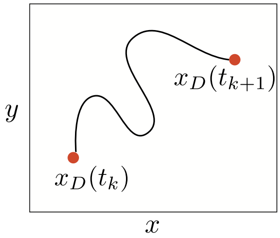


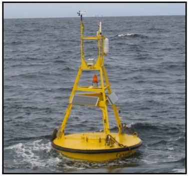
Lagrangian Data Assimilation

Suppose we want to estimate the Eulerian flow field $\mathbf{x}_F$, but the observations are of Lagrangian positions of passive drifters $\mathbf{x}_D$.



$$\dot{\mathbf{x}}_{D,i} = \mathbf{x}_F(\mathbf{x}_{D,i}, t)$$





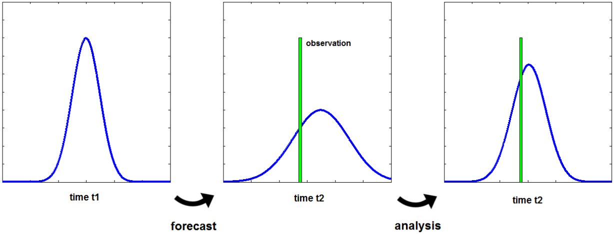
Lagrangian Data Assimilation

One approach to Lagrangian data assimilation:

- Append drifter position $\mathbf{x}_D$ to flow state vector $\mathbf{x}_F$: $\mathbf{x} = \begin{pmatrix} \mathbf{x}_F \\ \mathbf{x}_D \end{pmatrix}$
- Observation operator has simple, linear form: $\mathbf{H} = [\mathbf{0} \quad \mathbf{I}]$

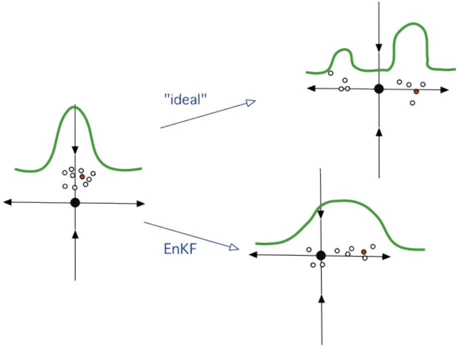
Sequential filters:

- Forecast (evolve previous estimate forward under dynamical system)
- Analysis (update current estimate with observation)



Traditional filters: Ensemble Kalman Filter (EnKF)

- Represent probability distribution with an ensemble of state vectors
- Evolve each ensemble member forward under model until next observation time
- When an observation is available, update each ensemble member according to the traditional Kalman analysis step
- Drawback: tends to impose Gaussianity at each assimilation step



Traditional filters: Particle Filter (PF)

- Represent probability distribution with *weighted* ensemble of state vectors, called particles
- When observation is available, update each particle's weight according to Bayes' Rule
- Need to resample to avoid weight converging on one particle
- Drawback: necessary number of particles increases exponentially with state dimension (curse of dimensionality)

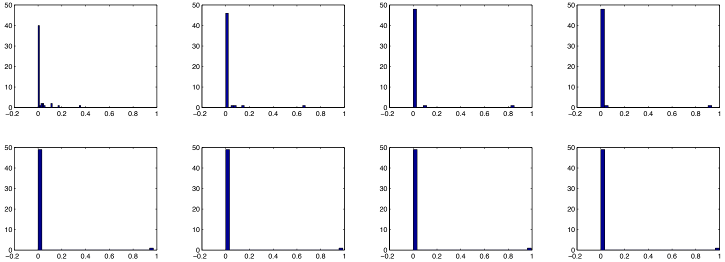
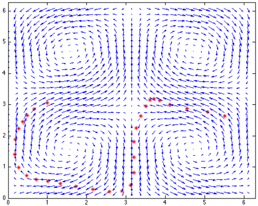


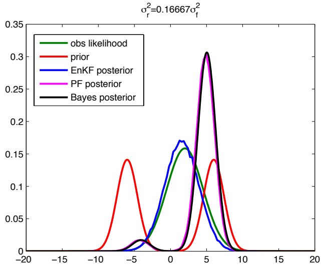
Figure: weight histograms: dim=1, N=50

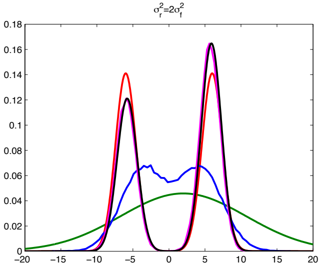
EnKF vs PF: Non-Gaussian Prior

- Lagrangian data assimilation leads to non-Gaussian priors
- Flow may solve linear system, but drifters solve nonlinear system:

$$\dot{\mathbf{x}}_F = f(\mathbf{x}_F) \text{ [linear or nonlinear]}$$
$$\dot{\mathbf{x}}_D = g(\mathbf{x}_F, \mathbf{x}_D) \text{ [always nonlinear]}$$







$N = 10^5$

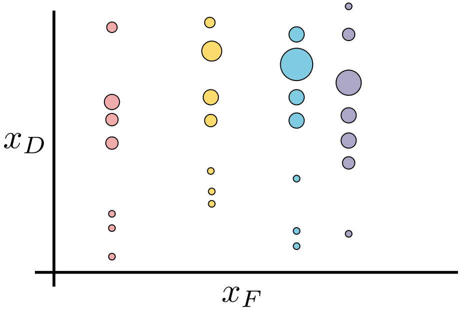
Hybrid PF-EnKF

- EnKF on high-dimensional Eulerian state x^F
- PF on low-dimensional, highly nonlinear Lagrangian part x^D

Ensemble:

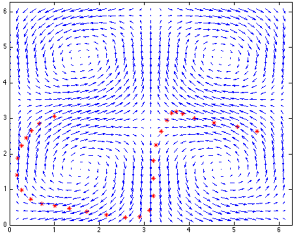
$$\{x_i^F, x_{i,j}^D, w_{i,j}\}_{i=1\ldots N_e, j=1\ldots M}$$

Update weights via standard particle filter update, and at resampling times, update x^F according to EnKF analysis.



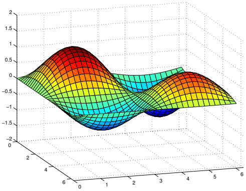
System:

$$\dot{u} = v - h_x$$
$$\dot{v} = -u - h_y$$
$$\dot{h} = -u_x - v_y$$



Solution with two modes:

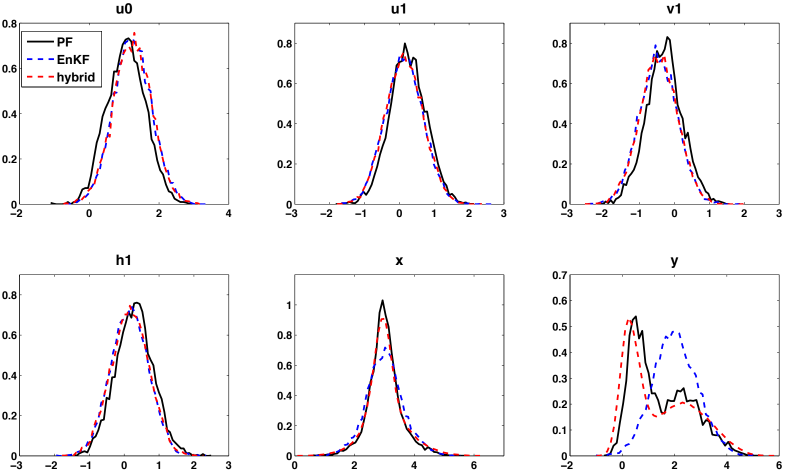
$$u(x,y,t) = -\sin(x)\cos(y)u_0 + \cos(y)u_1(t)$$
$$v(x,y,t) = \cos(x)\sin(y)u_0 + \cos(y)v_1(t)$$
$$h(x,y,t) = \sin(x)\sin(y)u_0 + \sin(y)h_1(t)$$



Results

$$u(x, y, t) = -\sin(x)\cos(y)u_0 + \cos(y)u_1(t)$$
$$v(x, y, t) = \cos(x)\sin(y)u_0 + \cos(y)v_1(t)$$
$$h(x, y, t) = \sin(x)\sin(y)u_0 + \sin(y)h_1(t)$$

PF: $N = 10^4$
EnKF: $N = 10^4$
Hybrid: $N_e = 10^4,\ M = 50$



Results

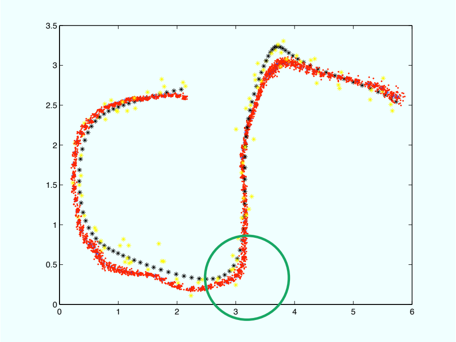


Figure: Ensemble Kalman filter

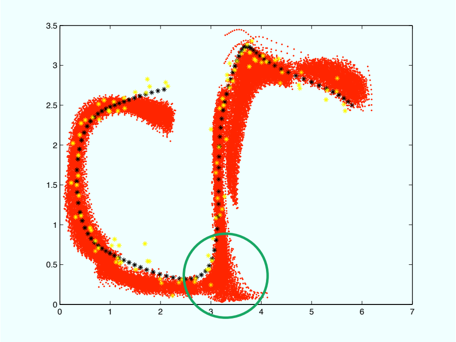


Figure: Hybrid filter

Summary and Future Work

- Hybrid filter combines advantages of PF and EnKF while avoiding disadvantages of each in Lagrangian DA case
- More computationally intensive than EnKF, but more accurate when drifters encounter saddle point
- Future: high dimensional nonlinear shallow water equations, drifter deployment experiments (Salman, Ide, Jones)

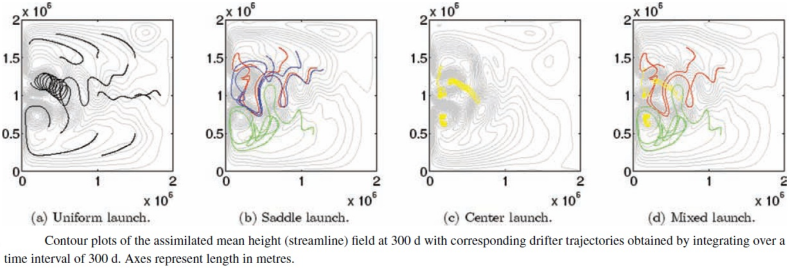


Figure: Salman et al., 2008

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