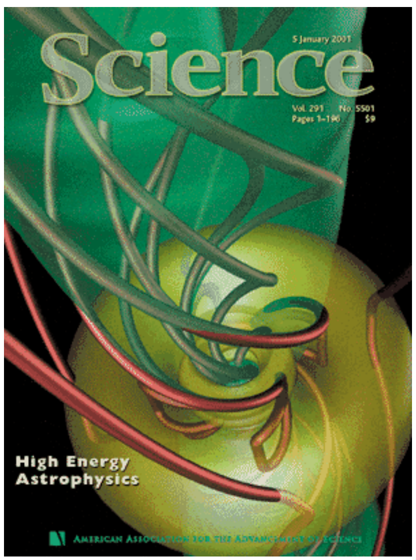
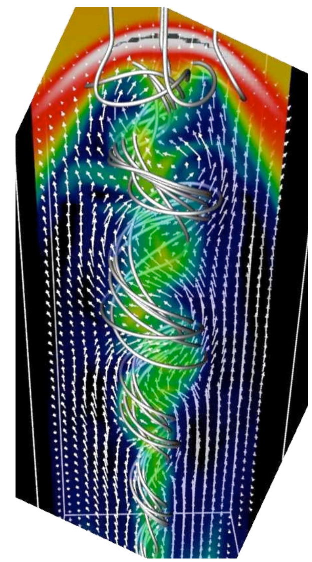
Strong, Time-Dependent Electromagnetic Fields in the Presence of Strong, Time-Dependent Gravity



Meier, Uchida & Koide (2001)

David Meier
Jet Propulsion Laboratory/Caltech



Nakamura, Uchida & Hirose (2001)

Co-Investigators:
M. Nakamura (JHU/STScI); S. Markoff (U. Amsterdam); P.C. Fragile (C of C);
D. Garofalo (JPL); P. Polko (U. Amsterdam)

Outline: Relativistic MHD of Black Hole Jets, Accretion, and Formation

- **Four topics, from the outer jet lobes to the hole formation**
  - M87 knots as MHD shocks in a Poynting-dominated jet (Nakamura, Garofalo, & Meier; 10 min)
  - Simulation of the hard accretion state as a radiatively-cooled Magnetically-Dominated Accretion Flow (Fragile & Meier; 5 min)
  - Numerical Constrained Transport as a Discrete Differential Geometry technique for evolving everything: EM & GR fields plus charge and matter sources (DLM; 10 min)

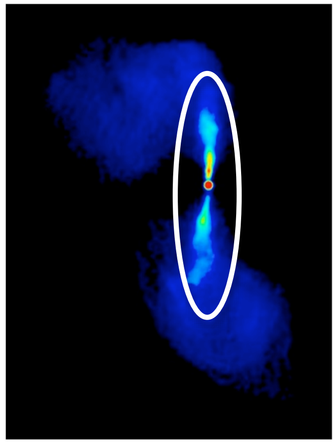
THE OUTER JET AND LOBES OF M87: WHAT THEY TELL US ABOUT JET DYNAMICS

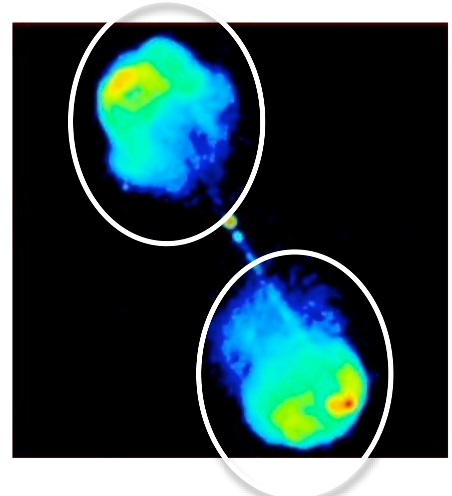
Nakamura, Garofalo, Meier

The Fanaroff & Riley Classification and Correlation

FR I (M 84 / 3C 272.1)

"1" emission region near galaxy



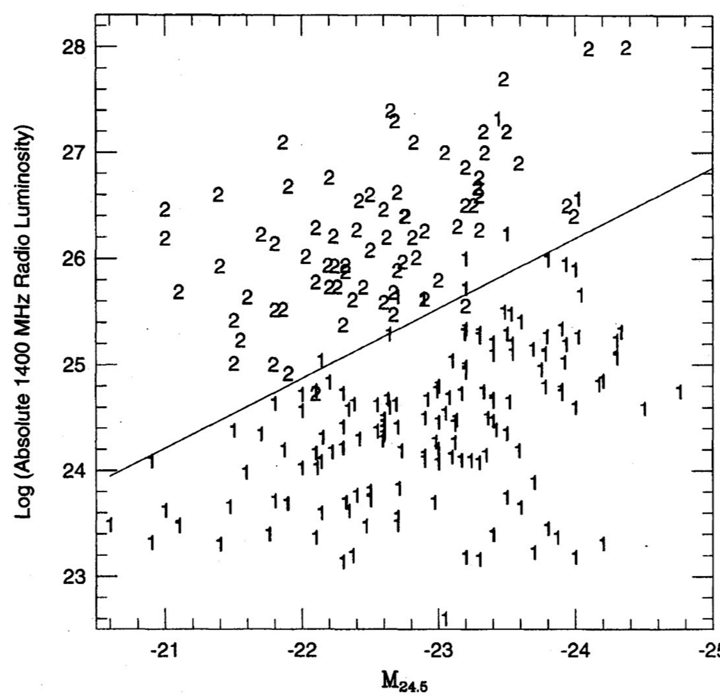


FR II (3C 47)

2 emission regions away from galaxy

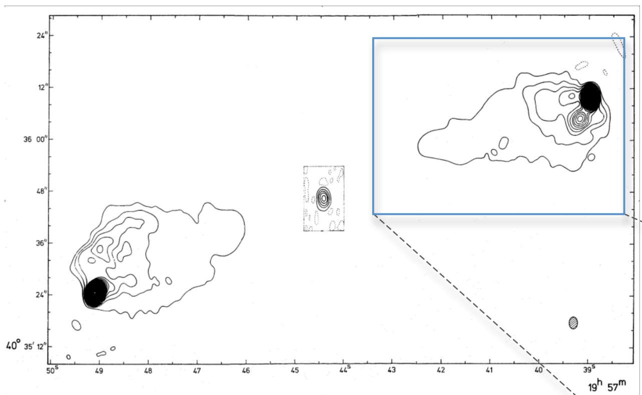
- The Fanaroff & Riley correlation:
  - FR Class I sources are low luminosity,
  - FR Class II are high luminosity, with the break at $P_{178} \sim 10^{25.3}$ W/Hz/Sr

- The FR I / FR II break is a strong function of galaxy OPTICAL luminosity ($\sim L_{opt}^2$) (Owen & Ledlow, AJ, 112, 9-22, 1996)



Cygnus A and the Blandford-Rees Hydrodynamic Model for Lobes & Hot Spots

Hargrave & Ryle(1974; 5km)

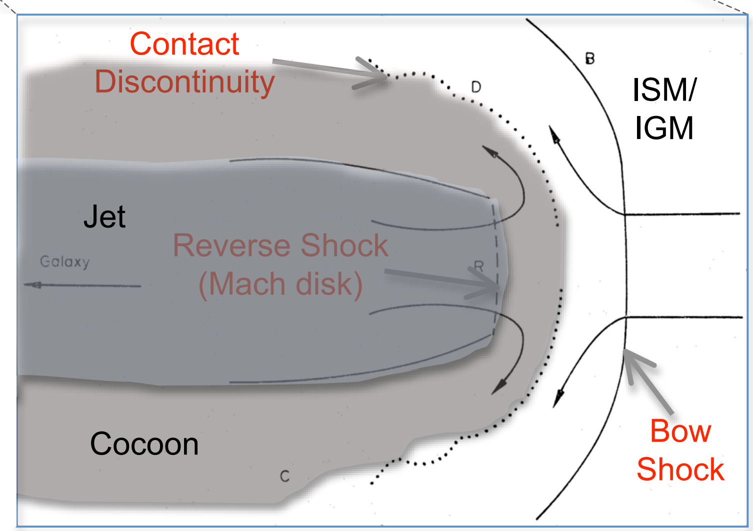


Perley, Dreher, & Cowan (1984; VLA)

Blandford & Rees' 1974 hydrodynamic model for the hot spots and lobes has withstood the test of time:

With only the HR74 map of Cyg A to go on, they deduced that FR II sources

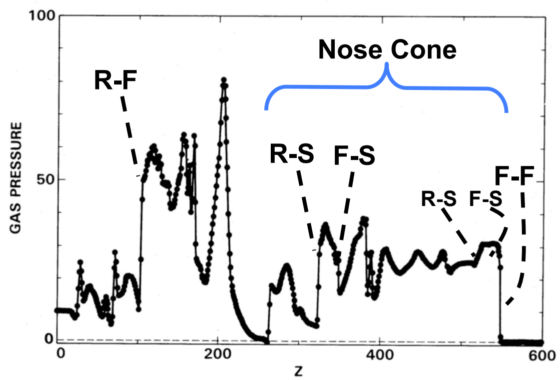
- were powered by jets
- produced a strong reverse "Mach disk" (hot spot) & forward "bow" shocks
- produced a hot cocoon of post-shock jet material that surrounded the jet
- **FR II sources, therefore look like hydrodynamic (HD) jets**

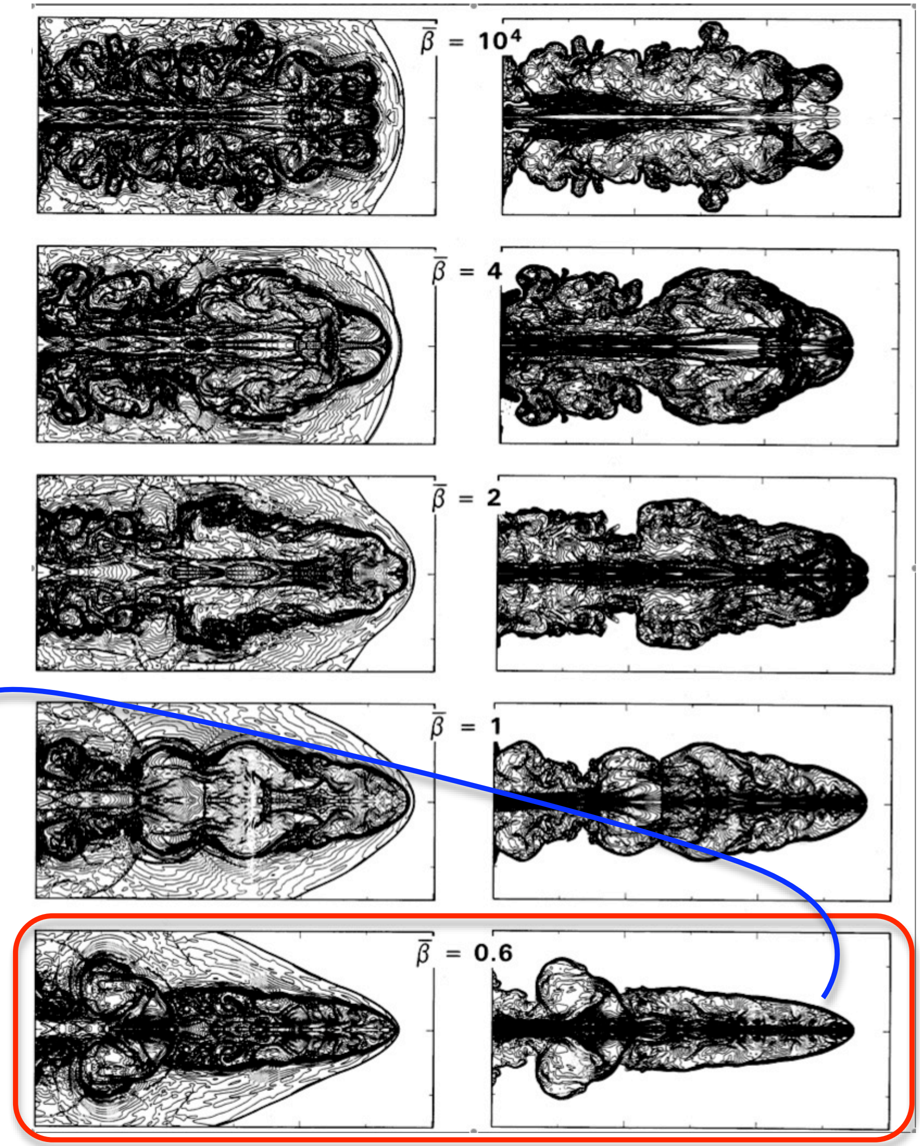


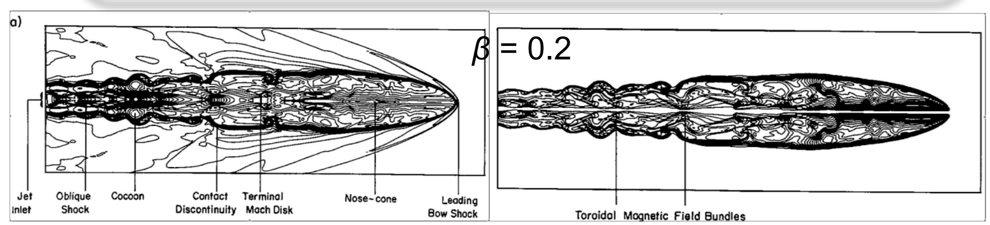
Simulations of ***MHD*** Jets

- 1^{st} 2-D simulations of magnetized jets performed in 1980s:
  - Lind, Payne, Meier, Blandford (1989)
  - Clarke, Norman, Burns (1986)

- Results
  - Jets with high $\beta_p = p_{gas}/p_{mag} \approx \frac{2}{3}(B_{eq}/B)^2 >> 1.0$ (HD jets) look like an FR II
  - Jets with low β (< 2.0) develop fast, leading "nose cones", forced forward by a strong toroidal magnetic field
  - Nose cone contains several slow shock pairs



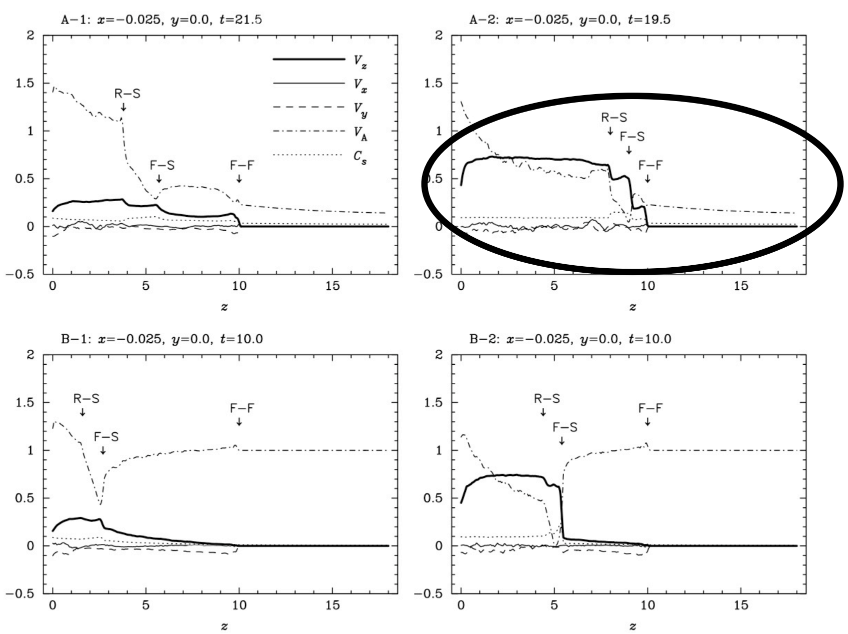


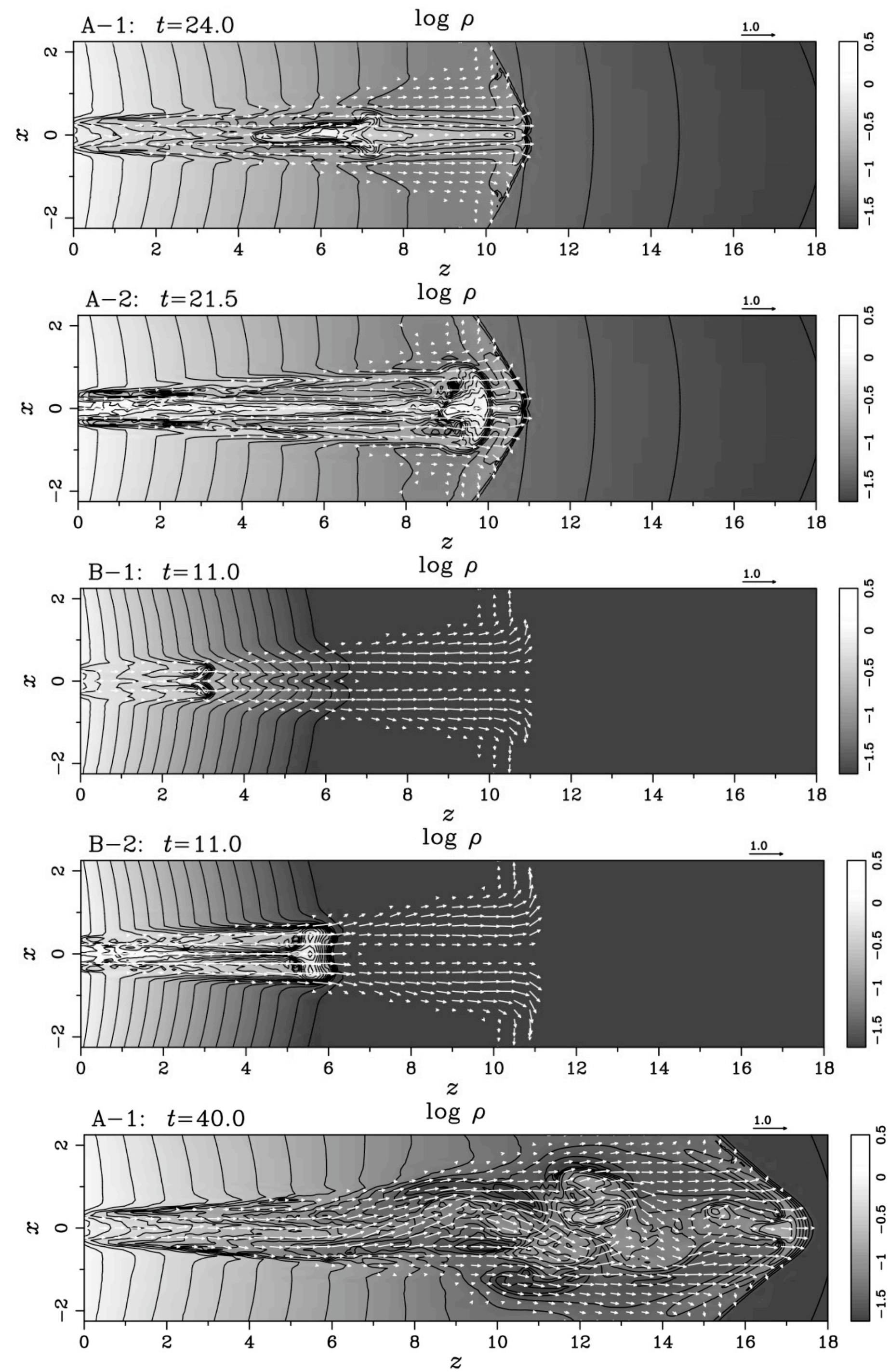


  - In general, an FR II radio source does NOT have the morphology of a magnetized jet with an equipartition ***toroidal*** magnetic field
  - **How about FR I sources?**

MHD Simulations of Jets (cont.)

- 3-D simulations of magnetized jets:
  - Nakamura & Meier (2004)
- Results
  - Even 3-D simulations show a shock system plus nose cone-like structure
  - "Nose cones" are facilitated by
  - Poynting flux domination (1; LPMB; Komissarov)
  - Steeper external pressure gradient (B)
  - At late times the slow shock pair develops a kink instability between the slow-mode shocks





Consequences of FR I Lobe Morphology: The Case of M87

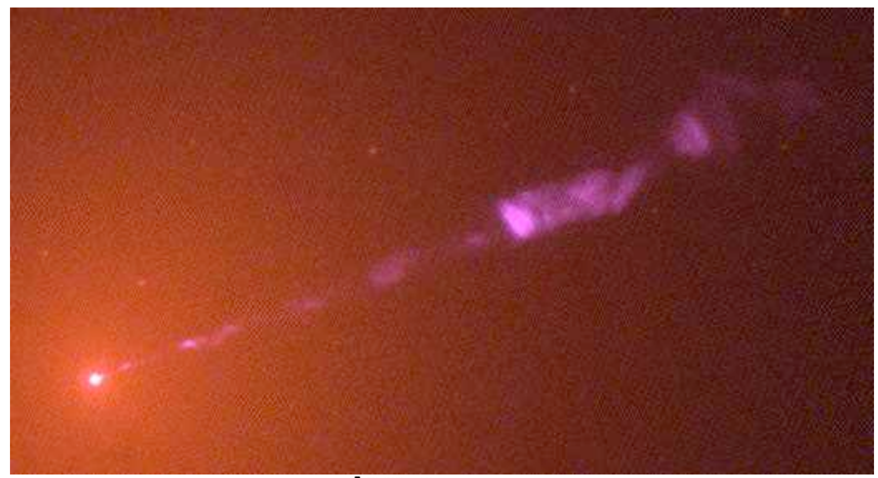
- If FR II jets are supersonic *hydrodynamic* jets, then what are FR I sources?
  - Model #1: Transonic FR II flows that spontaneously decelerate, inflate, decelerate, … (Bicknell 1985, 1995)
  - Model #2: Modest Mach number flows that decelerate and inflate by interacting with an external shock (Norman, Burns, Sulkanen 1988)
  - Model #3: *Magnetically* dominated jets that never became fully kinetic (Nakamura, Garofalo, Meier 2009)

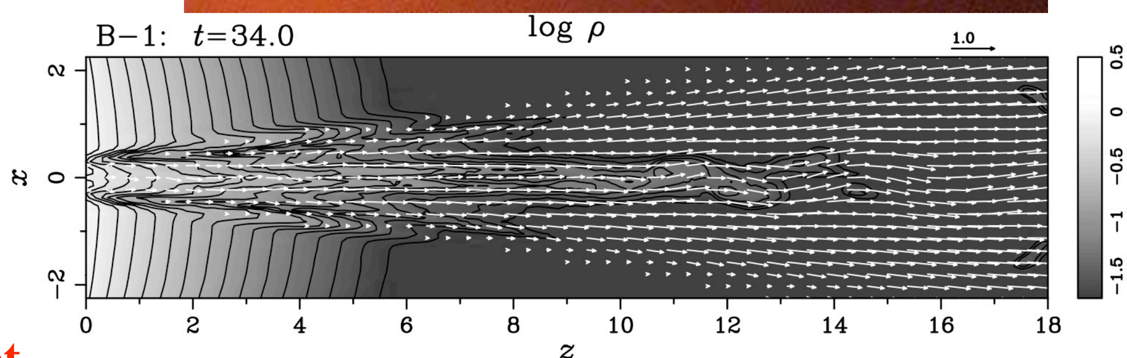
--- an extraordinary, detailed, paradigm-shifting, model

Motivation for MHD model:

- Two knots have very strong measured magnetic fields:
  - Knot HST-1 (Perlman et al. 2003; 10 mG)
  - Knot A (Stawarz et al. 2005; 100 μG < B < 1 mG)
- Inter-knot jet particle pressure << ambient external pressure (Sparks et al. 1996)
- A ⇔ C helical kink

⇒ **strong magnetic forces in jet**

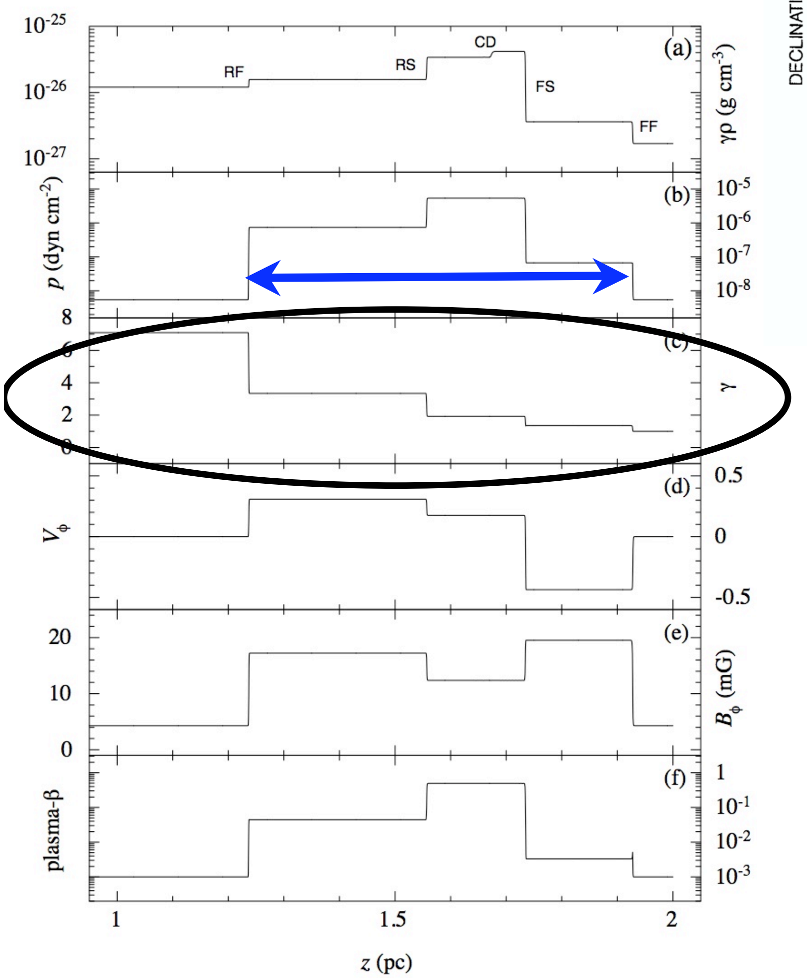


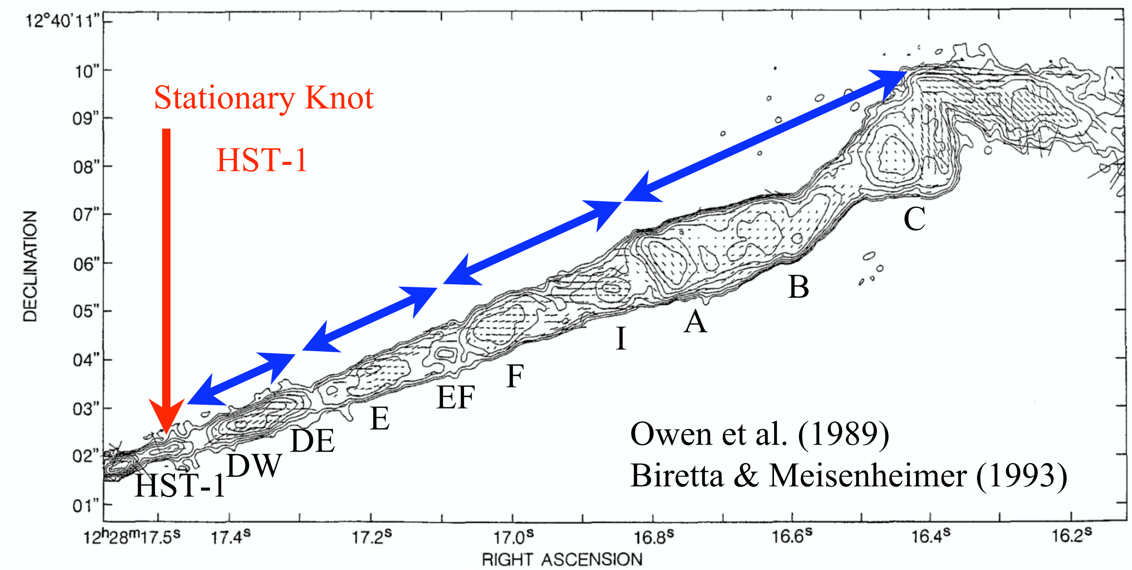


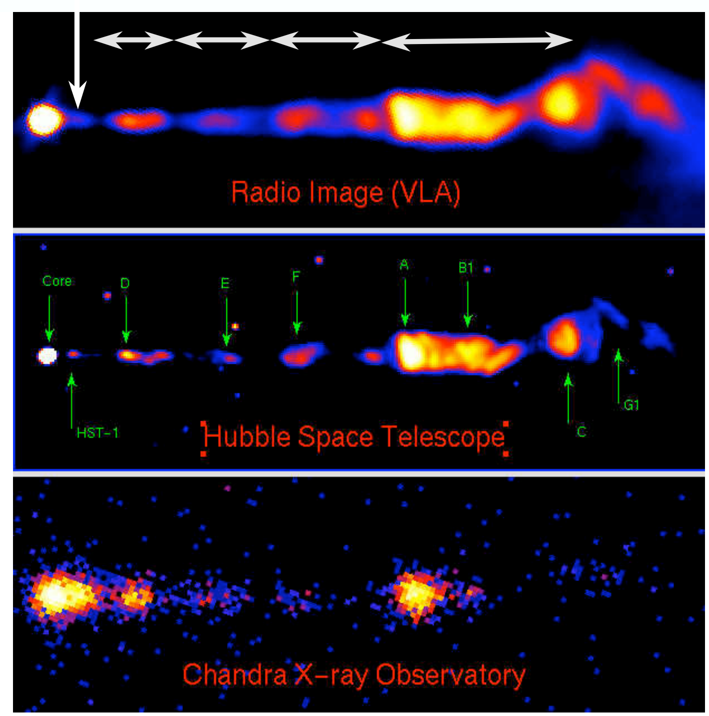
The M87 Jet as a Poynting-Dominated Flow

1-D Super-Fast, 4-Shock

MHD M87 *Jet* Simulation







HST-1

Sometime between 2005 December and 2006 February, the knot HST-1c split into two approximately equally bright features: a faster moving component (c1; $4.3c \pm 0.7c$) and a slower moving trailing feature (c2; $0.47c \pm 0.39c$).

HST-1 is essentially stationary ($< 0.25c$), and it appears to be the source of ***successive components, each of which splits into forward/reverse bright knots downstream***

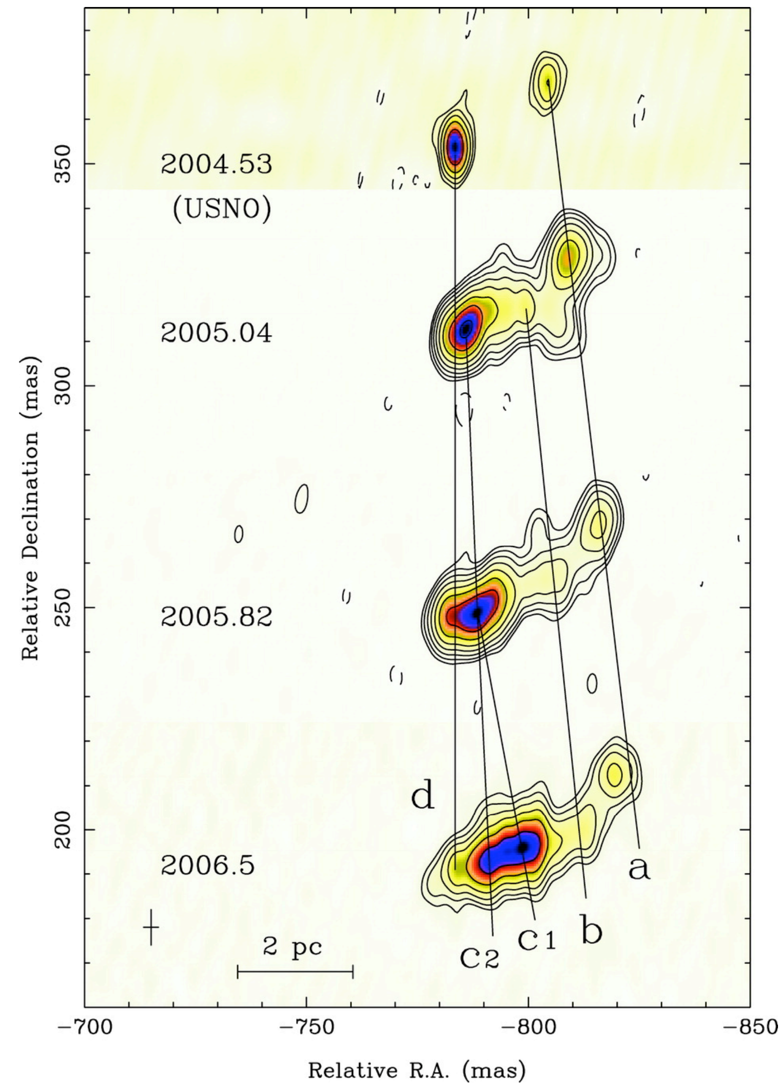
Our proposed model:

The superluminal components in M87 jet are

- ***relativistically propagating internal MHD shock fronts (not "blobs")***
- ***ejected from HST-1, not from the core itself!***

(Nakamura, Garofalo, & Meier 2009)

Working on complete M87 model: more papers to come …



Cheung, Harris, & Stawarz (2007)

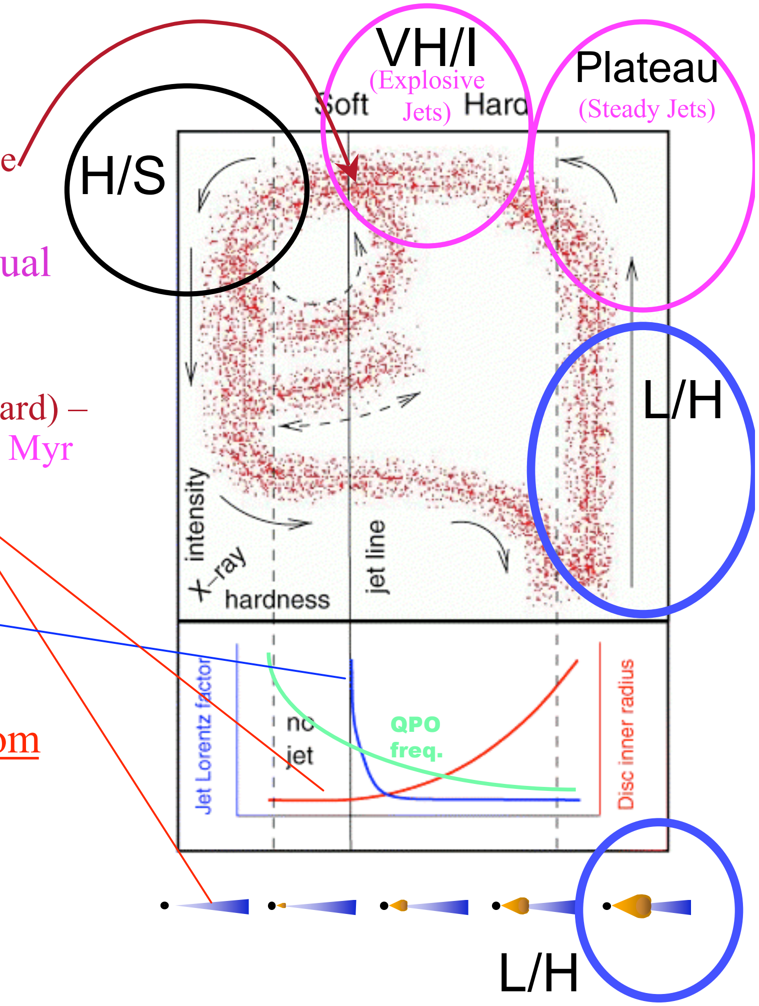
LAUNCHING JETS FROM THE <u>HARD STATE</u> ACCRETION DISK: HOW DOES A LOW-LUMINOSITY ACCRETION FLOW SET UP A STRONG POLOIDAL MAGNETIC FIELD AND LAUNCH A JET?

Fragile & Meier

Very Important Synthesis of Jet-Disk Connection (BH Accretion States DO MATTER)

- A new and very important color-magnitude plot: the FBG diagram (Fender, Belloni, Gallo 2004) for jet-producing binary X-ray sources
  - Like the HR diagram, but in X-rays, and color axis is reversed
  - HIGH and LOW refer to 2-10 keV X-ray flux
  - High/Soft state at upper left
  - Low/Hard state at lower right
  - Jet states are at top and right
  - Explosive jets occur only on transition from Hard state to Soft state
- A tremendous amount has been learned recently about how actual observed accreting black hole systems behave when they are producing jets
  - Black holes follow a prescribed path on the X-ray color (soft vs. hard) – magnitude (low vs. high intensity) diagram: takes days/hours, not Myr
  - Inner radius of cool disk decreases as spectrum becomes softer (the "truncated disk model")
  - Jet velocity increases as disk spectrum becomes softer
- Implication: In hard state, slow ($\Gamma < 2$) jet is NOT launched from vicinity of black hole, but from 10 -100 r_g instead

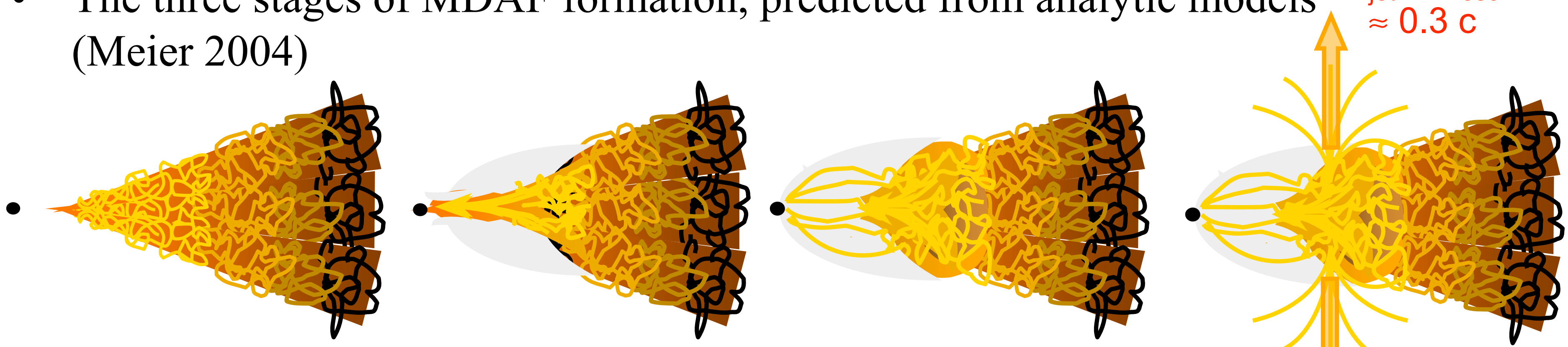
Color-magnitude diagram of X-ray source evolution, and QPO and jet production [Fender, Belloni, & Gallo 2004]



Studies of Cooled Black Hole Accretion Flows and Possible Development of Jet-Producing Magnetospheres

(Fragile & Meier 2009)

- The three stages of MDAF formation, predicted from analytic models (Meier 2004)



Regular non-radiative (ADAF) flow forms for $r > \sim 100\ r_g$. Relativistic electron synchrotron and Compton cooling are important when $T_e > \sim 10^{9.7}$ K ($r < \sim 100\ r_g$)

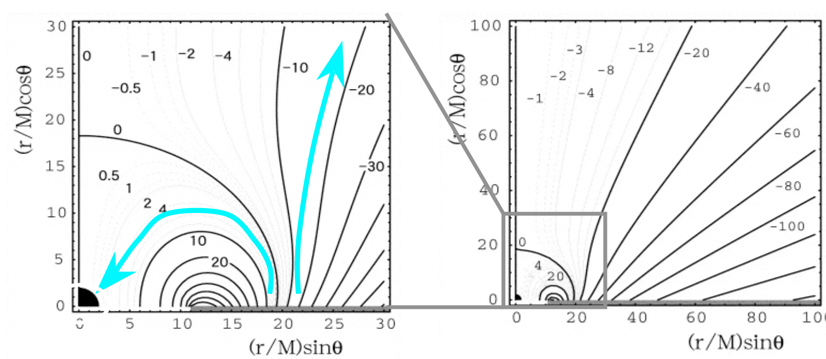
Cooling will reduce thermal pressure and therefore reduce plasma $\beta \equiv p_{gas} / p_{mag} \Rightarrow < 1$

(magnetically dominated)

Magnetic domination will turn off magneto-rotational instability that drives MHD turbulence, creating an inward-facing corona.

Open field lines will create conditions conducive to driving jets, but from outer edge of MDAF and maybe rotational QPOs.

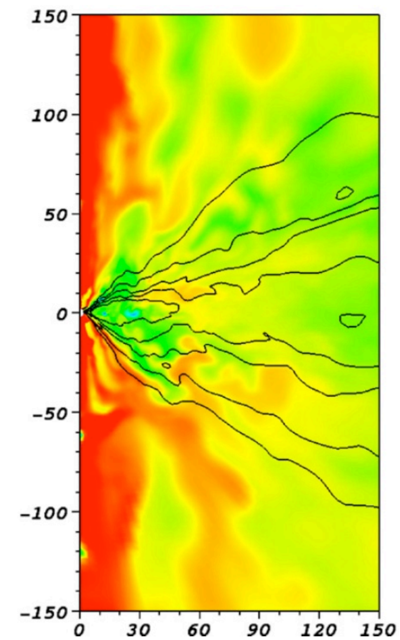
- Final structure should look similar to "black hole magnetosphere" models of Tomimatsu & Takahashi (2001) and Uzdensky (2004)

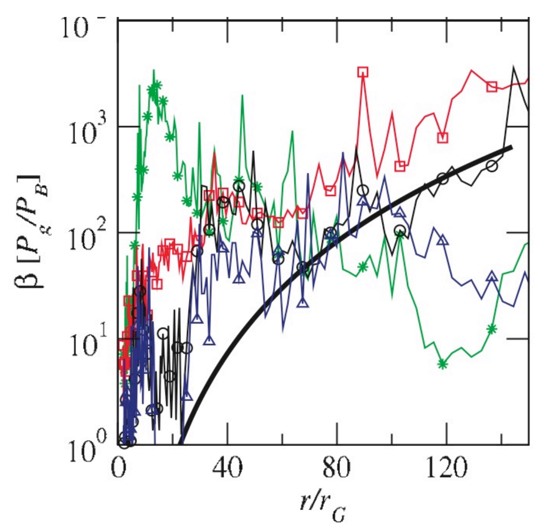


Tomimatsu & Takahashi (2001); Uzdensky (2004)

Studies of Cooled Black Hole Accretion Flows and Possible Development of Jet-Producing Magnetospheres
(continued)

- We are using Chris' COSMOS++ code (Anninos, Fragile, & Salmonson 2005) to test out each stage of this model
- We added Esin et al. (1996) cooling functions (Bremsstrahlung, Synchrotron, Comptonization) to COSMOS++
- Results
  - Cooling Does, indeed, produce a strong-field accretion flow ($\beta \rightarrow 1$) as flow approached black hole
  - Results quantitatively agreed with analytic predictions ("transition region" solution)
  - Choice of parameters did not allow strong MDAF, so now tooling up for $\beta < 1$ simulations
- Questions for new simulations
  - Does a true black hole magnetosphere form in some circumstances? If so, when and what controls its formation?
  - Do the resulting rotating black hole magnetospheres say anything about jet launching from hard state objects?





CONSTRAINED TRANSPORT:
A DISCRETE DIFFERENTIAL GEOMETRY ON A 4-DIMENSIONAL MANIFOLD

TOWARD

A FULLY INTEGRATED METHOD

FOR SIMULTANEOUSLY EVOLVING

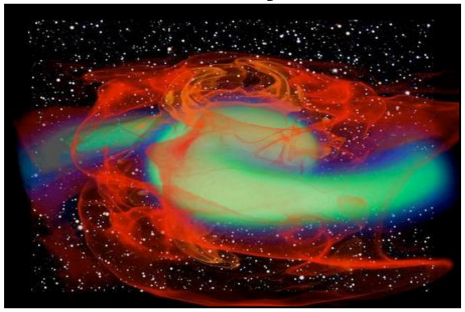
GR AND E&M FIELD PROBLEMS

ALONG WITH THEIR CONSTITUENT CONSERVATION LAWS

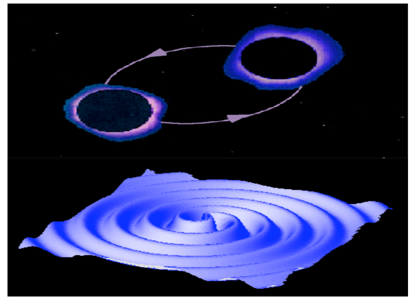
Meier (& Miller)

The Ultimate Goal: Simulate EM Gravitational Collapse

Neutron Star Binary Coalescence



Black Hole Binary Coalescence



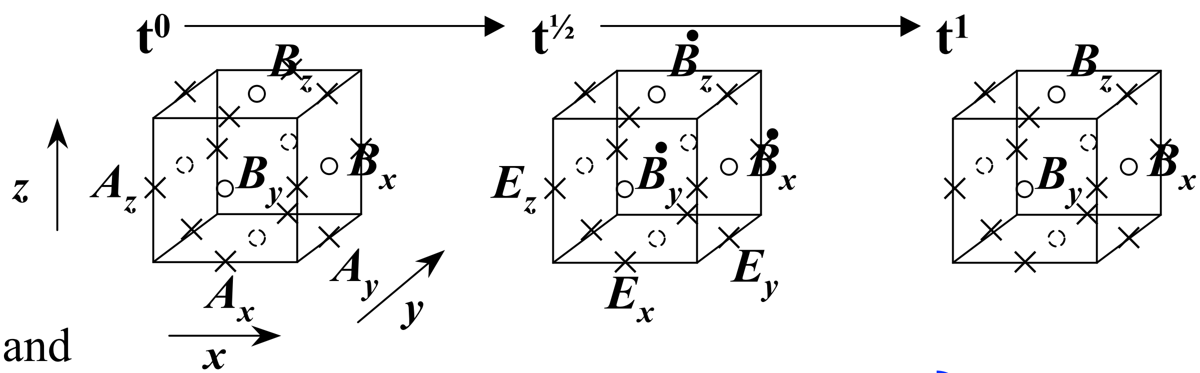
$\Rightarrow$

To solve the problem of Electromagnetic Gravitational Collapse, we need to evolve both the gravitational and electromagnetic fields and their sources (matter and charge)

Physics	Non-Relativistic Equations	Relativistic Equations
Gravity Field	$\nabla^2\psi = 4\pi G\rho$ $\quad [\,\psi = GM\_/r\,]$	$\mathbf{G} = 8\pi G\,\mathbf{T}/c^4$
Matter	$\partial\rho/\partial t + \nabla\cdot(\rho V) = 0$ $\partial(\rho V)/\partial t + \nabla\cdot(\rho V V) = -\nabla p + \boldsymbol{J}\times\boldsymbol{B}/c - \rho\nabla\psi$ $\partial(\rho\, e)/\partial t + \nabla\cdot(\rho\, e\, V) = -(p + e)\nabla\cdot V$	$\nabla\cdot(\rho\,\mathbf{U}) = 0$ $\nabla\cdot\mathbf{T} = 0$
EM Field	$\partial\boldsymbol{B}/\partial t + c\,\nabla\times\boldsymbol{E} = 0 \qquad \nabla\cdot\boldsymbol{B} = 0$ $\partial\boldsymbol{E}/\partial t - c\,(\nabla\times\boldsymbol{B}) = -4\pi\,\boldsymbol{J} \qquad \nabla\cdot\boldsymbol{E} = 4\pi\,\rho_q$	$\nabla\cdot{}^*\mathbf{F} = 0$ $\nabla\cdot\mathbf{F} = 4\pi\,\mathbf{J}/c$
Charge/Current	$\partial\rho_q/\partial t + c\,\nabla\cdot\boldsymbol{J} = 0$ $\boldsymbol{E} = -V\times\boldsymbol{B}/c$ (Ohm's law $\sigma \equiv 1/\eta \rightarrow \infty$)	$\nabla\cdot\mathbf{J} = 0$ $\mathbf{U}\cdot\mathbf{F} = ??$

Constrained Transport for MHD
(Evans & Hawley 1988)

- MHD constrained transport:
  evolving $\dot{\boldsymbol{B}} = -\mathrm{c}\,\nabla\times\boldsymbol{E}$ automatically maintains the constraint $\nabla\bullet\boldsymbol{B} = 0$.
- To do this, one staggers the grid in space *and time*



  – At $t = t^0$, $\boldsymbol{B} = \nabla\times\boldsymbol{A}$ and

$$
\begin{aligned}
\nabla\bullet\boldsymbol{B} &= B_x^{\,+} - B_x^{\,-} + B_y^{\,+} - B_y^{\,-} + B_z^{\,+} - B_z^{\,-} \\
&= (A_{z4}-A_{z2}) - (A_{y4}-A_{y2}) - (A_{z3}-A_{z1}) + (A_{y3}-A_{y1}) \\
&\quad + (A_{x4}-A_{x2}) - (A_{z4}-A_{z3}) - (A_{x3}-A_{x1}) + A_{z2}-A_{z1}) \\
&\quad + (A_{y4}-A_{y3}) - (A_{x4}-A_{x3}) - (A_{y2}-A_{y1}) + (A_{x2}-A_{x1}) = \nabla\bullet\nabla\times A = \mathrm{O}(\varepsilon_r)
\end{aligned}
\Bigg\}\ \textbf{Centrally-differenced diergence}
$$

  – Then at $t = t^{1/2}$, similarly, $\nabla\bullet\nabla\times\boldsymbol{E} = \mathrm{O}(\varepsilon_r)$, so $\nabla\bullet\dot{\boldsymbol{B}} = \mathrm{O}(\varepsilon_r)$

  – So, at $t = t^1$,

$$\nabla\bullet\boldsymbol{B}^1 = \nabla\bullet\boldsymbol{B}^0 + \Delta t\,(\nabla\bullet\dot{\boldsymbol{B}}^{1/2}) = \mathrm{O}(\varepsilon_r)$$

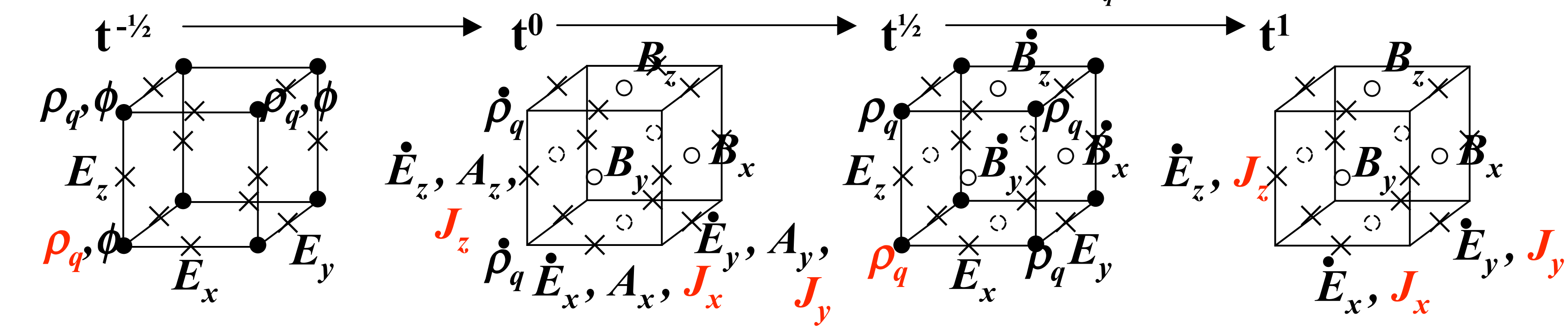
- So, if we put $\boldsymbol{A}$ on cube edges at $t=t^0$ and $\boldsymbol{E}$ on cube edges at $t=t^{1/2}$, <u>we can evolve $\boldsymbol{B}$ forward in time and keep $\nabla\bullet\boldsymbol{B} = \mathrm{O}(\varepsilon_r)$ *without any additional effort*</u>

CT for Electrodynamics (Yee 1966)

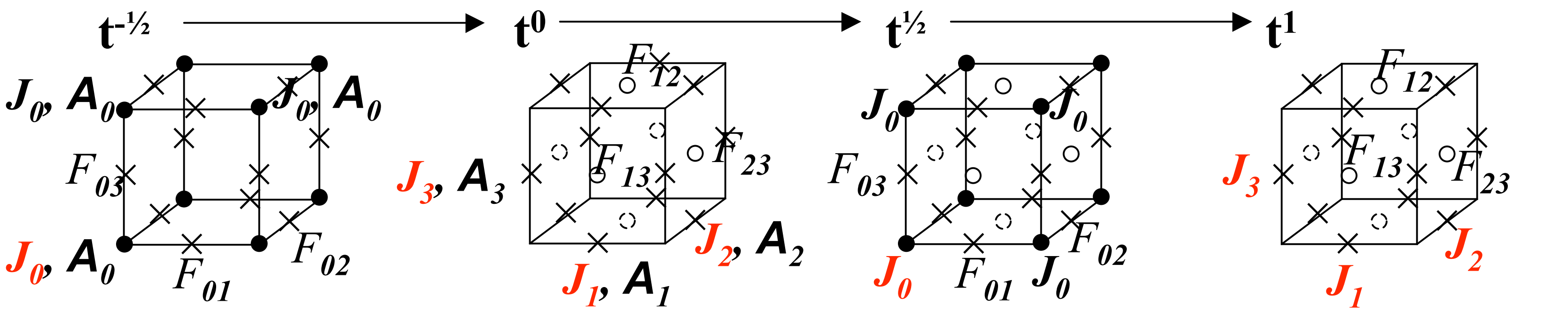
- Actually, a <u>more complete version of CT was invented by an ***engineer***</u>:
  the FDTD (finite-difference time-domain) algorithm used in antenna design and analysis
- In full electrodynamics, ***both*** of Maxwell's equations and ***both*** of the constraints must be propagated:

$$\dot{\boldsymbol{B}} = -c\,\nabla\times\boldsymbol{E} \qquad \nabla\bullet\boldsymbol{B} = 0$$
$$\dot{\boldsymbol{E}} = c\,\nabla\times\boldsymbol{B} - 4\pi\boldsymbol{J} \qquad \nabla\bullet\boldsymbol{E} = 4\pi\rho_q$$

  creating the need for three interlaced updates (one each for $\boldsymbol{E}$, $\boldsymbol{B}$, **and** ρ_q):

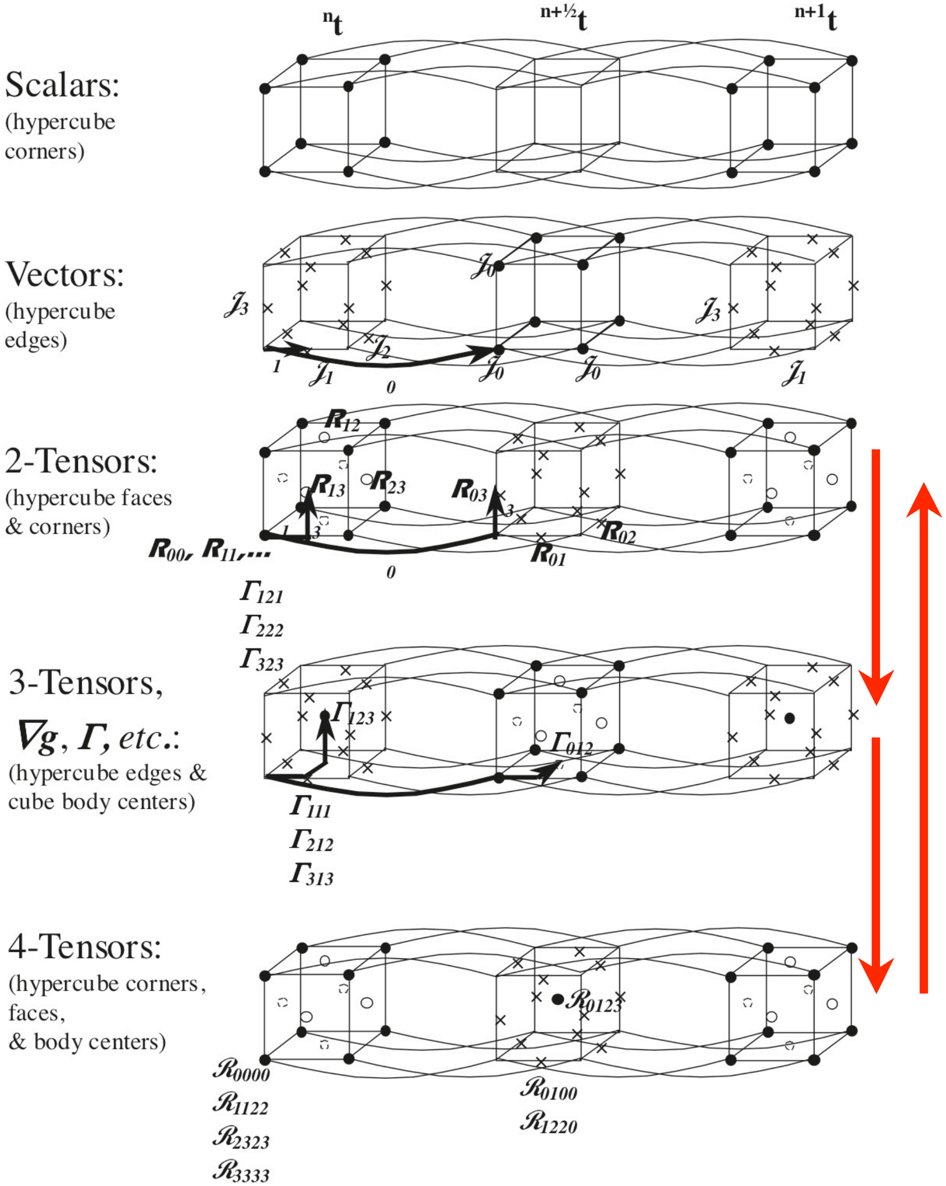


- In 4-D form, the Yee algorithm looks much simpler:

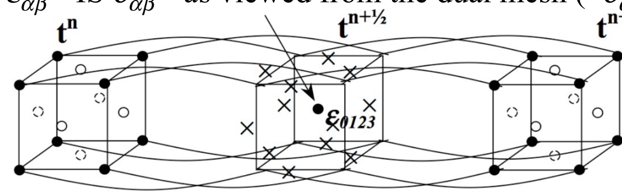


- Staggering the grid implicitly <u>satisfies the Bianchi identities</u> ($\nabla\times\nabla\phi = 0$; $\nabla\bullet\nabla\times\boldsymbol{A} = 0$) to machine accuracy, and <u>this implicitly transports the constraints</u>
- Furthermore: the <u>law of conservation of charge</u> ($\dot{\rho}_q = -\nabla\bullet\boldsymbol{J}$) <u>must be solved in a staggered grid manner in order to properly transport the inhomogeneous constraint and solve the E & M field</u>

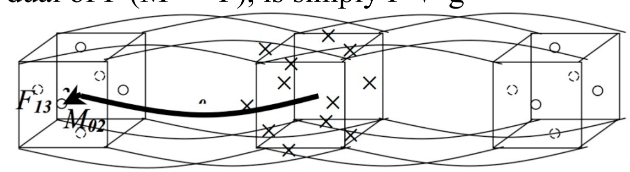
Staggered Grids in GR: A Centered-Differenced Discrete Differential Geometry (Meier 2004)



- **The electrodynamics CT problem suggests a natural, simple, and elegant method for staggering finite difference grids in 4-D**
- Special cases have interesting forms
  - Kronecker delta ($\delta_\alpha{}^\lambda$): $4 \times$ 1s at cell corners; $12 \times$ 0s at cube faces
  - Other identity tensors ($\delta_{\alpha\beta}{}^{\lambda\mu}$, $\delta_{\alpha\beta\gamma}{}^{\lambda\mu\nu}$): ± 1 at corners; 0 otherwise
  - Levi-Civita tensor ($\varepsilon_{\alpha\beta\gamma\delta}$): $\pm\sqrt{-g}$ at hypercube body centers; 0 otherwise
- Gives rise to the concept of a dual mesh
  - Shift origin to hypercube-centered point to create the dual mesh
  - $\varepsilon_{\alpha\beta}{}^{\lambda\mu}$ IS $\delta_{\alpha\beta}{}^{\lambda\mu}$ as viewed from the dual mesh ($*\delta_{\alpha\beta}{}^{\lambda\mu}$)



  - As viewed from the dual mesh the Maxwell tensor, the dual of F (M = *F), is simply F $\sqrt{-g}$



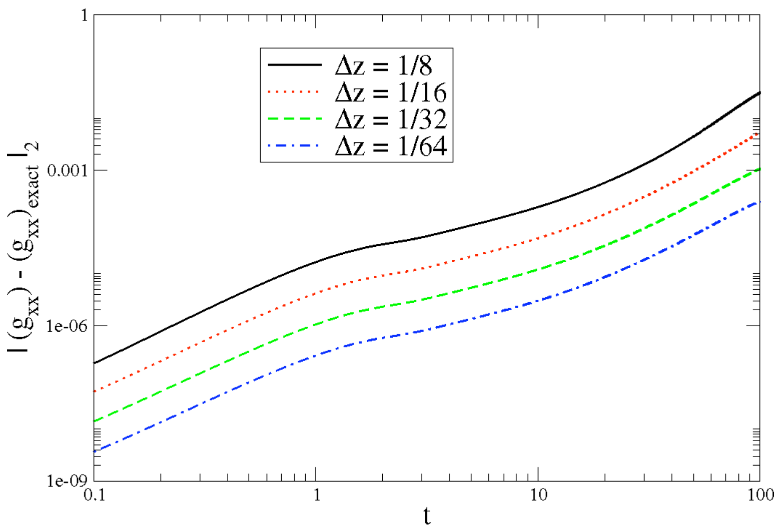
- **To paraphrase J. Wheeler, "A staggerd grid has deep geometric significance"**

Notes on Bianchi Identities in CT for EM & GR

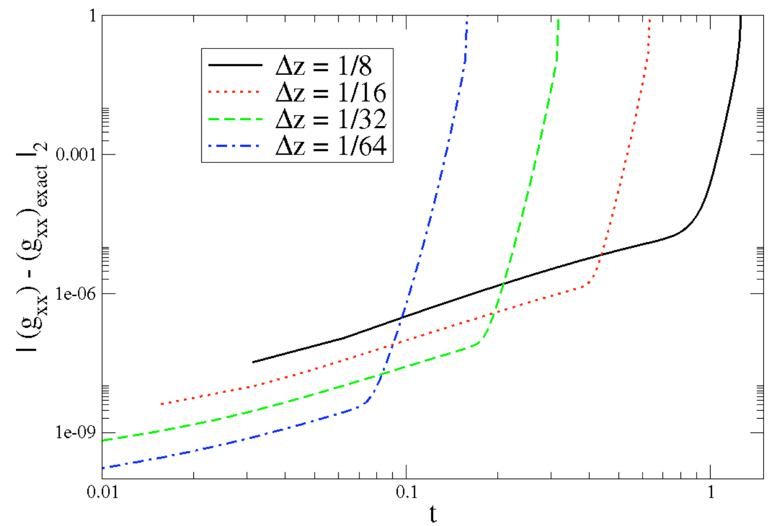
- Centrally-differenced CT is
  - Exact for E & M — to machine accuracy!
    - $(\sqrt{-g}\,(\sqrt{-g}\,F^{\alpha\beta})_{,\beta})_{,\alpha} = O(\varepsilon_r)$
  - Exact for GR — in *Rieman-normal* coordinates only ($\Gamma = 0$)
    - $R_{\alpha\beta[\gamma\delta\,,\,\varepsilon]} = O(\varepsilon_r)$
  - "Almost" exact for GR — in *global* coordinates
    - $R_{\alpha\beta[\gamma\delta\,;\,\varepsilon]} \sim \partial^2\Gamma + \Gamma\,\partial\Gamma + \Gamma\,\Gamma\,\Gamma$
    - $\Gamma\,\partial\Gamma$ terms <u>*also*</u> commute to machine accuracy!
    - $\Gamma\,\Gamma\,\Gamma$ terms DO NOT commute, BUT THEY NEARLY DO SO with adaptive gridding and high-order differencing in regions of large Γ
- In order to properly transport constraints without losses (to near machine accuracy), we need ALL OF THE FOLLOWING
  - $G^{\alpha\beta} = 8\pi\, T^{\alpha\beta}$
  - $T^{\alpha\beta} = T^{\beta\alpha}$ (**T** symmetry, since grid staggering ensures $G^{\alpha\beta} = G^{\beta\alpha}$)
  - $G^{\alpha\beta}{}_{;\beta} = 8\pi\, T^{\alpha\beta}{}_{;\beta}$
- That is, <u>we need the natural symmetries in all the tensors AND we need to apply the SAME DIVERGENCE OPERATOR to matter and GR fields alike</u>

Practicalities: Does CT Work for the GR Gauge Field?

- **<u>Tests with no sources</u>** (Miller & Meier 2005, unpub) :
  - Diagonal test (metric Gowdy cosmology):
    - CT is stable and convergent, AND nearly equivalent to best finely-tuned methods (BSSN)
  - Off-diagonal test in *Z* (gauge plane wave):
    - CT is stable and convergent
  - Off-diagonal test in *XY* (Bondi plane wave):
    - CT is stable and convergent <u>but only if Christoffel symbols are evolved **as a set in a srongly hyperbolic manner** along with the metric!</u>
  - CT by itself, therefore, does not guarantee stability



Fully Hyperbolic CT:
Evolve Christoffels;
Evolve Metric using Christoffels



Partially Hyperbolic CT:
Evolve Metric; Compute Christoffels Using <u>Centered</u> Spatial Derivatives of Metric

CT with Field Sources

- Maxwell Field equations

$$\nabla \bullet \mathbf{F} = 4\pi\, \mathbf{J}$$

- Field sources

$\mathbf{J}$ = charge-current 4-vector

- Field Bianchi identities

$$\nabla \bullet (\nabla \bullet \mathbf{F}) = 0$$

- Implied conservation law

$$\nabla \bullet \mathbf{J} = 0$$

<u>The conservation of charge is a direct result of E & M</u>

- Einstein Field equations

$$\mathbf{G} = 8\pi\, \mathbf{T}$$

- Field sources

$\mathbf{T}$ = stress-energy-momentum tensor

- Field Bianchi identities

$$\nabla \bullet \mathbf{G} = 0$$

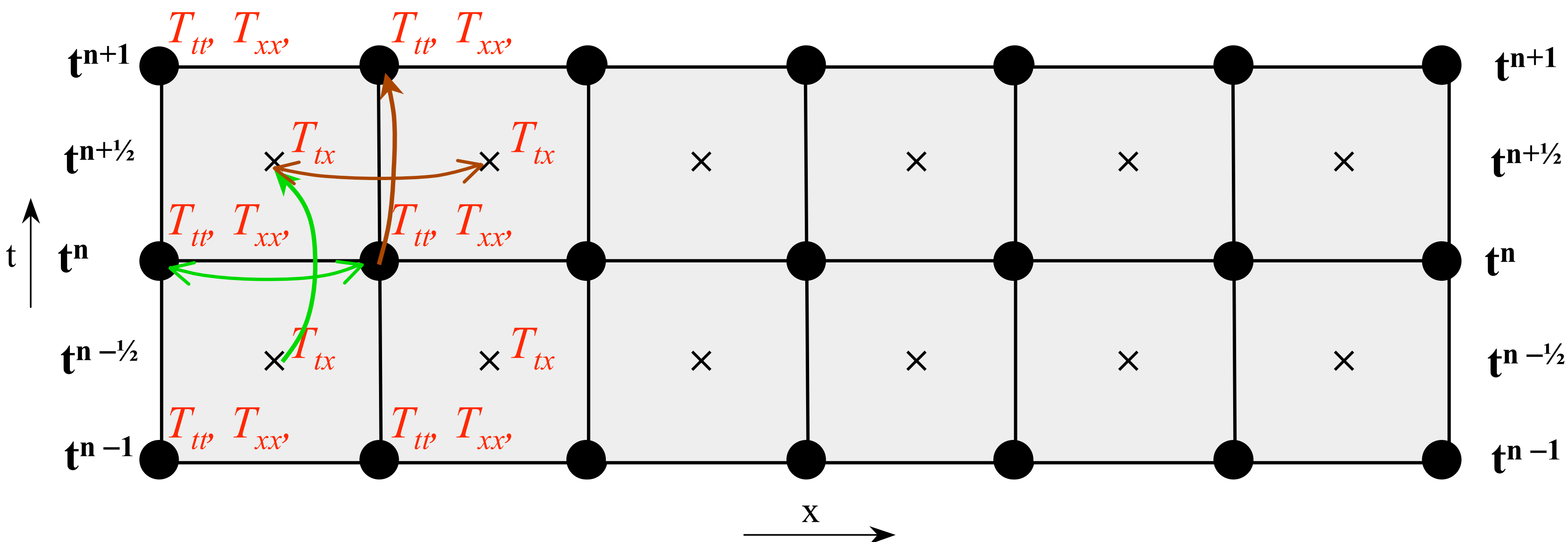
- Implied conservation law

$$\nabla \bullet \mathbf{T} = 0$$

<u>The conservation of momentum and energy is a direct result of GR</u>

And, just as there is a 4-D staggered-grid technique for integrating the conservation of current,
there also should be a 4-D staggered-grid technique for integrating the conservation of energy and momentum.

Staggered-Grid Algorithms for Fluid Dynamics



- **Consider $\nabla \bullet \mathbf{T} = 0$ in flat space**
- $\mathbf{T} = [\rho + (p + \varepsilon)/c^2]\, \mathbf{U}\mathbf{U}^{\mathrm{T}} + p\boldsymbol{\eta}$

$$= \begin{pmatrix} T_{tt} & T_{tx} & T_{ty} & T_{tz} \\ T_{tx} & T_{xx} & T_{xy} & T_{xz} \\ T_{ty} & T_{xy} & T_{yy} & T_{yz} \\ T_{tz} & T_{xz} & T_{yz} & T_{zz} \end{pmatrix}$$

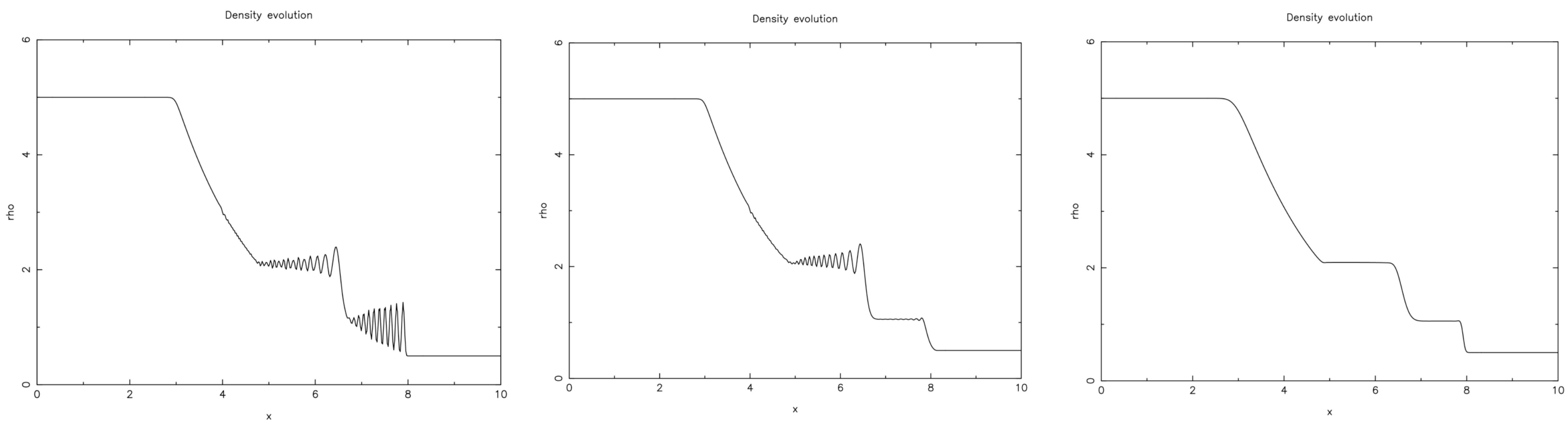
← *Conserved variables*

Fluxes

- Note that the stress-energy-momentum tensor is symmetric
- That is, the energy flux T_{xt} equals the momentum conserved variable T_{tx}
- So, once the T_{ij} are computed, the T_{tj} and T_{tt} updates fshould ollow immediately with no additional effort

Does CT Work for Fluid Dynamics?

- **<u>Tests with no fields</u>** (Meier 2009, unpub) : Simple shock tube
  - Lax-Wendroff test (fully centered differencing):
    - CT works, but contains oscillations both at shock and contact discontinuity
  - Lax-Wendroff plus artificial bulk viscosity:
    - CT works, but <u>oscillations still persist</u> at contact discontinuity
  - Nakamura 2-step hyperbolic scheme (LW+Godonov):
    - Works, but <u>centered differencing must be discarded FOR ALL EQUATIONS.</u>
  - **<u>Just like the field evolution tests, strongly-hyperbolic evolution algorithms must be used for all sets of evolution equations</u>**



Bottom Line: Hyperbolic CT Needed

- As a discrete differential geometry, CT has tremendous power and capability
- Central-differenced CT works fine for evolving the sourceless electromagnetic field
- However, central-differenced CT is unstable in its evolution of the Einstein field and produces undesirable results in the evolution of its conservation laws
- To proceed further we need to completely recast CT with hyperbolic, not central, differential operators
- Mark Miller's 8th-order difference operators look very interesting