Review of Some Fast Algorithms for Electromagnetic Scattering

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CSCAMM Lecture

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Acknowledgements

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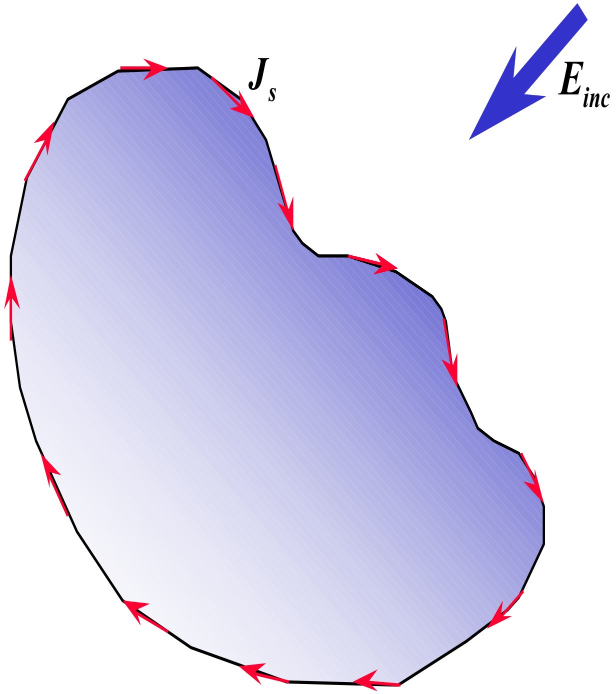
Outline

- Fast algorithm (MultiLevel Fast Multipole Algorithm)
- Parallelization of MLFMA (ScaleME)
- Applications of MLFMA
- Extension to layered media
- Low-frequency breakdown problem for small objects
- Conclusions

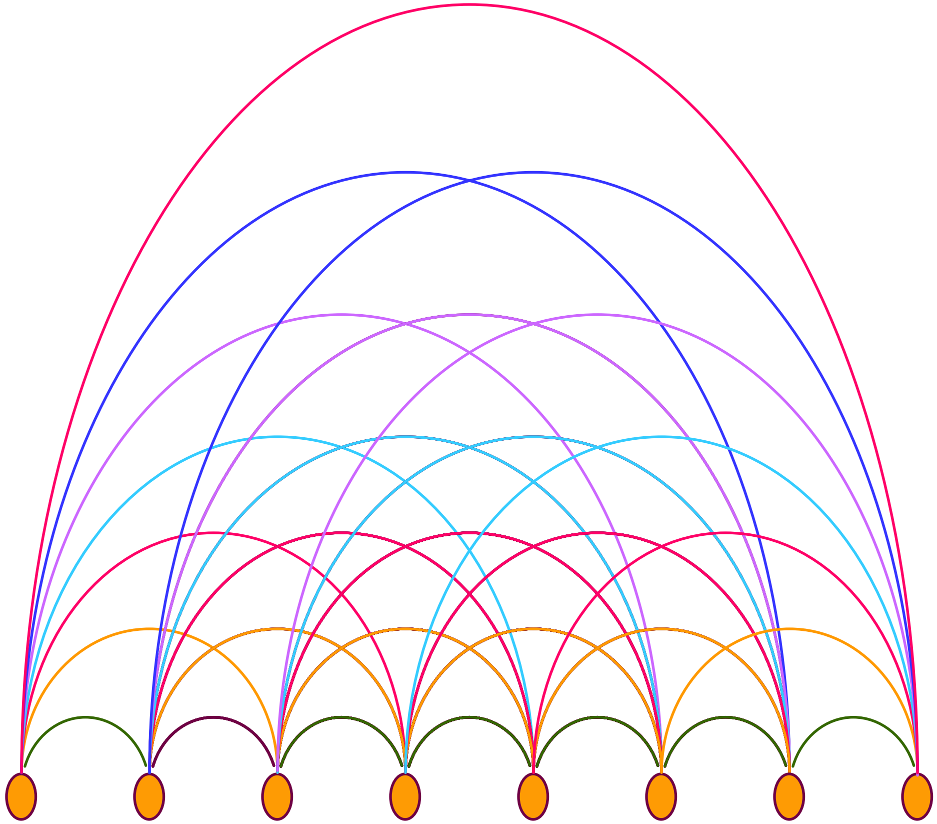
Integral Equation for Scattering Problems

- Currents are induced on a scatterer illuminated by a source.
- The induced currents adjust themselves to cancel the incident field.
- Hence, every current element needs to talk to each other.



A One-Level Link

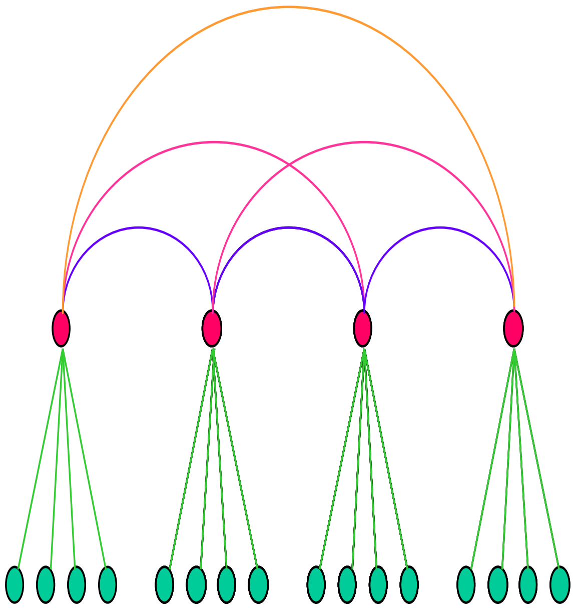
- A one-level matrix-vector multiply where all current elements talk directly to each other.
- The number of "links" is
- proportional to N^2 where N is the number of current elements.



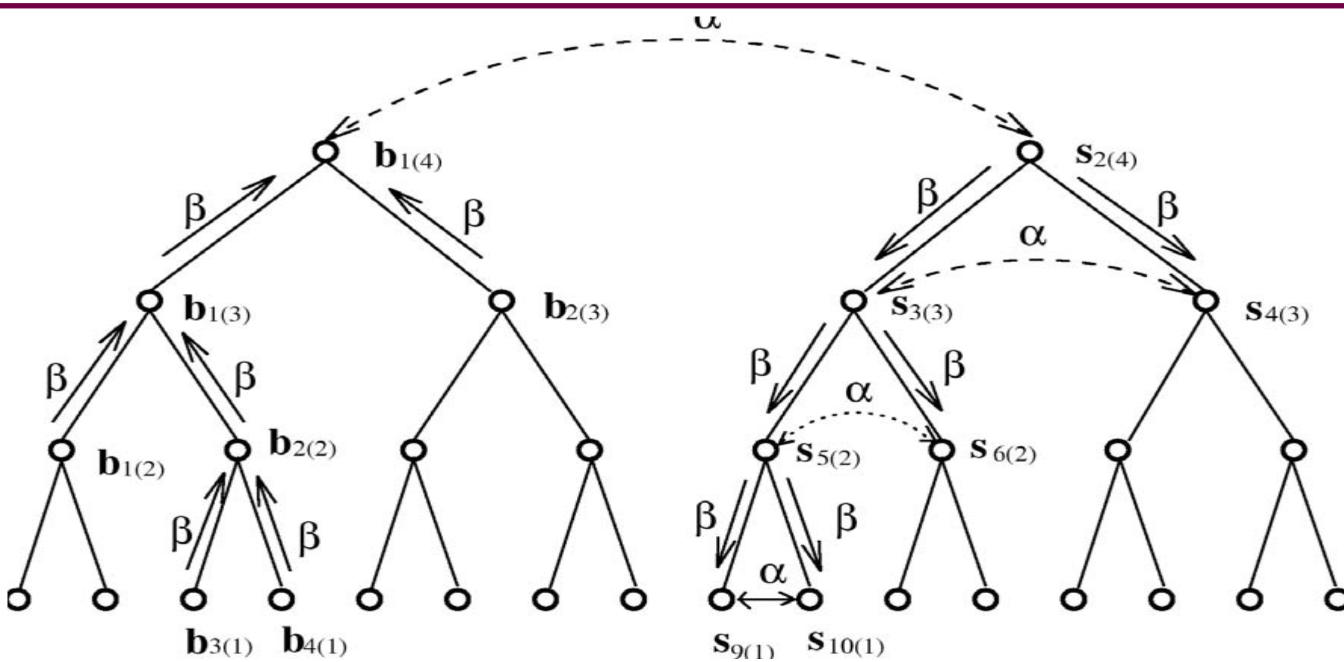
A Two-Level Link

- A two-level matrix-vector multiply where "hubs" are established to reduce the number of direct "links" between the current elements.
- This could reduce the complexity of a matrix-vector multiply.
- Mathematically, this is achievable by the factorization of an element of the matrix *A*:

$$A_{ij} = \overline{V}^{t}_{il} \bullet \overline{\overline{T}}_{ll'} \bullet \overline{V}_{l'j}$$



A Multilevel Tree Structure—Multilevel Fast Multipole Algorithm

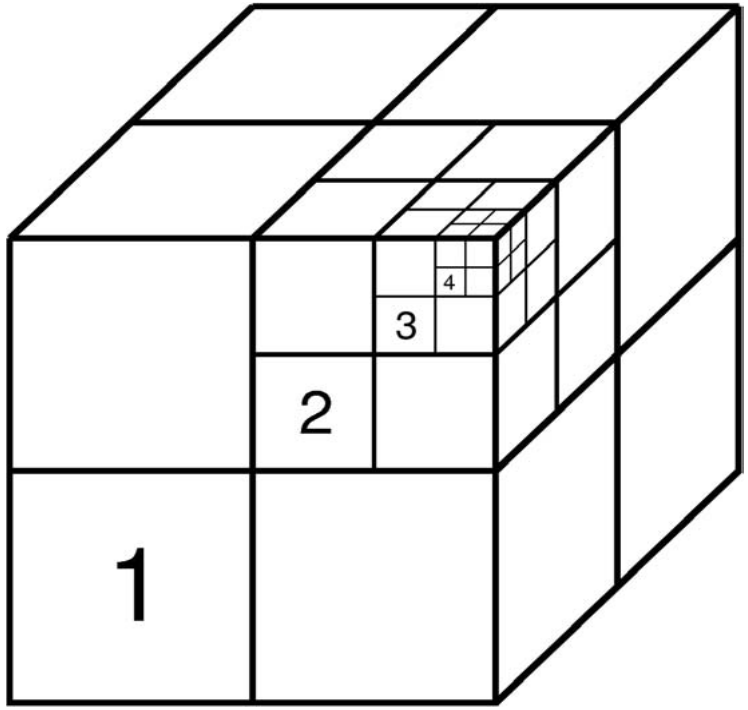


- **A tree structure showing the aggregation and the disaggregation procedure to form a multilevel algorithm.**
- **In this case, the matrix *A* needs to be factorized as a product of many matrices:**

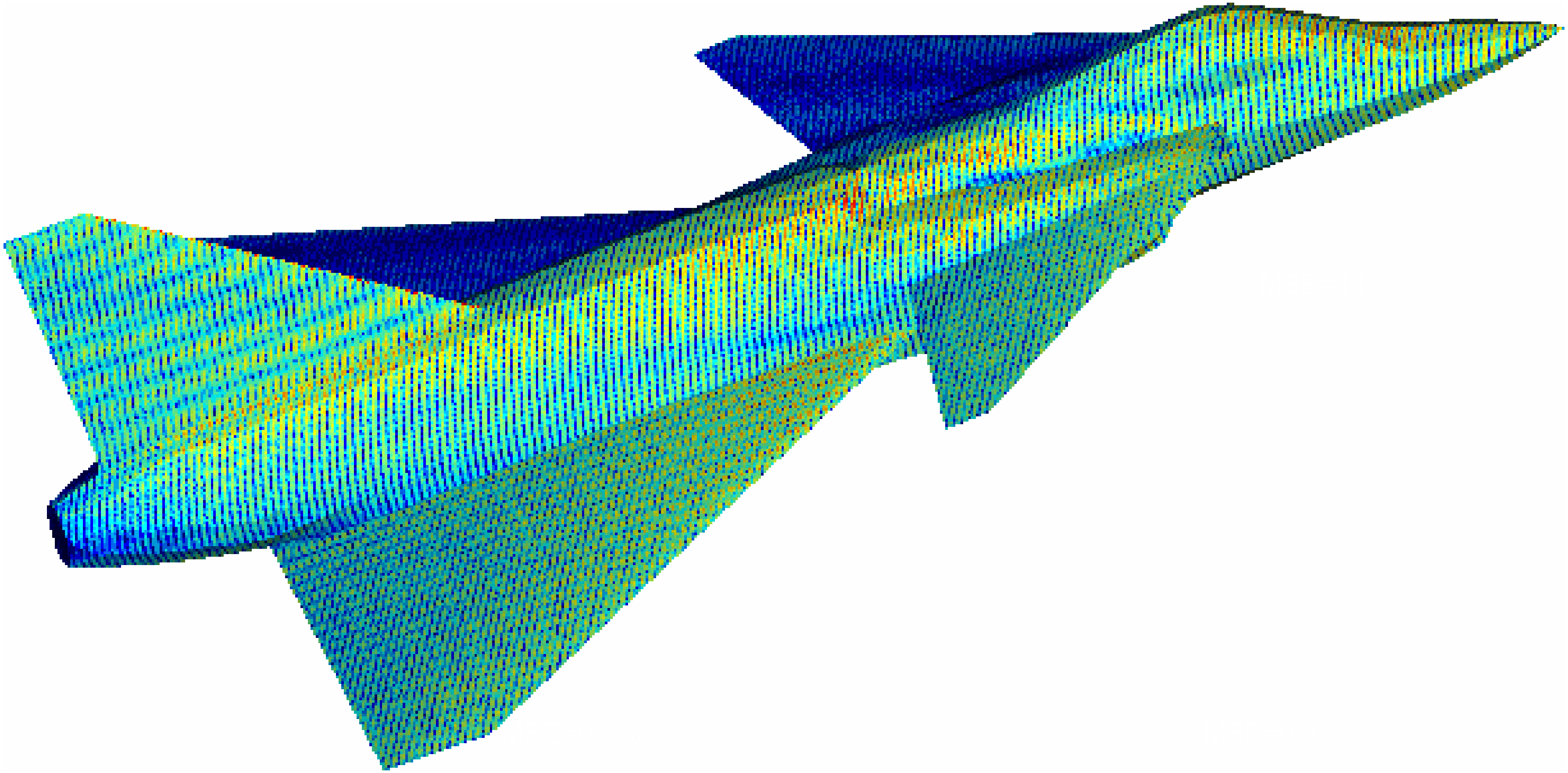
$$A_{ij} = \overline{V}^t_{il_1} \bullet \overline{\overline{\beta}}_{l_1 l_2} \bullet \overline{\overline{\beta}}_{l_2 l_3} \Lambda \overline{\overline{\beta}}_{l_{L-1} l_L} \bullet \overline{\overline{T}}_{ll'} \bullet \overline{\overline{\beta}}_{l_L l_{L-1}} \Lambda \overline{\overline{\beta}}_{l_3 l_2} \bullet \overline{\overline{\beta}}_{l_2 l_1} \bullet \overline{V}_{l'j}$$

An Oct-Tree Structure in 3D Space MLFMA

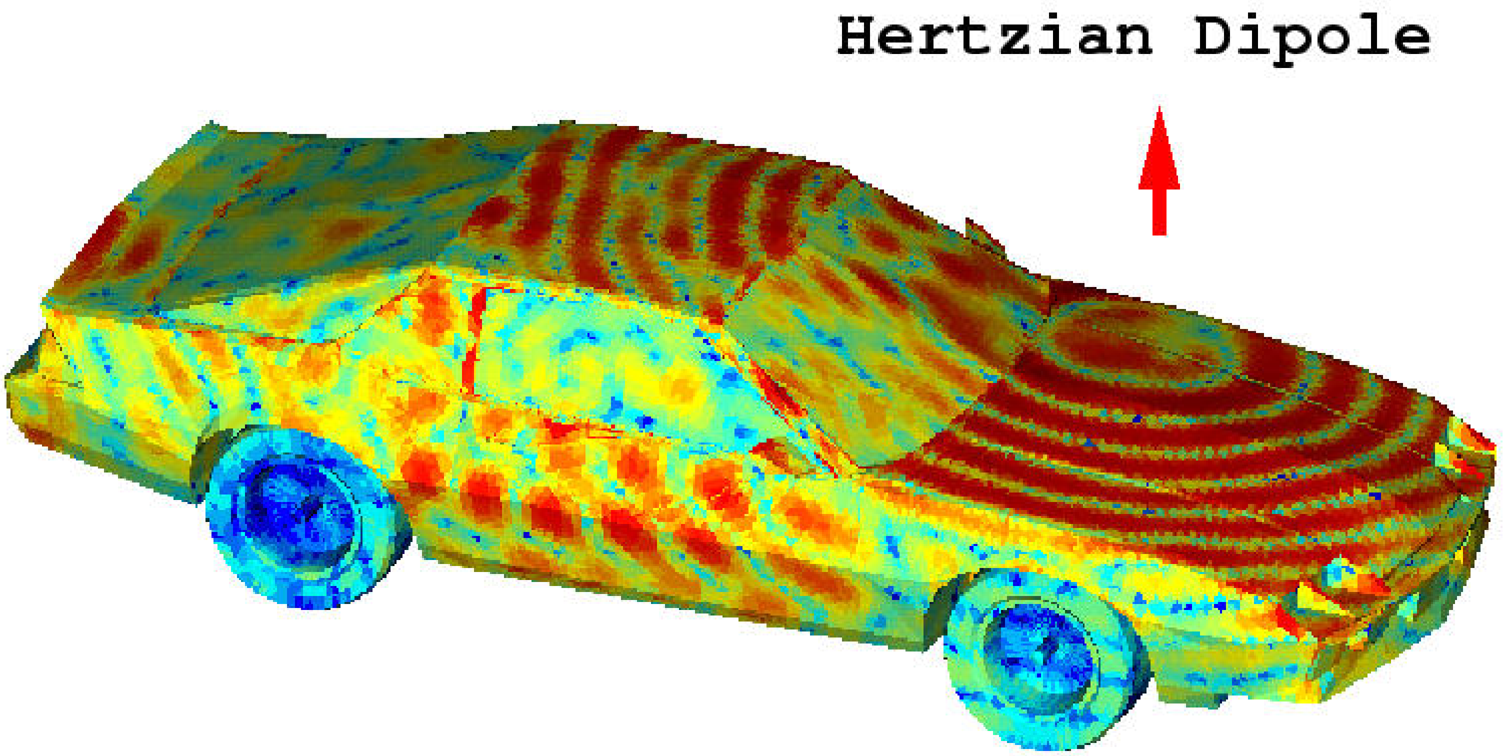
- **By using the OCT-TREE in three dimensions, a matrix-vector multiply for three-D objects is achieved in O(N log N) operations.**
- **A scattering problem can be solved in N_{iter} N log N.**



VFY218 at 2 GHz, V-pol

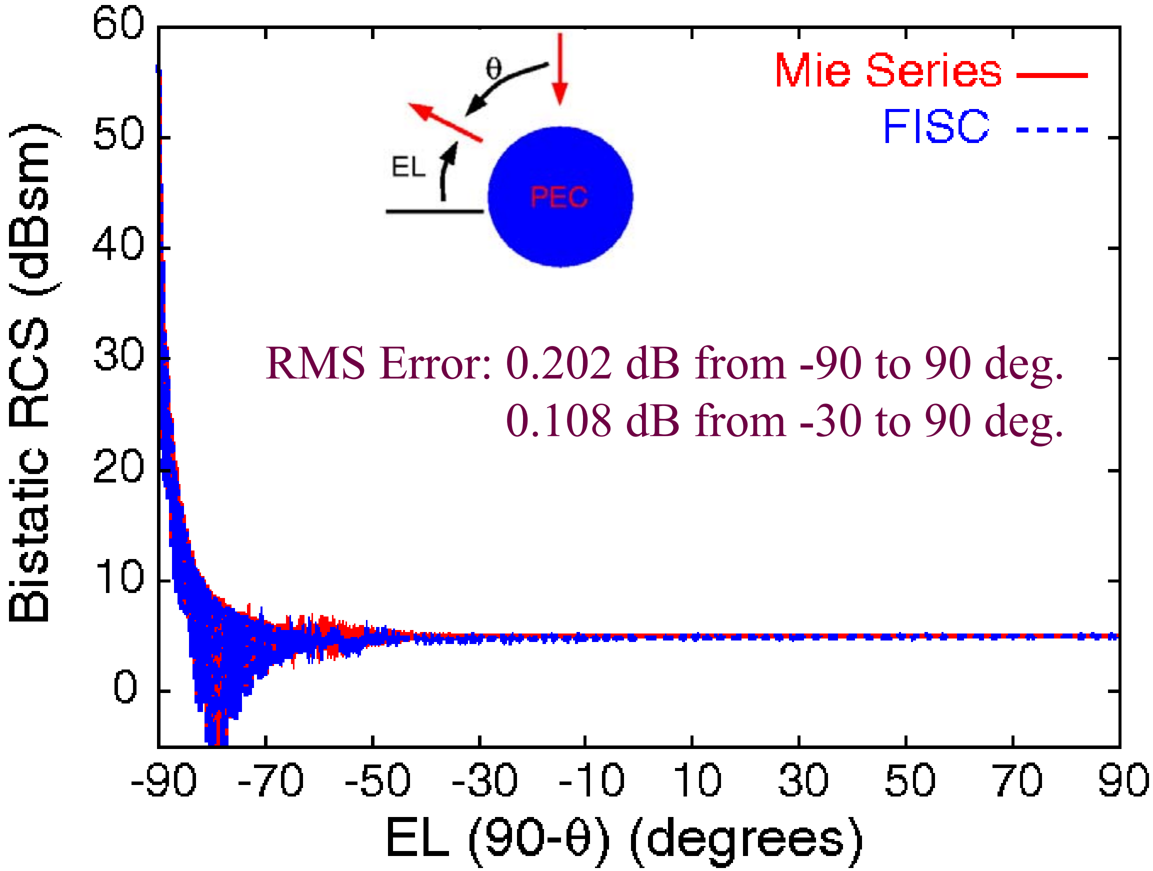


83 Camaro at 1 GHz by FISC



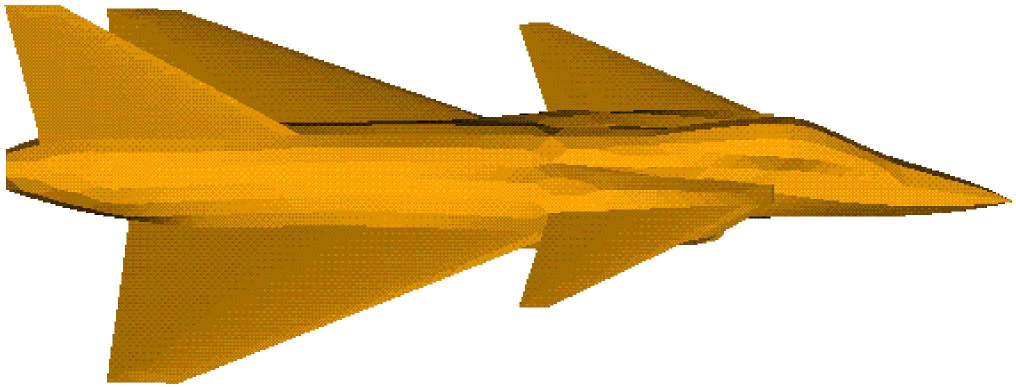
- Irradiation of a 83 Camaro at 1 GHz by a Hertzian dipole.

D=120λ, N=9,633,792



32 nodes of Origin2000, 26.7 GB of memory, 1.5 hrs. for filling matrix, 13.0 hrs. for 43 iterations in GMRES-15 to reach 0.001 residual error, 3 minutes for 1800 points of RCS. (the accuracy setting is not as high as previous example due to memory limit)

VFY218 at 8 GHz



	Nodes	Facets	Unknowns
Original	2,844	5,684	8,526
8 GHz	3,330,308	6,660,612	9,990,918

	Length	Width	Height
Inch	609”	350”	161”
8 GHz	412λ	237λ	109λ

The longest edge is 0.3λ, the average is 0.2λ, and the surface area is 115,789 λ^2

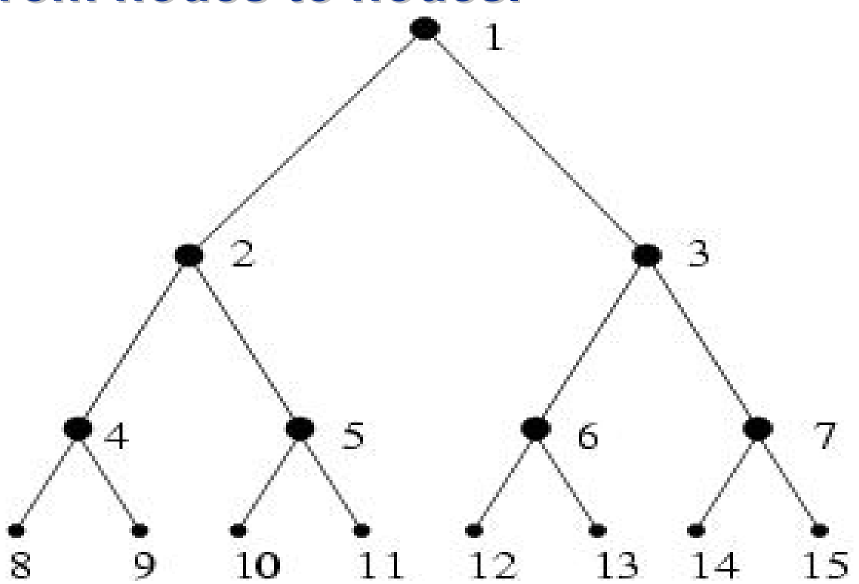
10-level MLFMA is used

ScaleME: Landmarks to Date

- Solved a demo problem over a heterogeneous network of 4 DEC Alphas and 2 Sun Ultra workstations. (April 1999)
- 602,112 unknowns on a 16-node PC cluster running Linux on AMD K6-2 processors. (Total cost of the cluster was ***$15,000.***) (May, 1999)
- 4 million unknowns on an SGI Origin 2000 (July, 2000)
- 10 million unknowns on SGI Origin 2000 (May 2001)
- 20 million unknowns on 10 SUN Blade Cluster (April 2003).

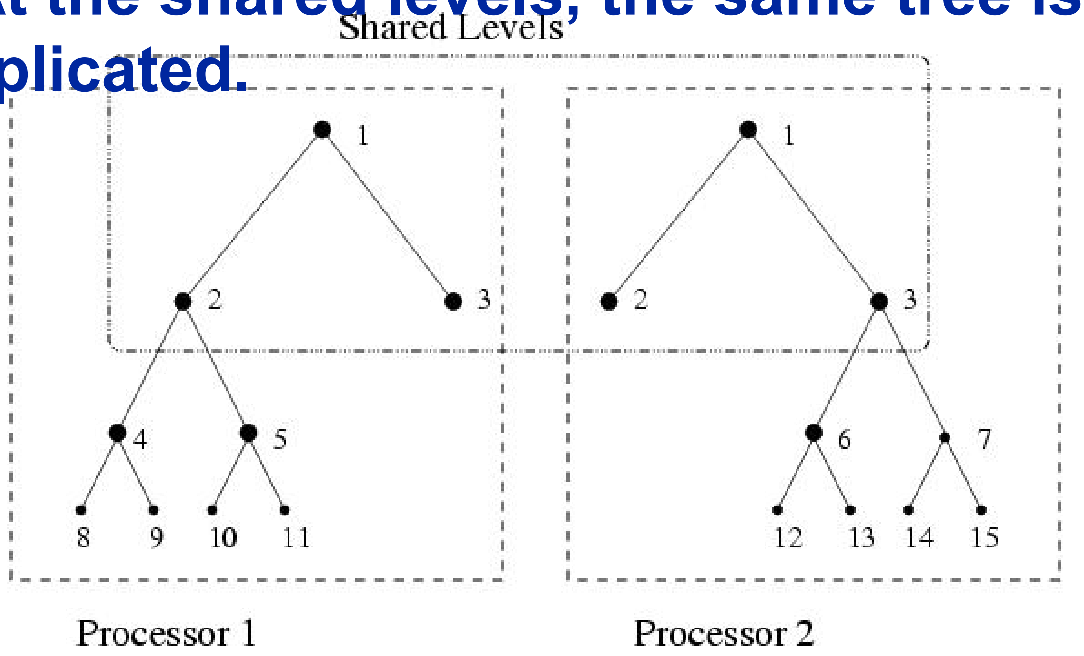
Essential Ideas

- A simple way to parallelize MLFMA, which is a tree code, is to split the workload according to te workload at each node.
- However, this gives rise to exorbitant communication cost.
- Hence, a two prong approach is used—the bottom part of the tree is split according to workload at each node, but the top is split according to message length being passed from nodes to nodes.



Essential Ideas - Illustrated

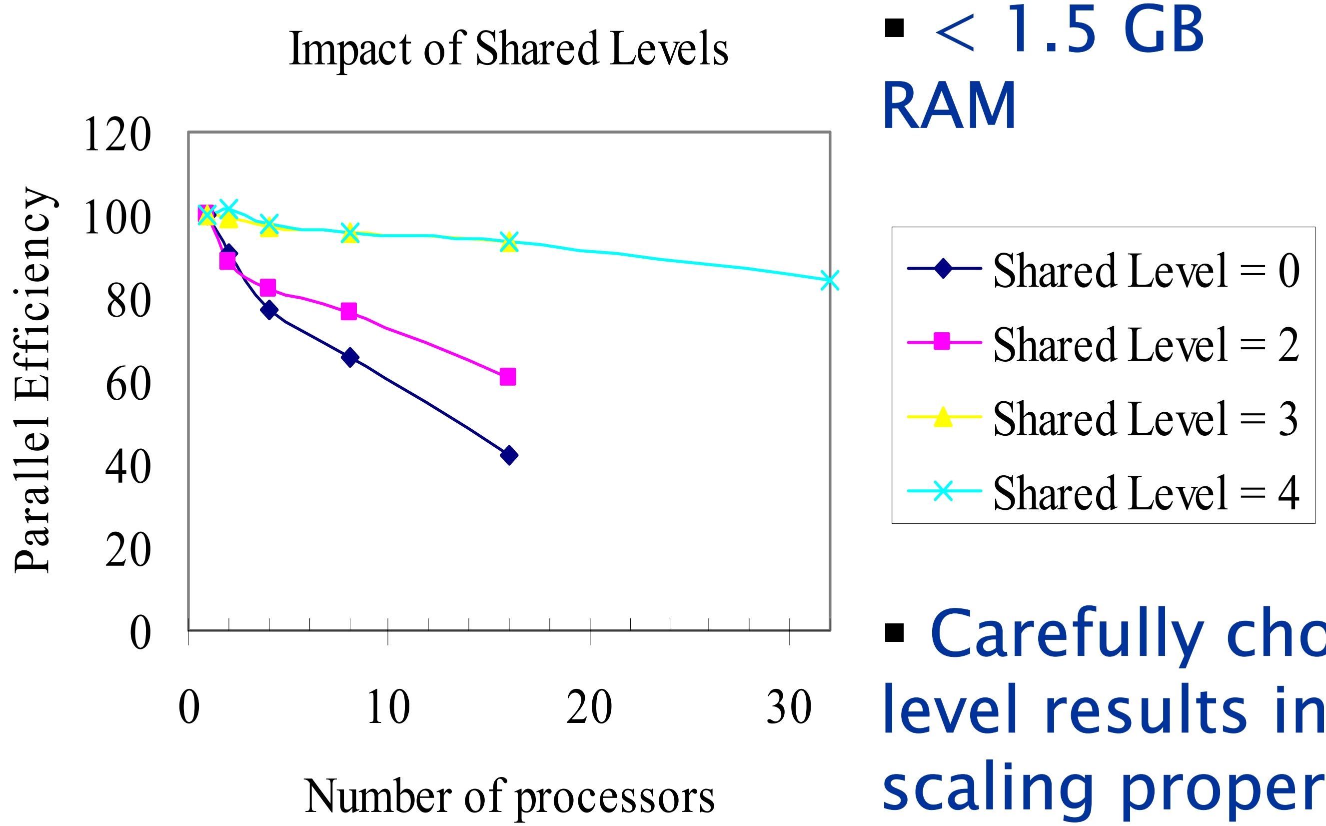
- We called the top levels of the tree shared levels.
- At the shared levels, the same tree is replicated.



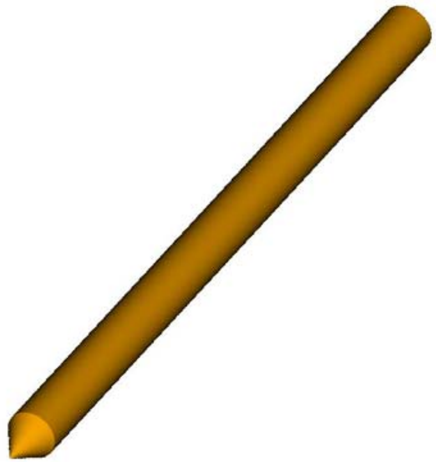
- Each processor gets half the radiation/receiving patterns of the boxes numbered 1, 2 and 3.

Matrix-Vector products: Pencil at 4 GHz

Number of levels = 8



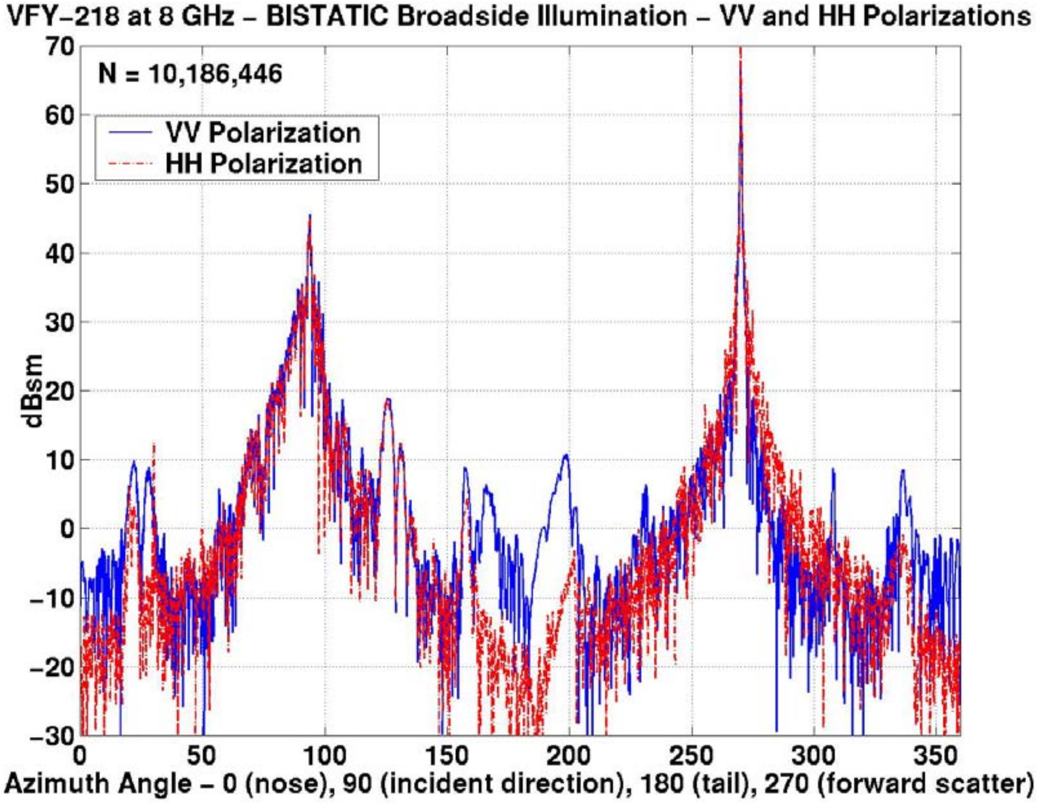
- N = 291,774
- < 1.5 GB RAM



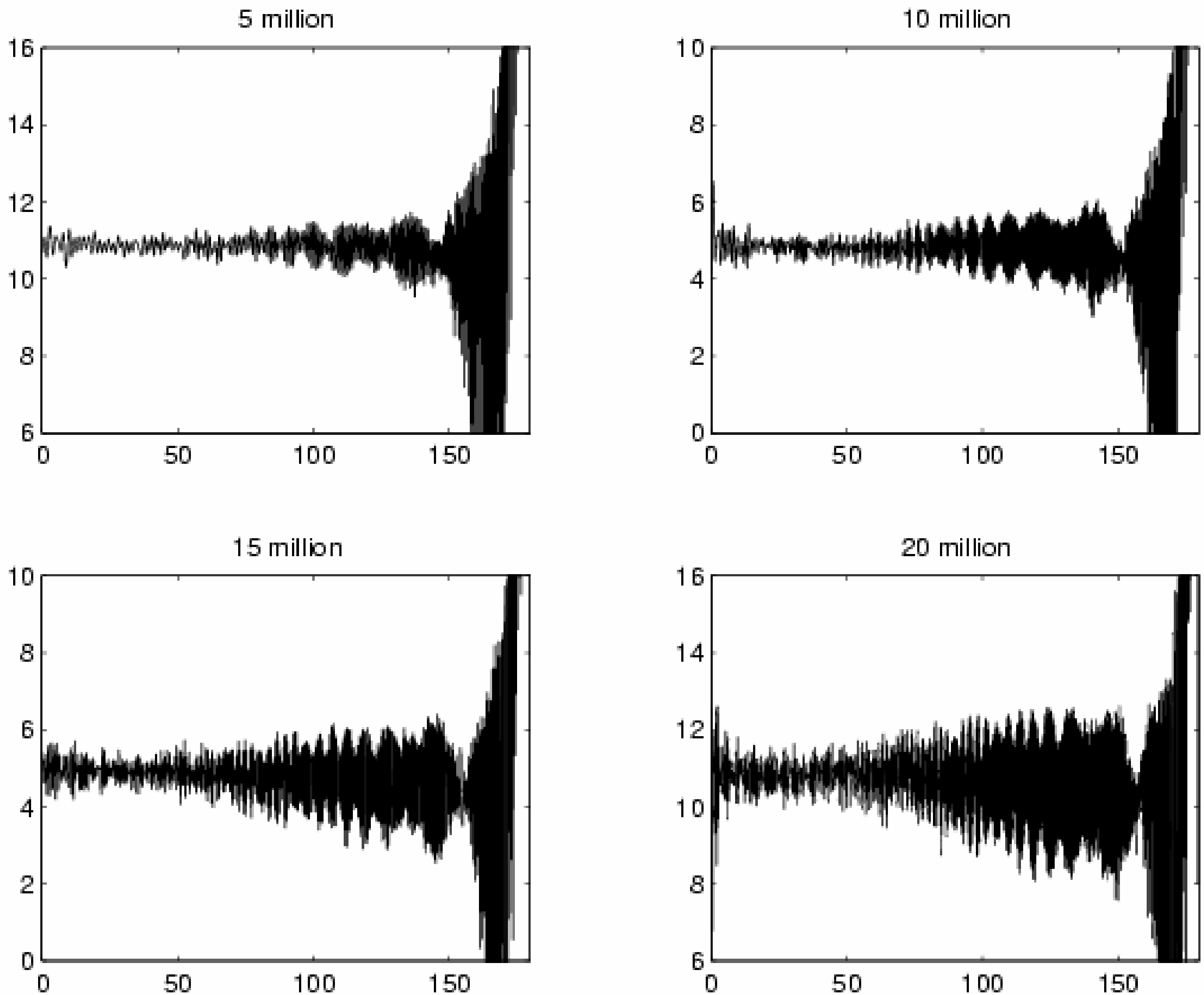
- Carefully chosen shared level results in impressive scaling properties

Very Large Scale Problem – VFY-218

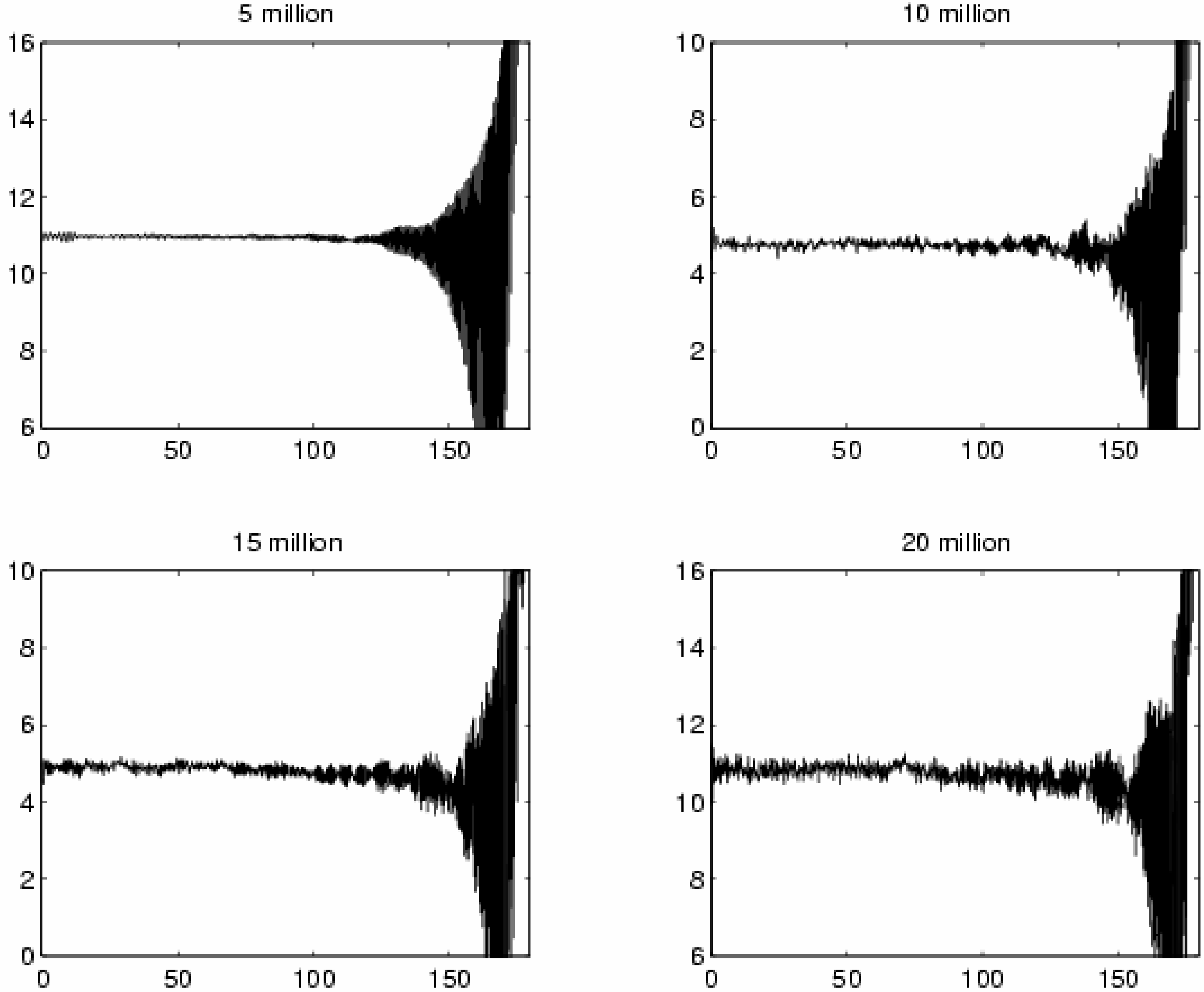
- Frequency = 8 GHz; N = 10,186,446
- Time for matrix-vector products: 119 s on 126 processors
- Total solution time: 7 hrs and 25 mins (2 rhs)



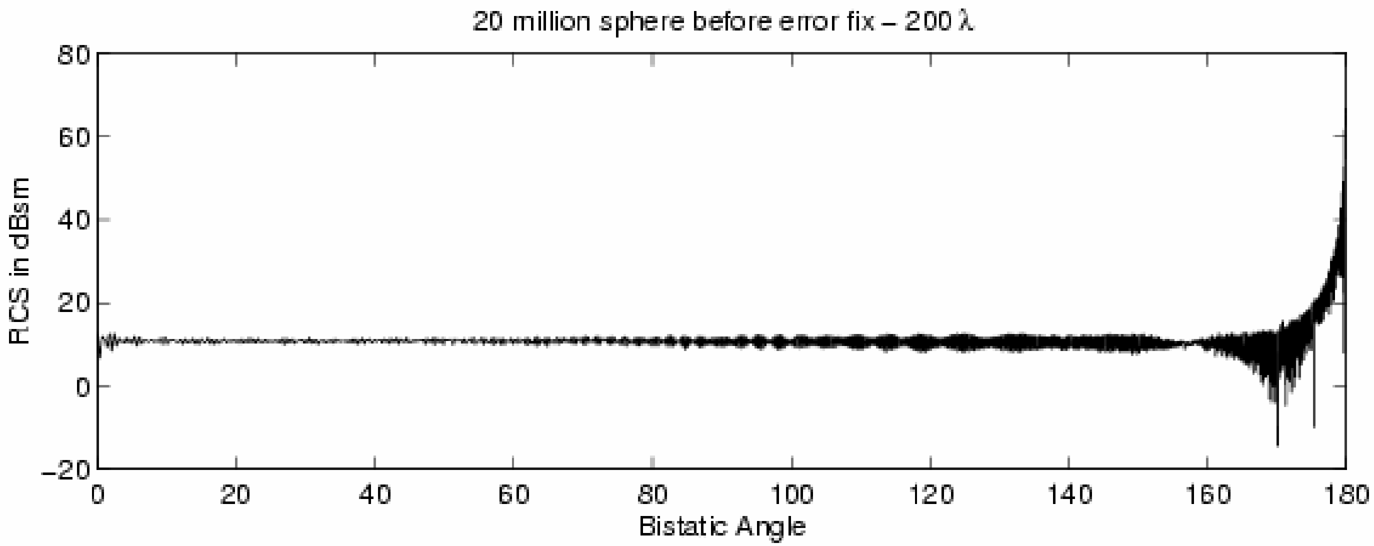
Ultra Large Scale Problems
ScaleME Pre-Bug Fix

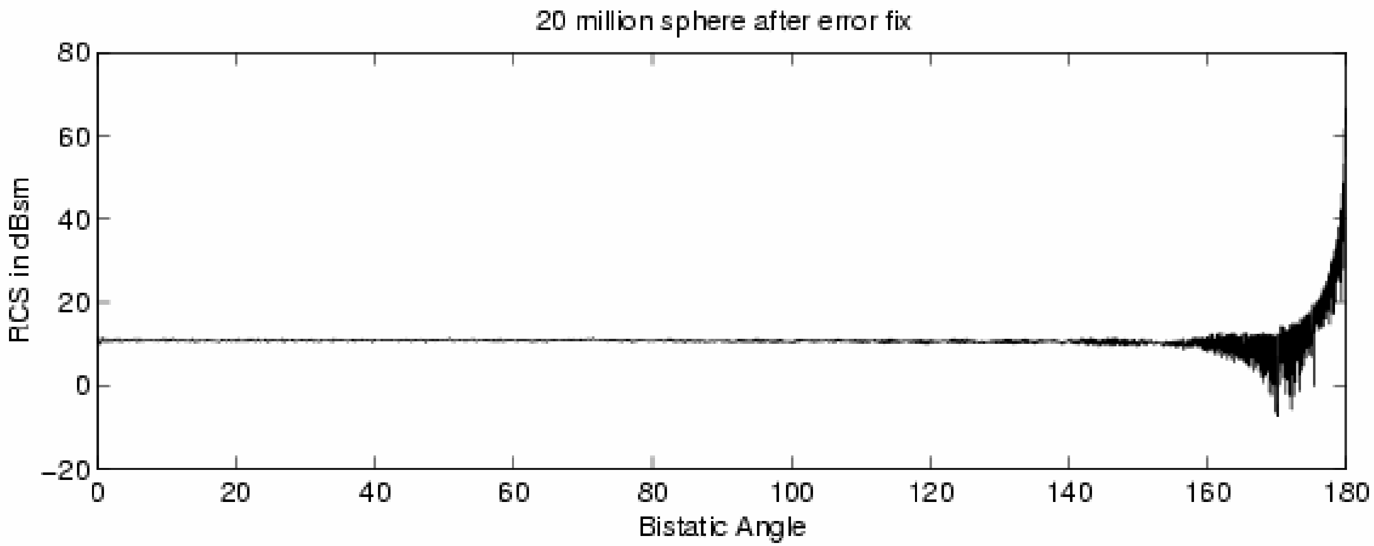


ScaleME Post-Bug Fix

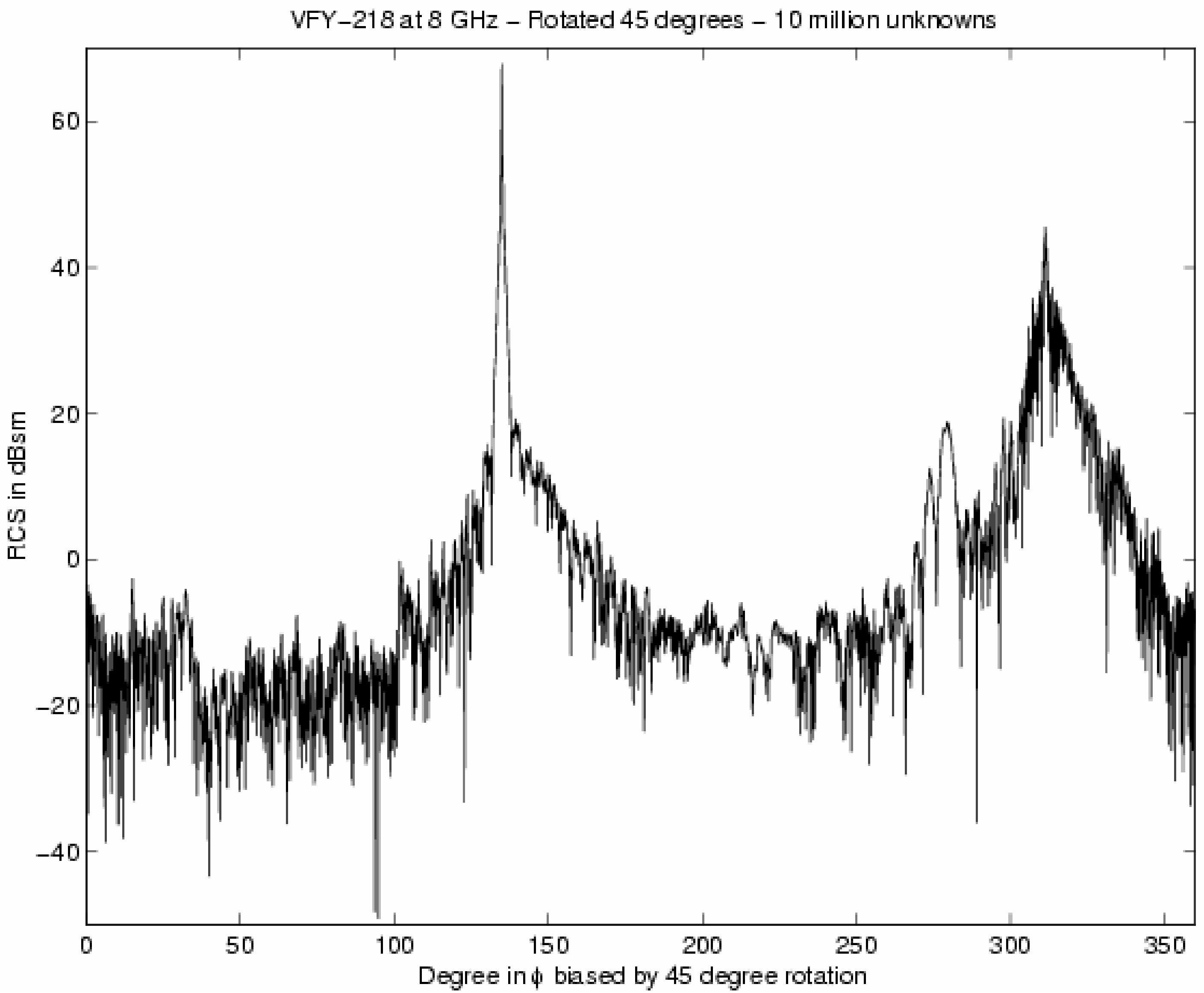


ScaleME Noise Bug Data – 20 Million



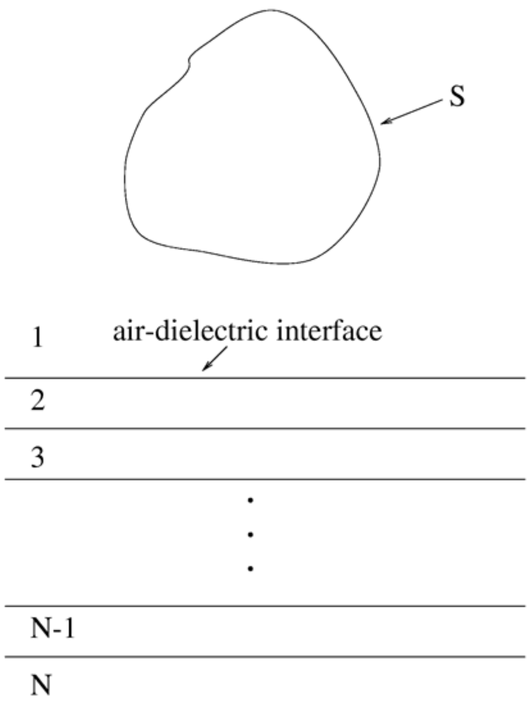


VFY-218 – 10 Million Unknowns 10 Gbytes Savings



Fast Algorithm for Analyzing Layered Medium Structure

Consider the scattering from a PEC scatterer S on top of a multi-layered medium.



The surface current $\mathbf{J}(\mathbf{r}')$ can be obtained by solving the following electrical field integral equation (EFIE)

$$\int_S \overline{\mathbf{G}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') d\mathbf{r}' = -\mathbf{E}^{inc}(\mathbf{r})$$

where $\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')$ is the dyadic Green's function for layered medium and $\mathbf{E}^{inc}$ denotes the incident electric field in the presence of the layered medium.

Using Method of Moments, the integral equation can be converted into a matrix equation.

Basic Idea - Green's Function

In the following talks, it is assumed that the Green's function can be written in the following form

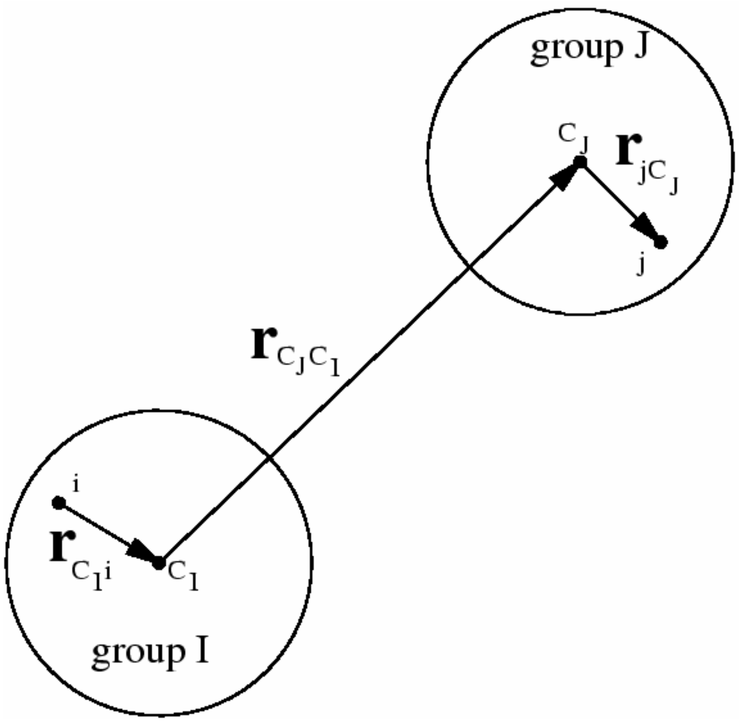
$$g(\mathbf{r}_j, \mathbf{r}_i) = \int d\hat{k} W(\hat{k}) e^{i\mathbf{k}\cdot\mathbf{r}_{ji}}$$

Here, $e^{i\mathbf{k}\cdot\mathbf{r}_{ji}}$ is called the inhomogeneous plane wave and it points to $\hat{k}$. Therefore, the Green's function $g(\mathbf{r}_j, \mathbf{r}_i)$ can be considered as the summation of the inhomogeneous plane waves which is propagating toward $\hat{k}$ and weighted by $W(\hat{k})$.

Note: Double integral is required for 3D problems.

Basic Idea - Grouping

Before proceeding, the source and observation points are grouped in the following manner:



Here, $\mathbf{r}_{C_I}$ and $\mathbf{r}_{C_J}$ denote the center of the source and observation groups and $\mathbf{r}_{ji}$ can be written as

$$\mathbf{r}_{ji} = \mathbf{r}_{jC_J} + \mathbf{r}_{C_JC_I} + \mathbf{r}_{C_Ii}$$

The integral can be expressed

$$g(\mathbf{r}_j, \mathbf{r}_i) = \int d\hat{k} W(\hat{k}) e^{i\mathbf{k}\cdot\mathbf{r}_{jC_J}} \cdot e^{i\mathbf{k}\cdot\mathbf{r}_{C_JC_I}} \cdot e^{i\mathbf{k}\cdot\mathbf{r}_{C_Ii}}$$

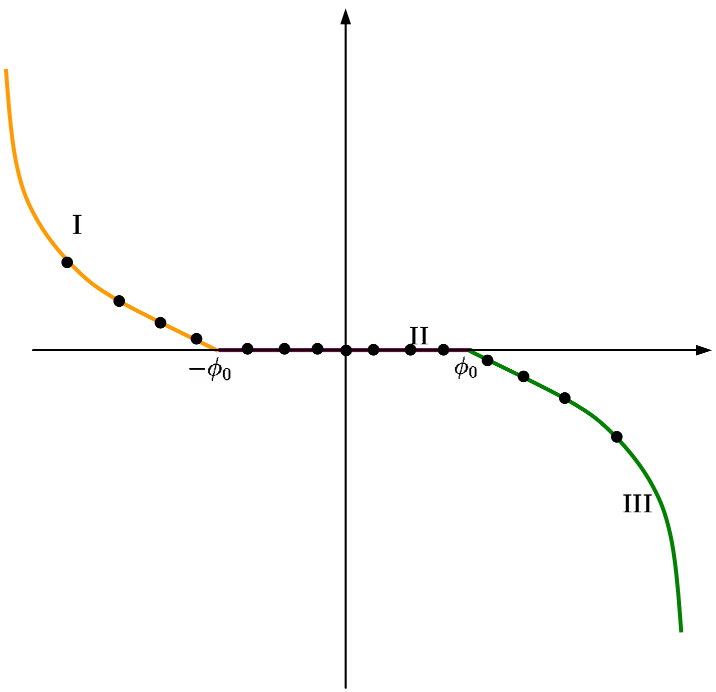
Basic Idea - Numerical Integration

- **Numerical Integration**
  - **Choice of quadrature rule**
    - Path I and III: The integrand shows exponential decay along it. Gauss-Laguerre rule is used.
    - Path II: The integrand is oscillatory. The trapezoidal or Gauss-Legendre rule is used.
  - **Integration formula**

    Assume $\boldsymbol{\Omega}_{\mathbf{q}}$ as the quadrature samples on SDP and $\mathbf{w}_{\mathbf{q}}$ as the weights for the integral, we can write the integral as the following summation

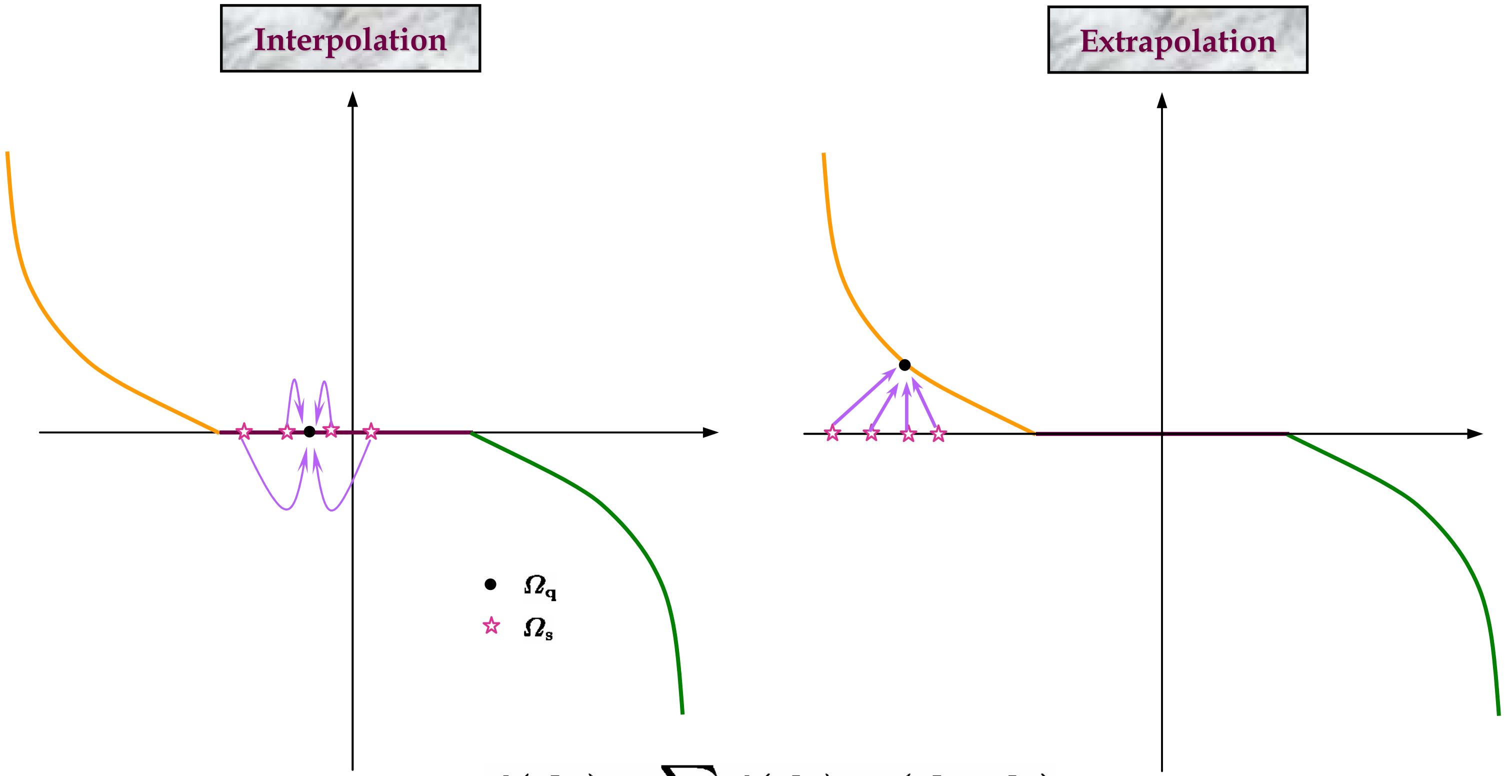
$$g(\mathbf{r}_j, \mathbf{r}_i) = \sum_{\mathbf{q}} \underbrace{e^{i\mathbf{k}(\boldsymbol{\Omega}_{\mathbf{q}})\cdot \mathbf{r}_{jC_J}}}_{\boldsymbol{\beta}_{jC_J}(\boldsymbol{\Omega}_{\mathbf{q}})} \cdot \underbrace{\mathbf{w}_{\mathbf{q}} e^{i\mathbf{k}(\boldsymbol{\Omega}_{\mathbf{q}})\cdot \mathbf{r}_{C_J C_I}}}_{\boldsymbol{\alpha}_{C_J C_I}(\boldsymbol{\Omega}_{\mathbf{q}})} \cdot \underbrace{e^{i\mathbf{k}(\boldsymbol{\Omega}_{\mathbf{q}})\cdot \mathbf{r}_{C_I i}}}_{\boldsymbol{\beta}_{C_I i}(\boldsymbol{\Omega}_{\mathbf{q}})}$$

note: $g(\mathbf{r}_j, \mathbf{r}_i)$ is expressed as a summation of the inhomogeneous plane waves.



Basic Idea - Diagonalization

- **Illustration of Interpolation and Extrapolation**



$$\beta(\Omega_q) = \sum_s \beta(\Omega_s) \cdot \mathcal{I}(\Omega_q, \Omega_s)$$

Basic Idea - Diagonalization

Expressing it into the matrix product form, we have

$$g(\mathbf{r}_j, \mathbf{r}_i) = \left[\cdots \; \beta_{jC_J}(\Omega_s) \; \cdots\right] \cdot \begin{bmatrix} \ddots & & \\ & \mathcal{T}_{C_J C_I}(\Omega_s) & \\ & & \ddots \end{bmatrix} \cdot \begin{bmatrix} \vdots \\ \beta_{C_I i}(\Omega_s) \\ \vdots \end{bmatrix}$$

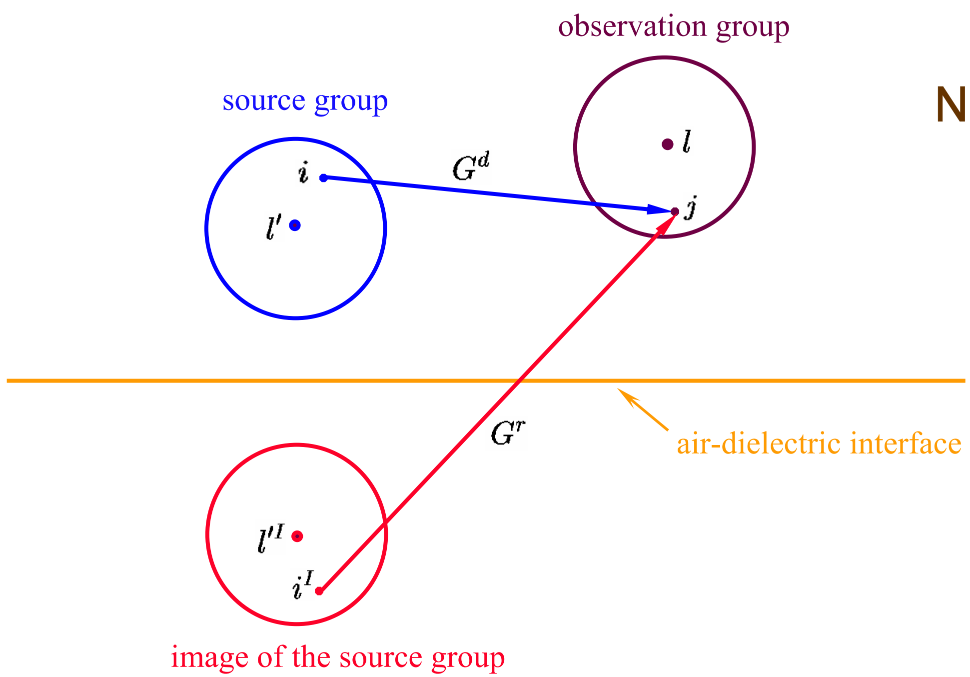
Therefore, the Green's function $g(\mathbf{r}_j, \mathbf{r}_i)$ is evaluated by a summation of the homogeneous plane waves and the translation operator is diagonal.

$\beta_{jC_J}(\Omega_s)$ and $\beta_{C_I i}(\Omega_s)$ are both sampled on the real axis and can be re-used for different integral path. The number of samples are proportional to the size of the group, due to the quasi band-limited property.

3D Layered Medium Problems - Grouping

G^d is the free space Green's function and the source point $\mathbf{r}_i$ and observations point $\mathbf{r}_j$ can be grouped.

G^r can be considered as the interaction between a source located at $\mathbf{r}_{i^I}$ the mirror image of $\mathbf{r}_i$, and the observation point at $\mathbf{r}_j$. Therefore, we can also group the $\mathbf{r}_{i^I}$ to form an image group.



Not Image Theorem!

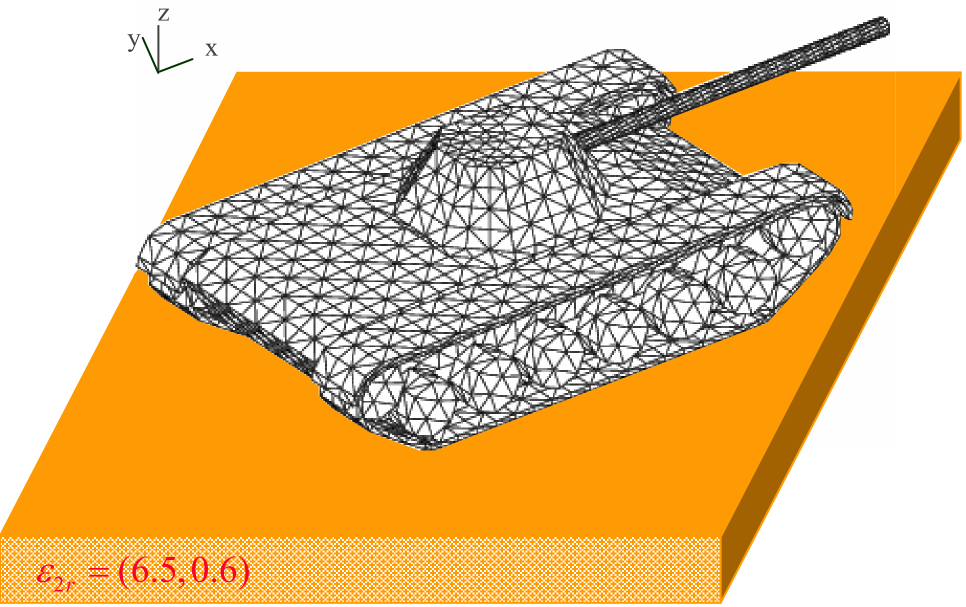
3D Layered Medium Problems - Diagonal Form

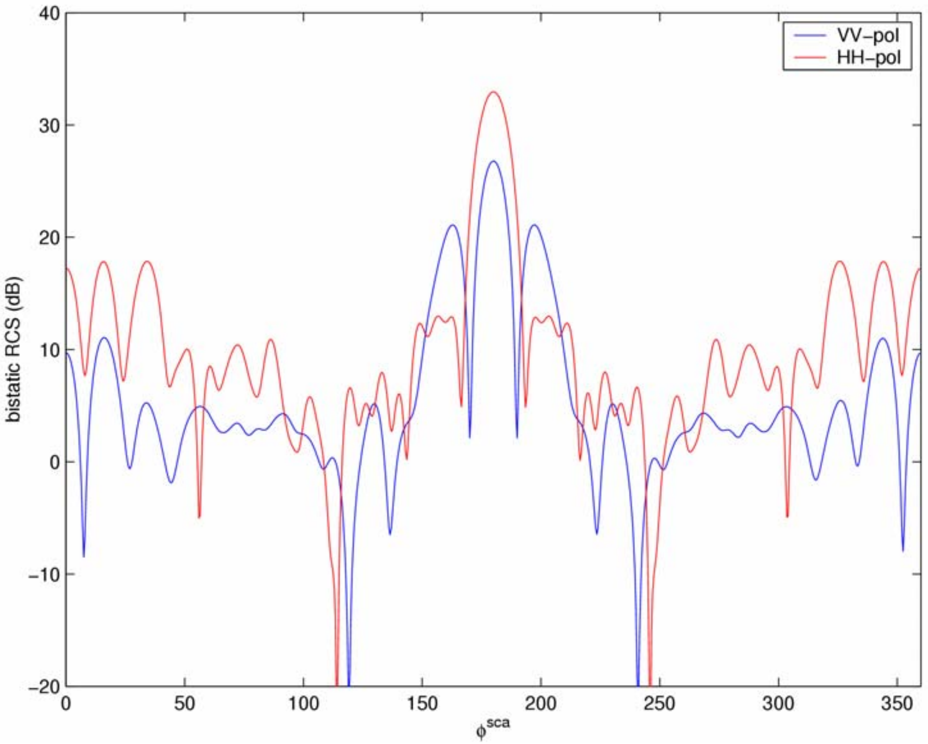
- **The diagonalized form of the Green's function can be written as**

$$
\begin{aligned}
G(\mathbf{r}_j,\mathbf{r}_i) = & \sum_{\mathbf{s}} \boldsymbol{\beta}_{jl}(\Omega_{\mathbf{s}}) \cdot \mathcal{T}_{ll'}(\Omega_{\mathbf{s}}) \cdot \boldsymbol{\beta}_{l'i}(\Omega_{\mathbf{s}}) && \longleftarrow G^d \\
& + \sum_{\mathbf{s}} \boldsymbol{\beta}_{jl}(\Omega_{\mathbf{s}}) \cdot \mathcal{T}_{ll'^I}^{rSDP}(\Omega_{\mathbf{s}}) \cdot \boldsymbol{\beta}_{l'i}(\Omega_{\mathbf{s}}) && \longleftarrow G^{rSDP} \\
& + \sum_{\mathbf{p}} \boldsymbol{\beta}_{jl}(\Omega_{\mathbf{p}}) \cdot \mathcal{T}_{ll'^I}^{rP}(\Omega_{\mathbf{p}}) \cdot \boldsymbol{\beta}_{l'^I i^I}(\Omega_{\mathbf{p}}) && \longleftarrow G^{rP} \\
& + \sum_{\mathbf{n}} \boldsymbol{\beta}_{jl}(\Omega_{\mathbf{n}}) \cdot \mathcal{T}_{ll'^I}^{rB}(\Omega_{\mathbf{n}}) \cdot \boldsymbol{\beta}_{l'^I i^I}(\Omega_{\mathbf{n}}) && \longleftarrow G^{rB}
\end{aligned}
\quad \Bigg\} G^r
$$

3D Layered Medium Problems -- Numerical Results

- **Tank on top of the ground**





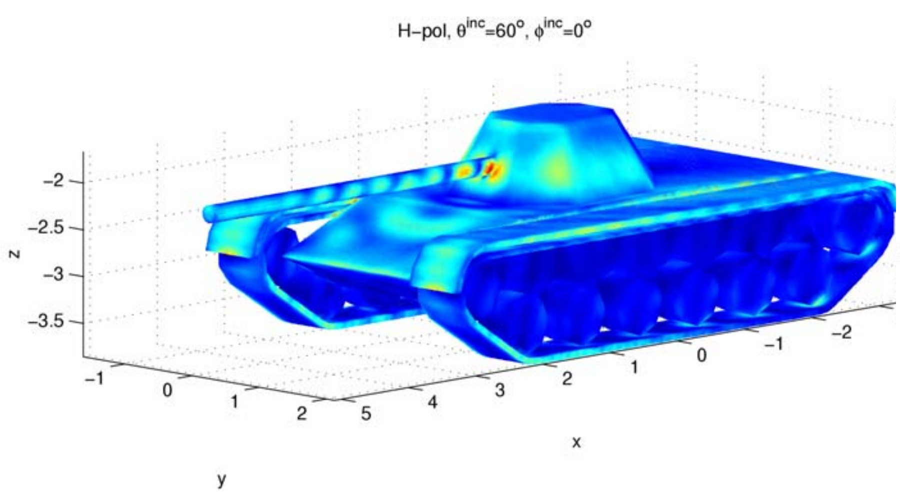
$$\left(\theta^{inc}, \phi^{inc}\right) = \left(60^o, 0^o\right)$$

L = 8.50m
W = 3.74m
H = 2.20m

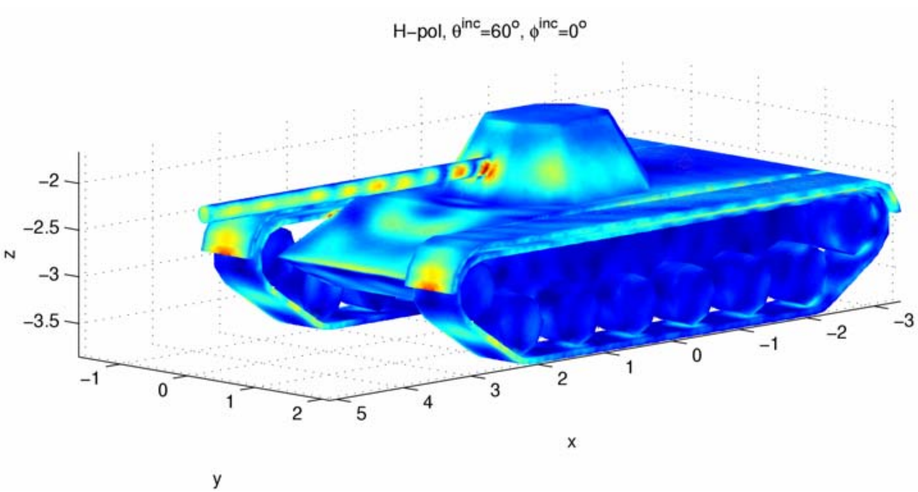
f = 400MHz
N= 133,578
6 level ML-FIPWA

3D Layered Medium Problems -- Numerical Results

- **Current Distribution on Tank (H-pol)**



Free space



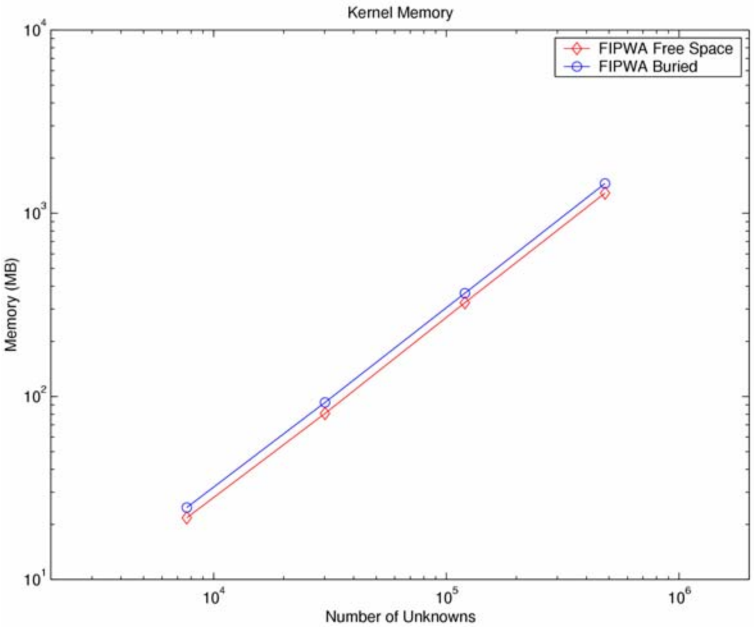
Sit on the ground

f = 400MHz
N = 133,578
6 level ML-FIPWA

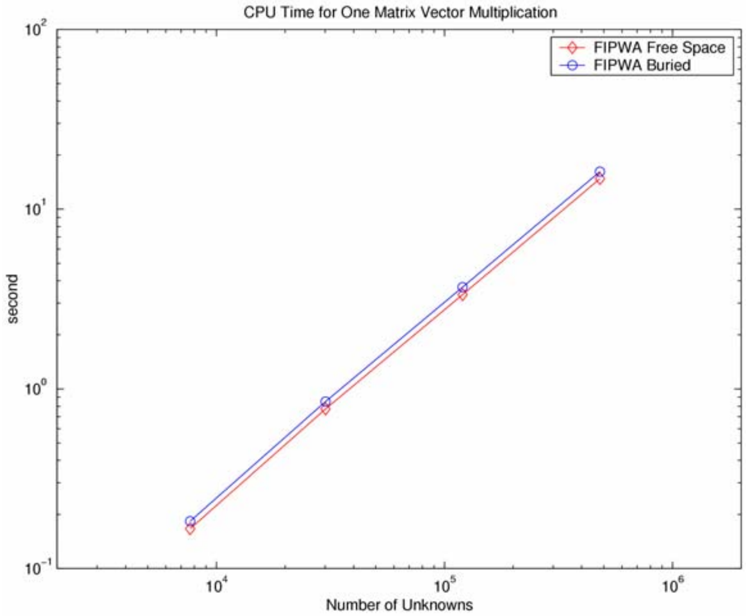
3D Layered Medium Problems -- Numerical Results

- **Computational Complexity for the Bunker problem**

Memory requirement



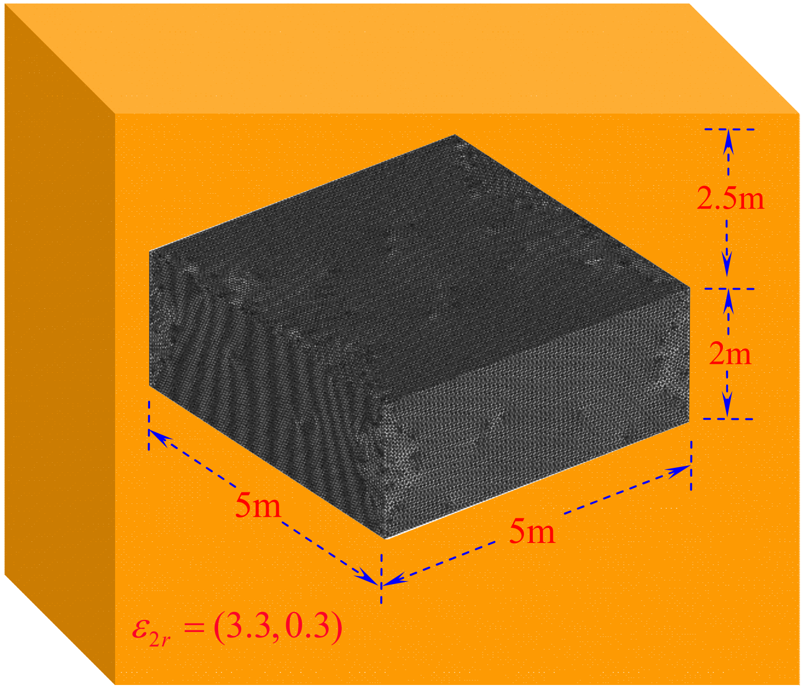
matrix vector multiply time

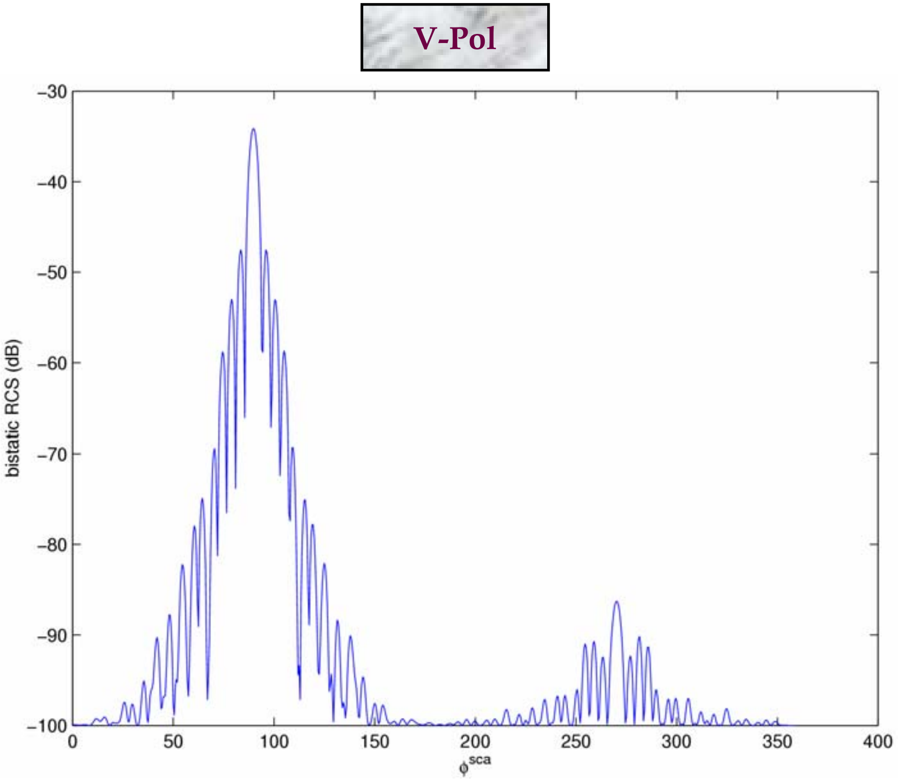


- A 600 MHz DEC Alpha 21164 workstation is used;
- memory requirement increases by 13% compared to the free space code;
- CPU time for matrix-vector multiply increases by 10% compared to the free space one.

3D Layered Medium Problems -- Numerical Results

- **Underground Bunker**





$$\left(\theta^{inc}, \phi^{inc}\right) = \left(60^o, -90^o\right)$$

f = 900MHz
N = 1,074,588
8 level ML-FIPWA

	Solution time	Memory
FIPWA	18 hours (SGI Origin 2000)	3.94 GB
Full matrix (est.)	~11 years	~9.3TB

Very Small Problem Compared to Wavelength

- **The Need for Low Frequency Simulation**
  - Detail object geometry variation
  - Detail material property variation
  - The need to interface with circuit theory
  - Growing interest in nano-technology

Electromagnetics at Low Frequency

Decoupling of Electric and Magnetic Fields at DC

The Maxwell's equations can be written as follows when right at zero frequency

$$\nabla \times \mathbf{E} = 0, \quad \nabla \cdot \varepsilon \mathbf{E} = \rho = \lim_{\omega \to 0} \nabla \cdot \mathbf{J} / i\omega$$

$$\nabla \times \mathbf{H} = \mathbf{J}, \quad \nabla \cdot \mu \mathbf{H} = 0$$

It is easy to see that the electric and magnetic fields are decoupled at DC and the current can be decomposed as

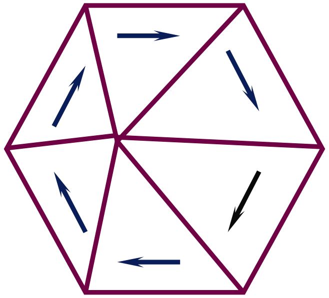
$$\mathbf{J} = \mathbf{J}_{sol} + \mathbf{J}_{irr}$$

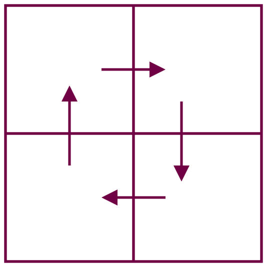
divergence-free → $\mathbf{H}$

curl-free → $\mathbf{E}$

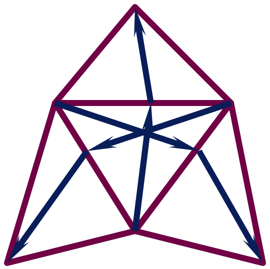
Low Frequency--Formulations

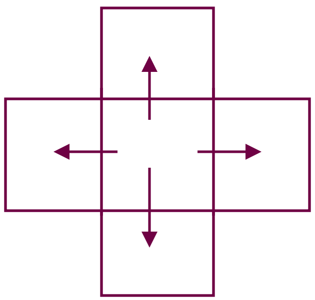
- **Loop Basis: divergence free**





- **Star Basis: quasi-curl-free.**



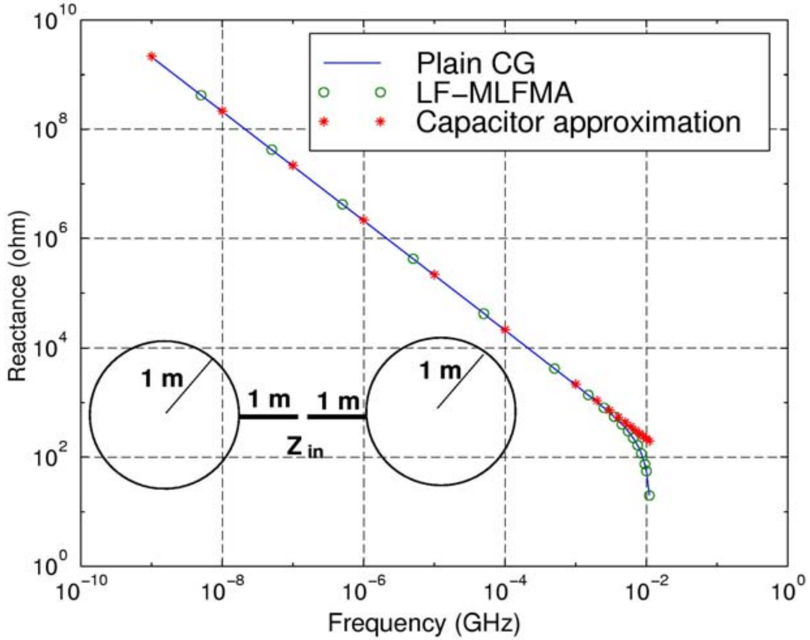


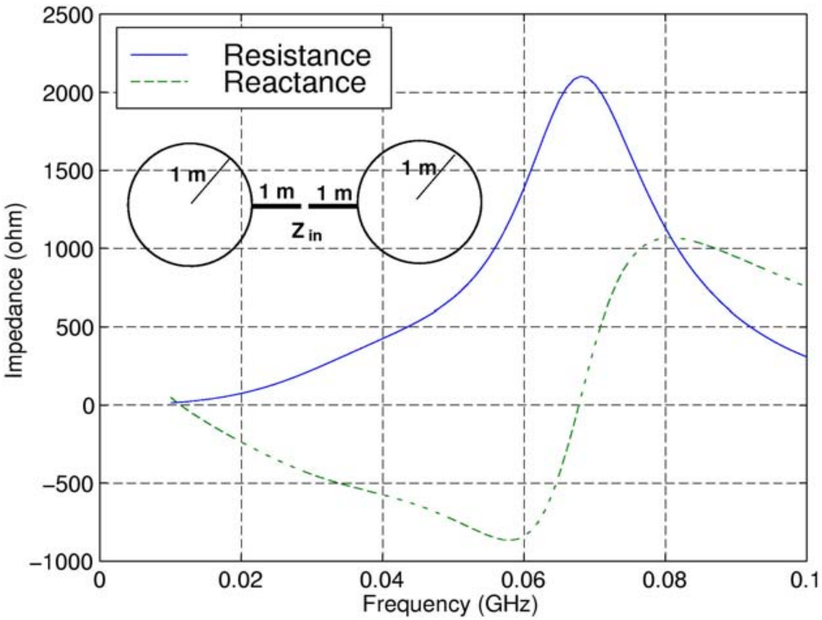
- **Tree Basis: RWG basis with the basis along a cut removed. The cut prevents the rest of the RWG basis (tree basis) from forming any loop.**

> LS or LT formulation isolates the contribution of vector potential and scalar potential.
>
> Information of vector potential will not be lost due to machine precision.

Hertzian Dipole from Zero to Microwave Frequencies

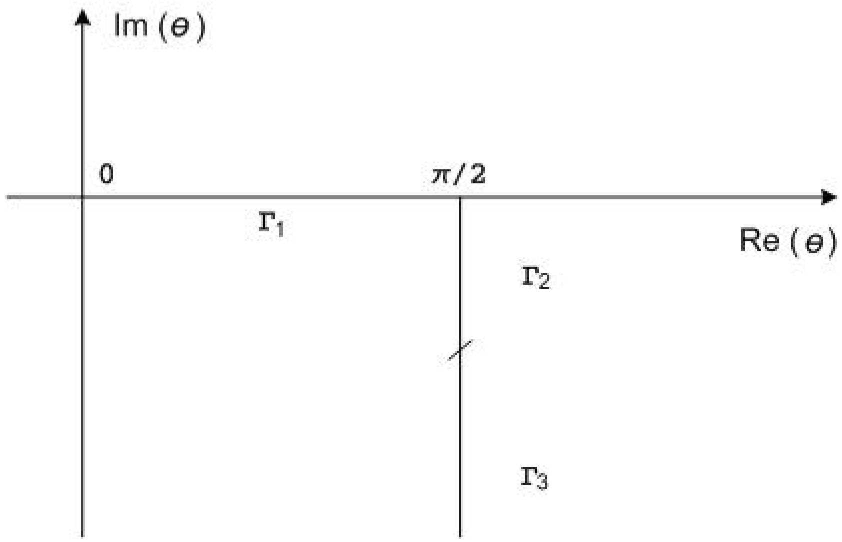
- **Input admittance/impedance of a Hertzian dipole at very low frequencies and at higher frequencies**





LF-FIPWA: Fast InhomogeneousPlane Wave Algorithm

- Integration path can be divided into three segments.
- Plane waves are therefore categorized into three kinds



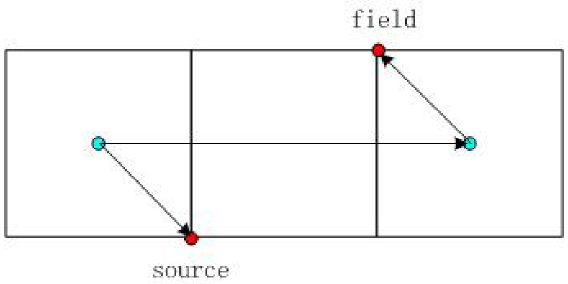
$\mathcal{G}_1$ Propagating Waves

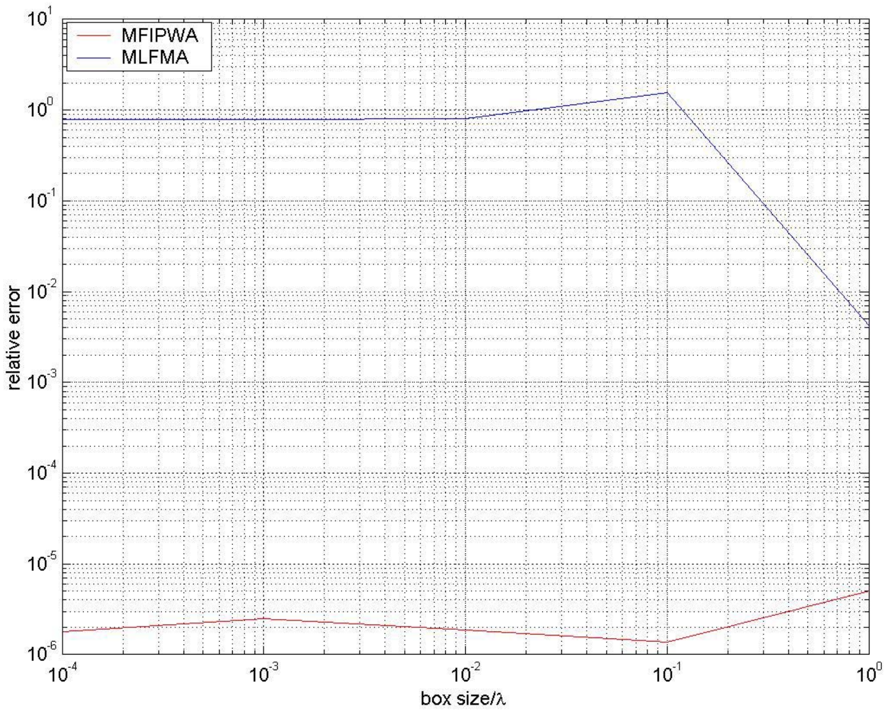
$\mathcal{G}_2$ Evanescent Waves

$\mathcal{G}_3$ Evanescent Waves

- Evanescent Waves on Γ_3 are less significant when it goes to $-i\infty$
- Truncation of Γ_3 determines the accuracy

Dyadic Green's Function

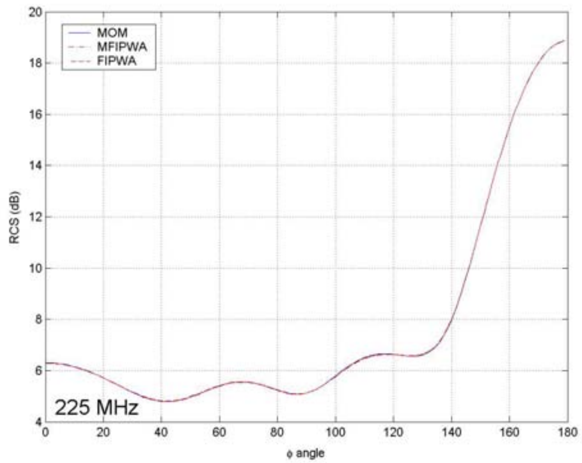




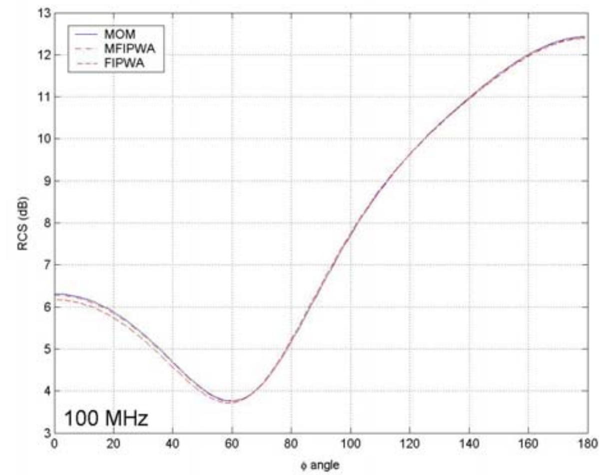
- G_{xx} **is computed**
- **The source point and the field point are arranged to check the worst possible interaction accuracy**

- **MLFMA has very bad accuracy**
- **MFIPWA can provide less than 10^(-5) for single precision operation**

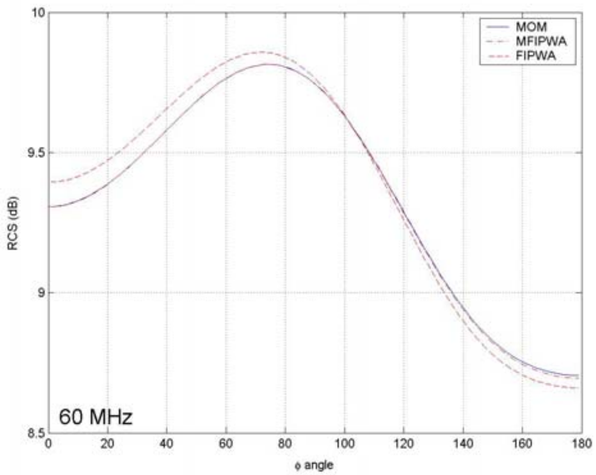
RCS of the Unit Sphere



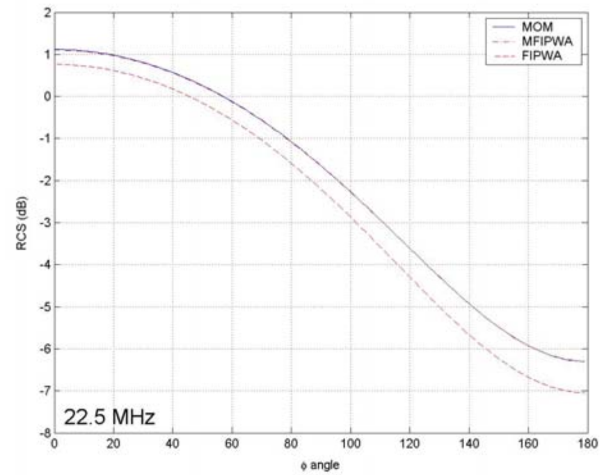
Smallest box size=0.187 wavelength



Smallest box size=0.0833 wavelength



Smallest box size=0.05 wavelength



Smallest box size=0.0187 wavelength

Low-Frequency Multilevel Fast Multipole Algorithm

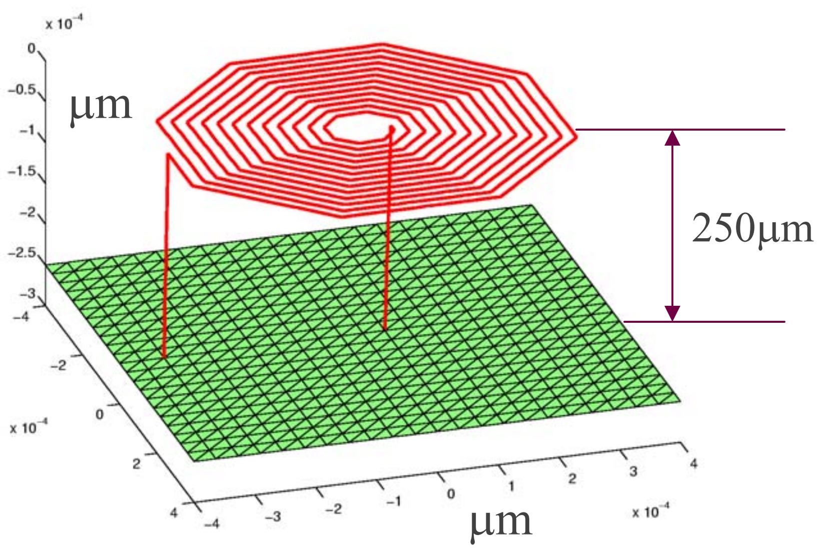
- **Wire Spiral Inductor with Ground Plane**

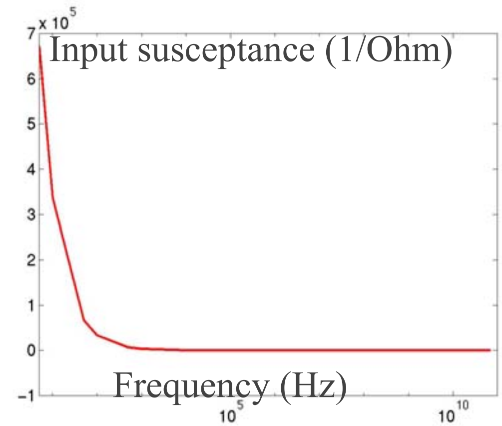
12 Turns

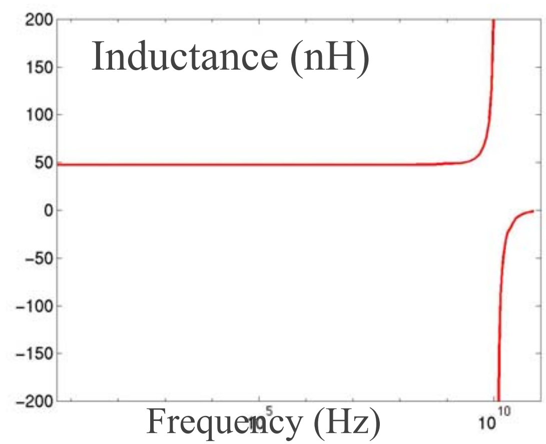
Radius of wire = 2.76 µm

Track spacing = 24 µm

Diameter of the inductor = 680 µm

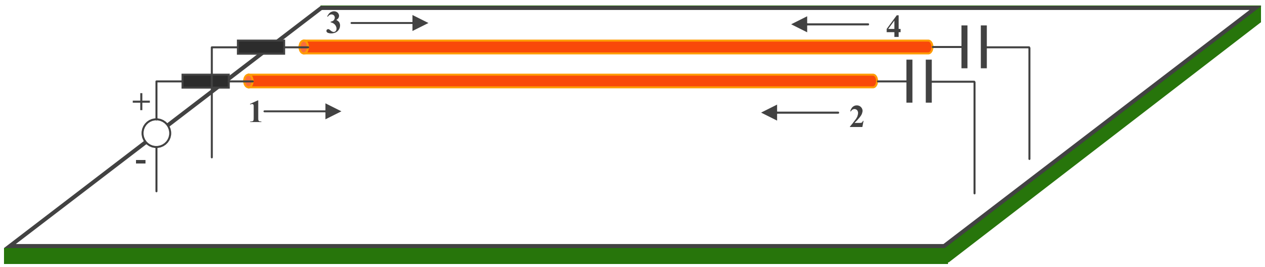


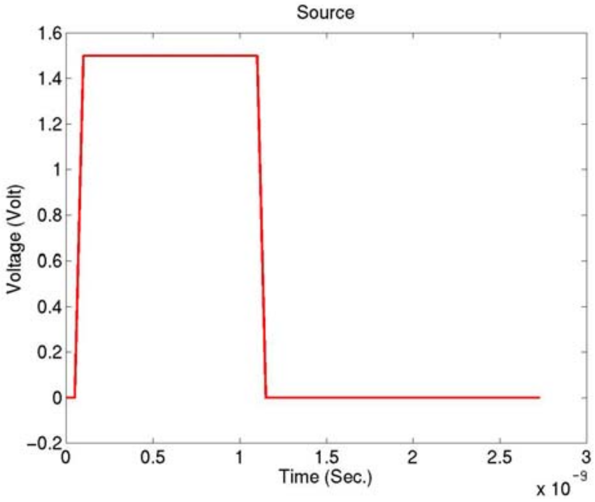




Cross-talk Simulation Suggested by Mazumder at Intel

- **Setup for the Simulation**

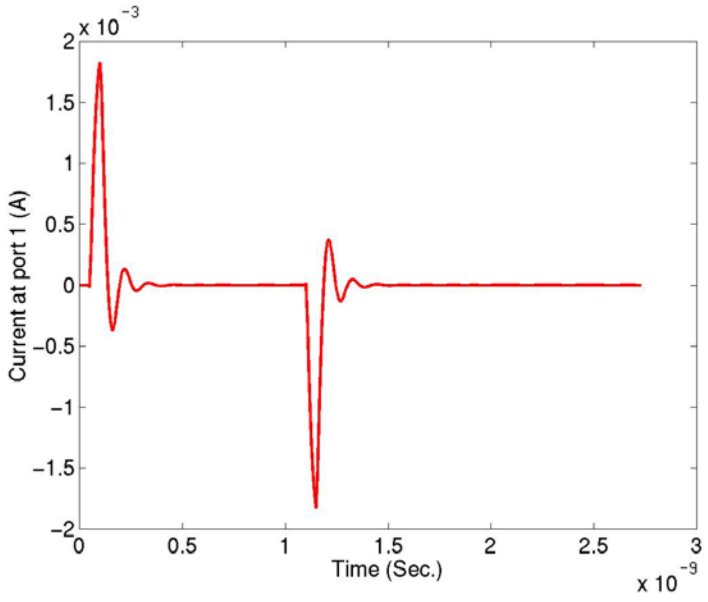


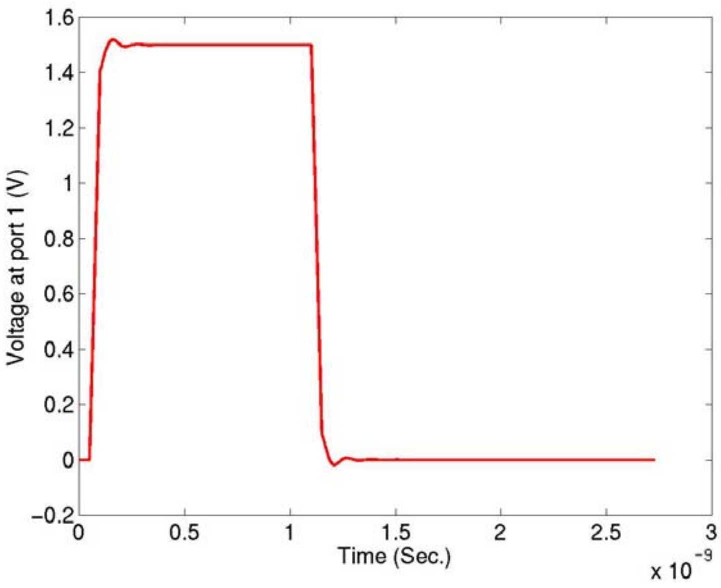


Resistor: 50 (ohm)
Capacitance: 50E-15 Farad
Radius of the wire: 0.4 um
Length of the wires: 2000 um
Distant between axes of wires: 1.6 um
Conductivity of the wire: 2.38E7
Conductivity of the plate: 2.893E7
Thickness of the plate: 2.0 um
Height of the wire: 60um
Rising/falling time: 50 ps

Cross-talk Simulation

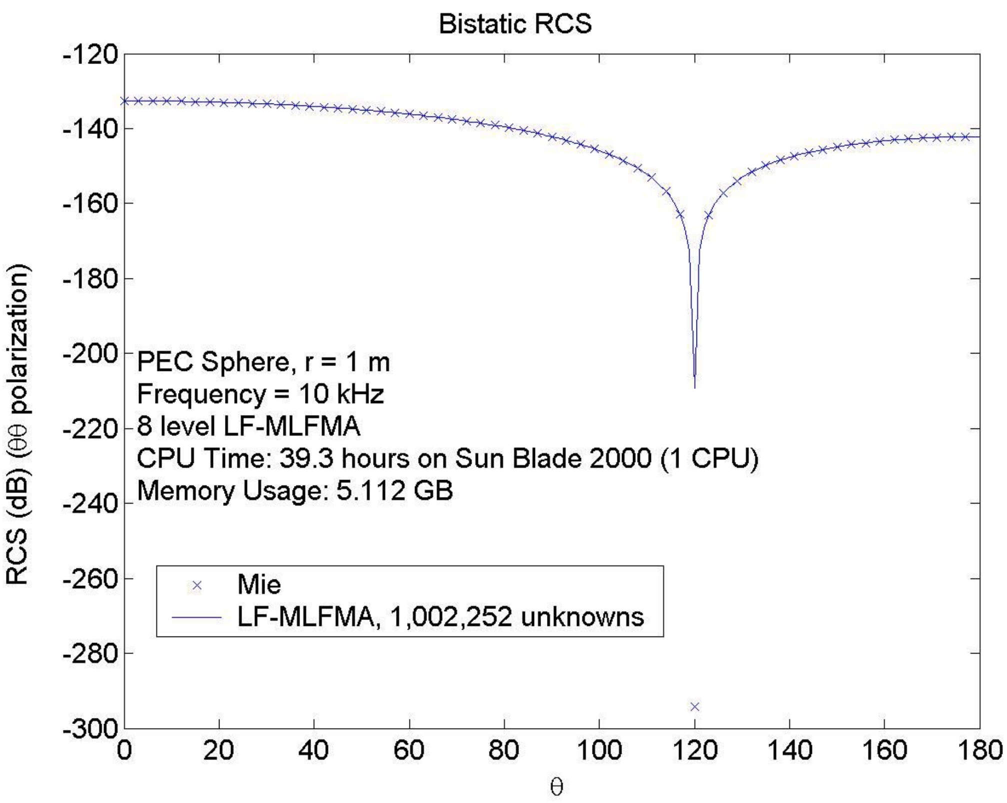
- **Current and Voltage at Port 1**



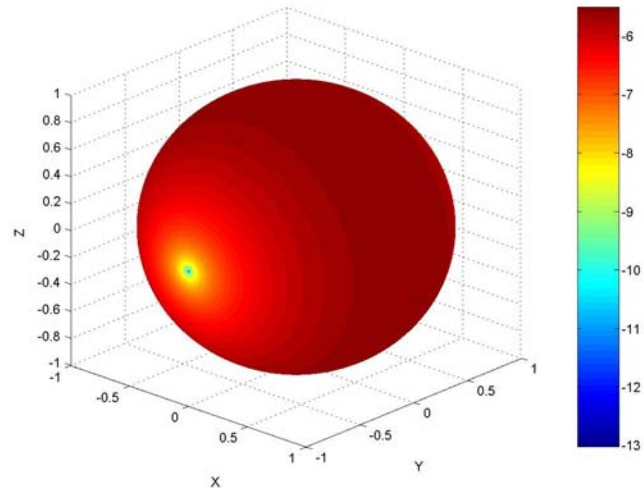


Large-Scale Low-Frequency Computation

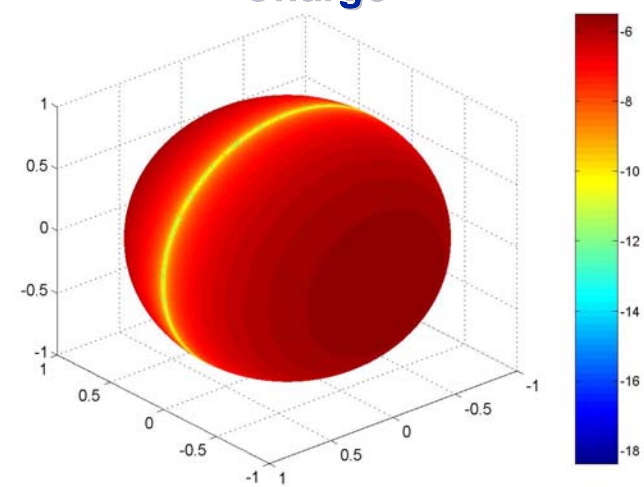
A PEC Sphere with Over One Million RWG Unknowns



Current



Charge

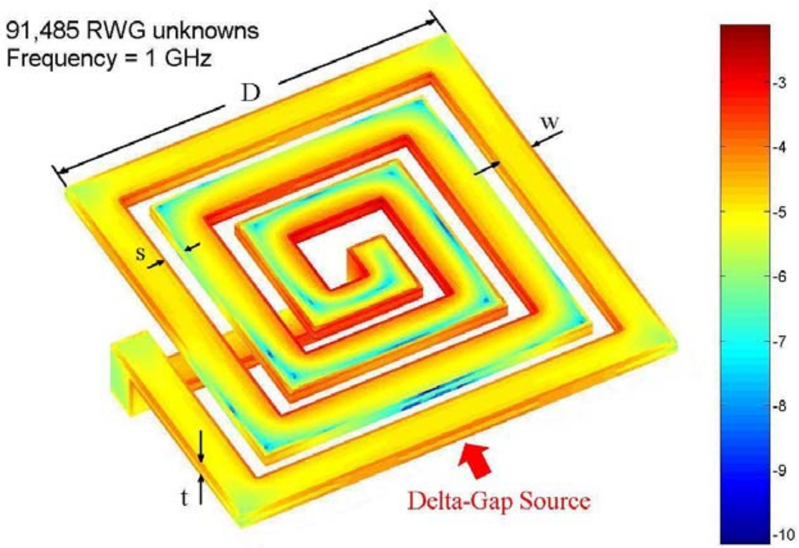


Large-Scale Low-Frequency Computation (cont'd)

On-Chip Spiral Inductor

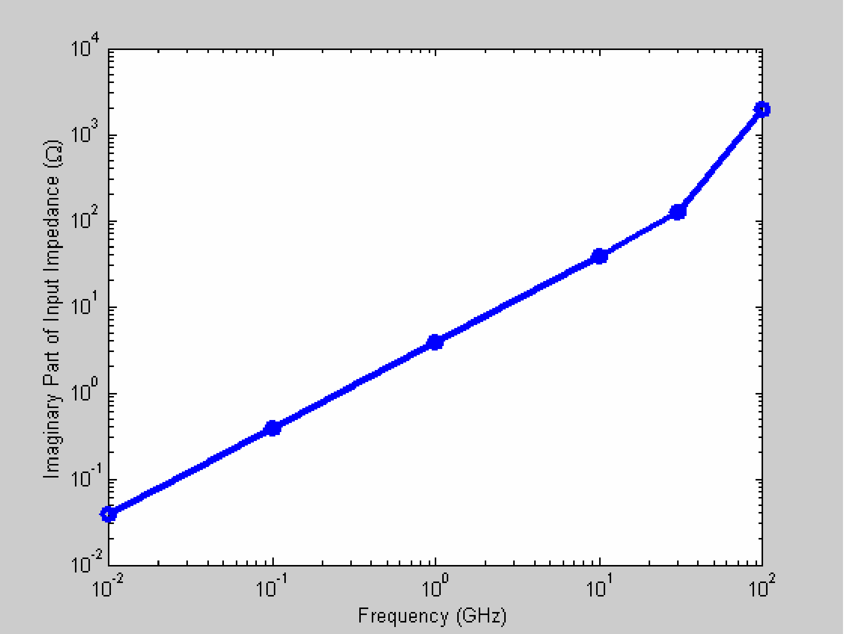
Current Magnitude Distribution in Logarithmic Scale at 1 GHz with 91,485 RWG Unknowns

D=95 um, w = 10 um, t=3 um, s = 5 um



Computed on a Sun Blade 2000 (1 CPU)
Total CPU Time: 5.25 hours
Total Memory Usage: 599 Mb

Imaginary Part of Input Impedance with 9,894 RWG Unknowns

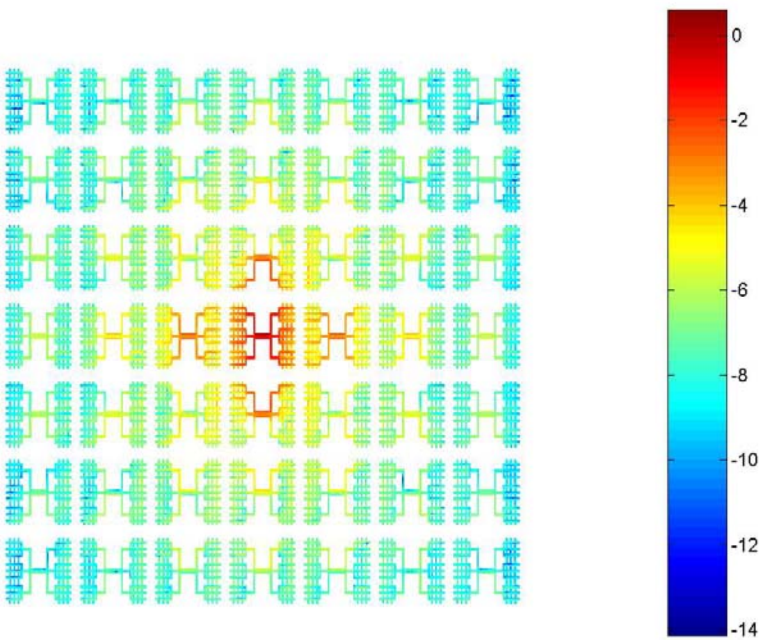


CPU Time/Freq. Pt. : 25 minutes
Total Memory Usage: 155 Mb

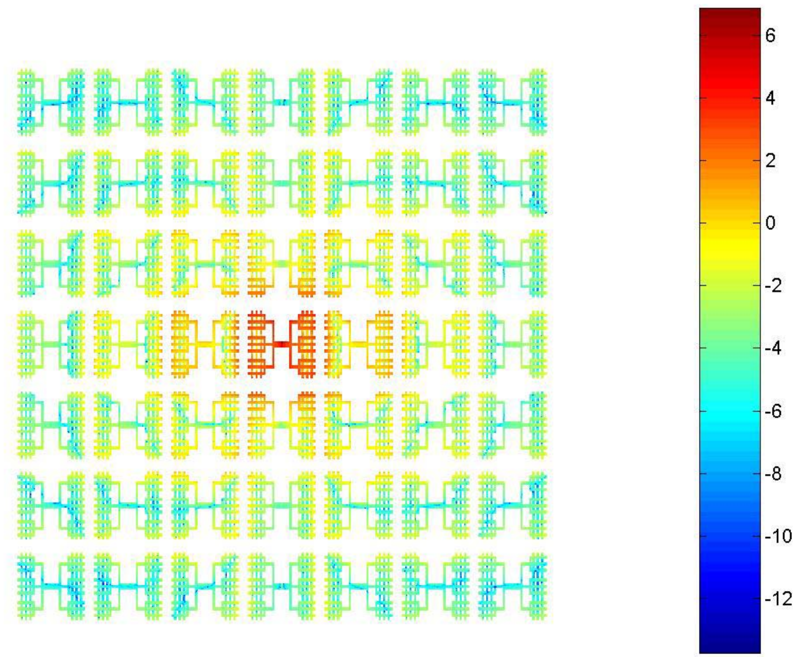
Numerical Results (cont'd)

731,031 Unknowns

Current



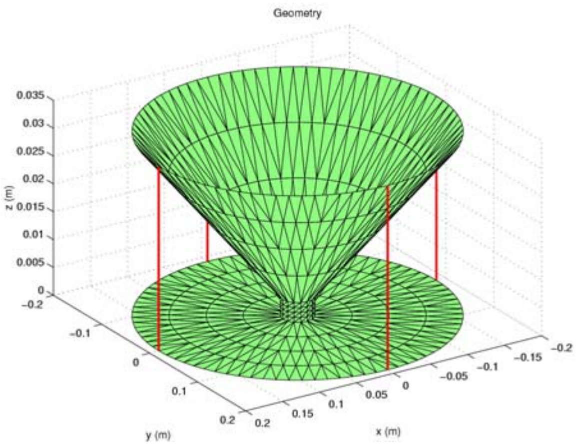
Charge



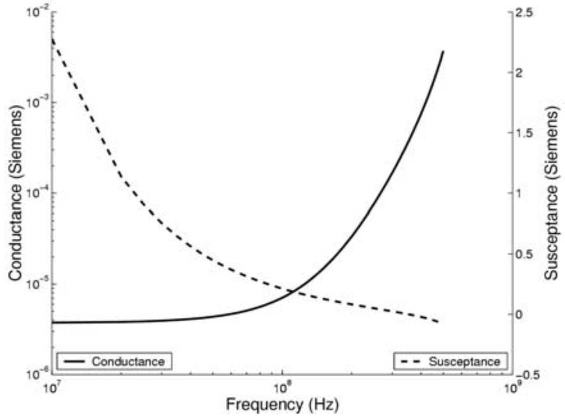
7 x 7 Array

Small Antenna Results

- **Input Conductance and Susceptance of a Conical Antenna with Four Wires**



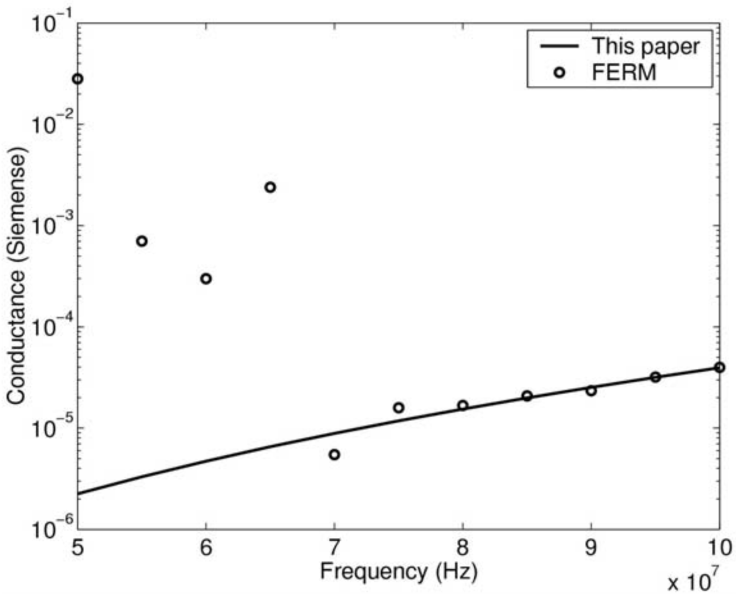
  - **At very low frequencies, the structure is like an inductor.**

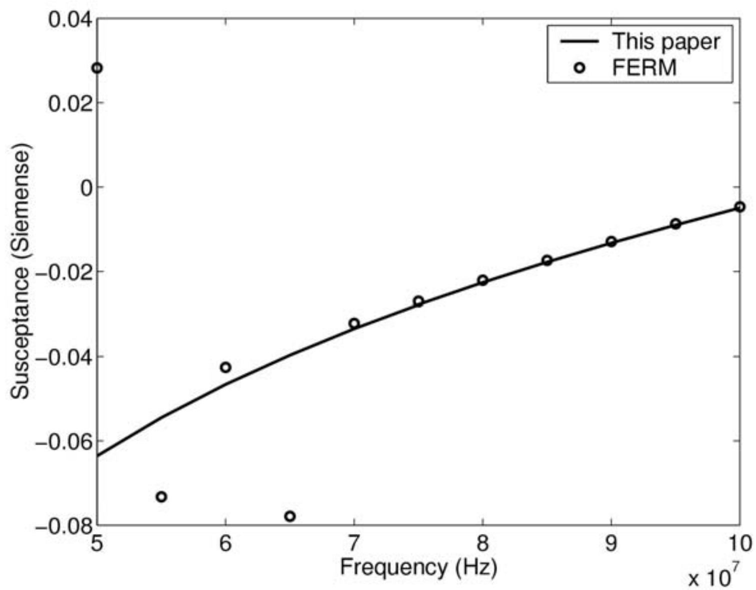


  - **When the frequency goes high, the distributed inductance becomes bigger and make the structure resonate.**

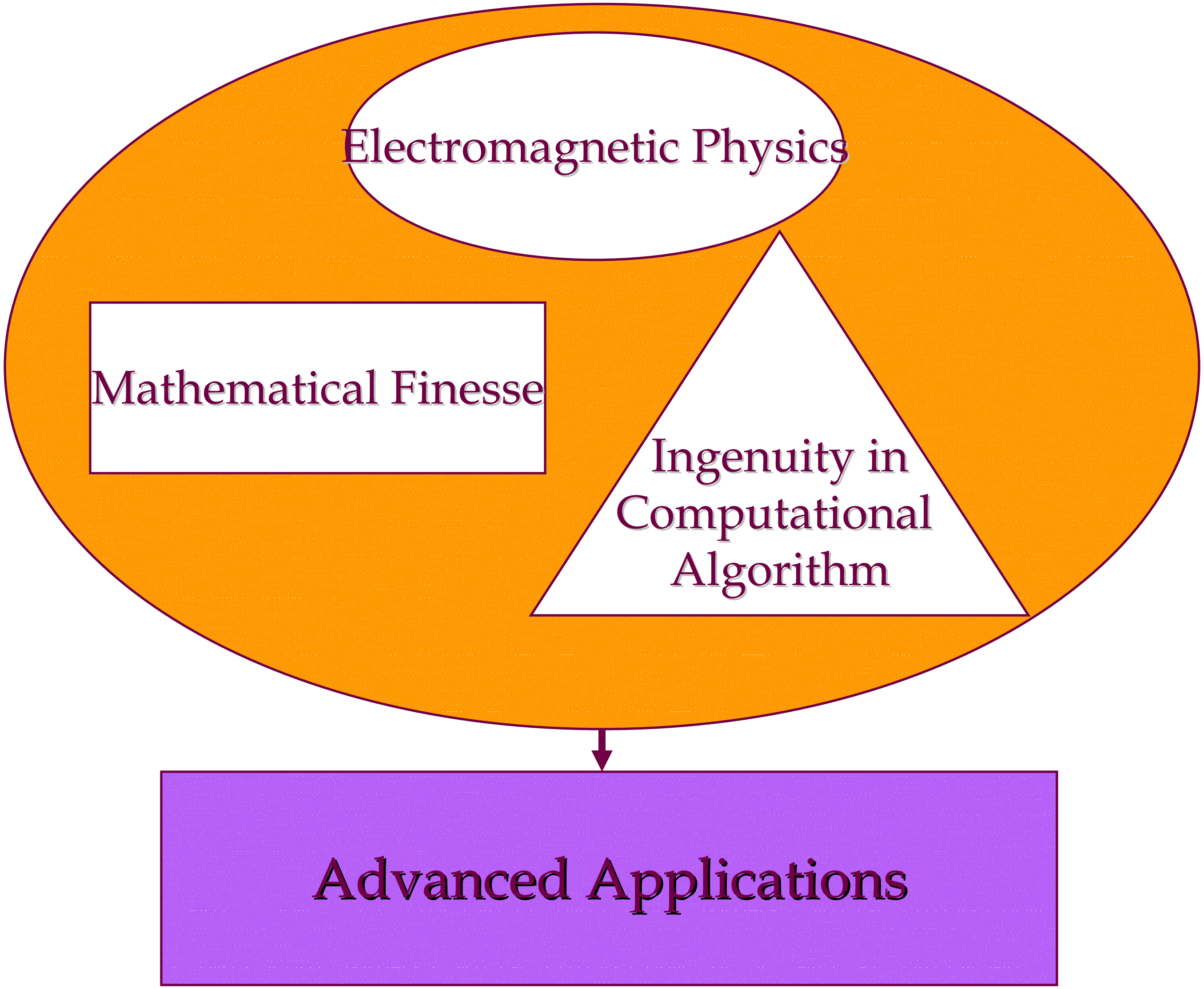
Small Antenna Results

- **Comparison of our Code and FERM**
  - FERM does not work at low frequencies





Computational Electromagnetics ---A New Age Analysis Tool



Conclusions

- EM Simulation is itself a science that is a melange of electromagnetic physics, mathematics, and computer science.
- The field is challenging, interesting, and portends high impact.
- EM simulation will become a new-age analysis tool, and is very important for many branches of electrical engineering.