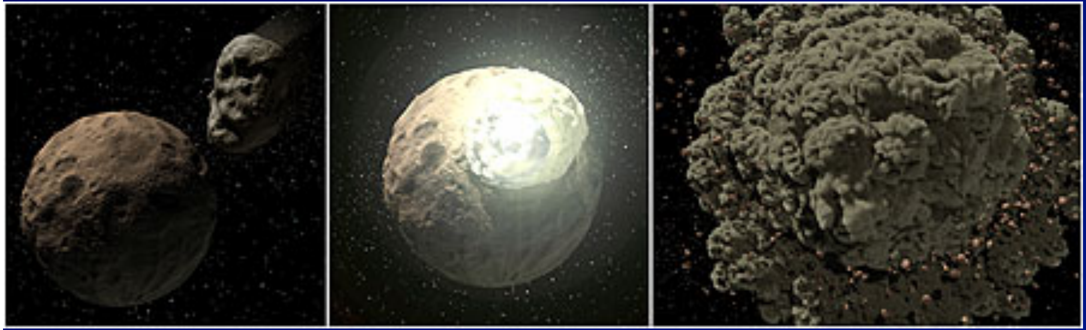
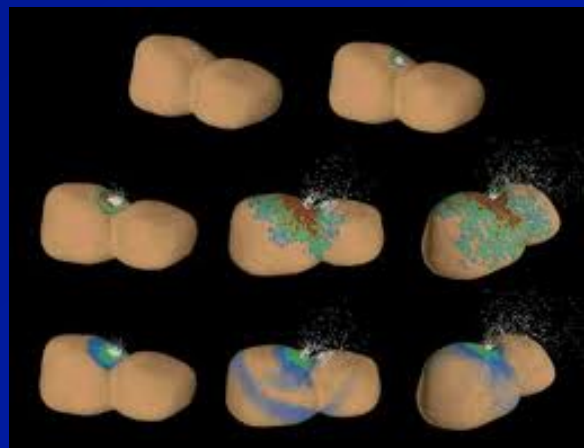
2011 Interdisciplinary Summer School: Granular Flows



IMPACTS

Patrick Michel
Leader of the group of Planetology



Asteroid Mathilde (50 km)

1.3 g/cm^3

C-type
low albedo
(<0.1)

Asteroid Eros (23 km)

2.7 g/cm^3

S-type
high albedo
(> 0.15)

Note: even two bodies of
same spectral type
can be very different!

Asteroid Itokawa (350 m)

S-type

1.9 g/cm^3

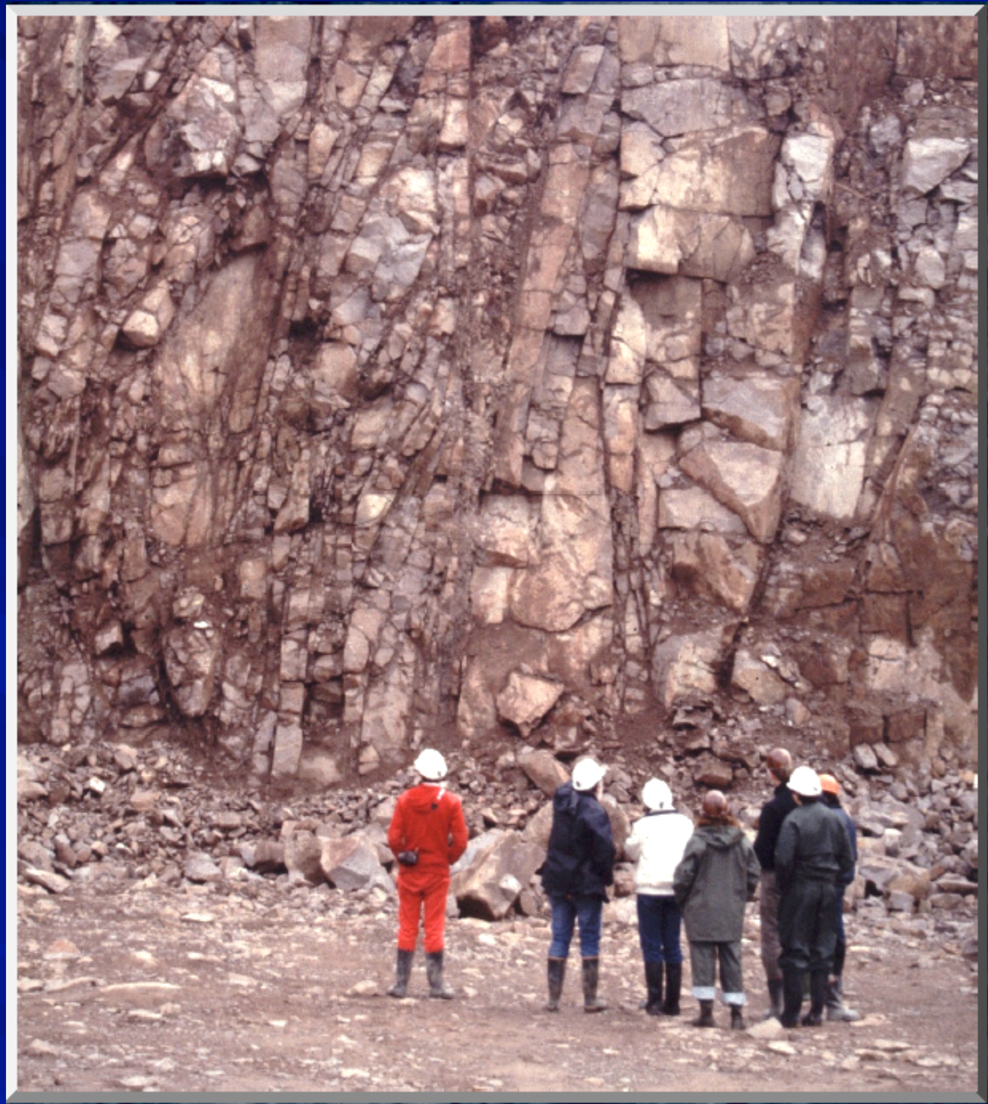
Great diversity of structures

⇒bulk density:
smaller for lower albedo objects

Presence of regolith on all these bodies

Rock fragmentation:

A Modeling Challenge



Numerical methods

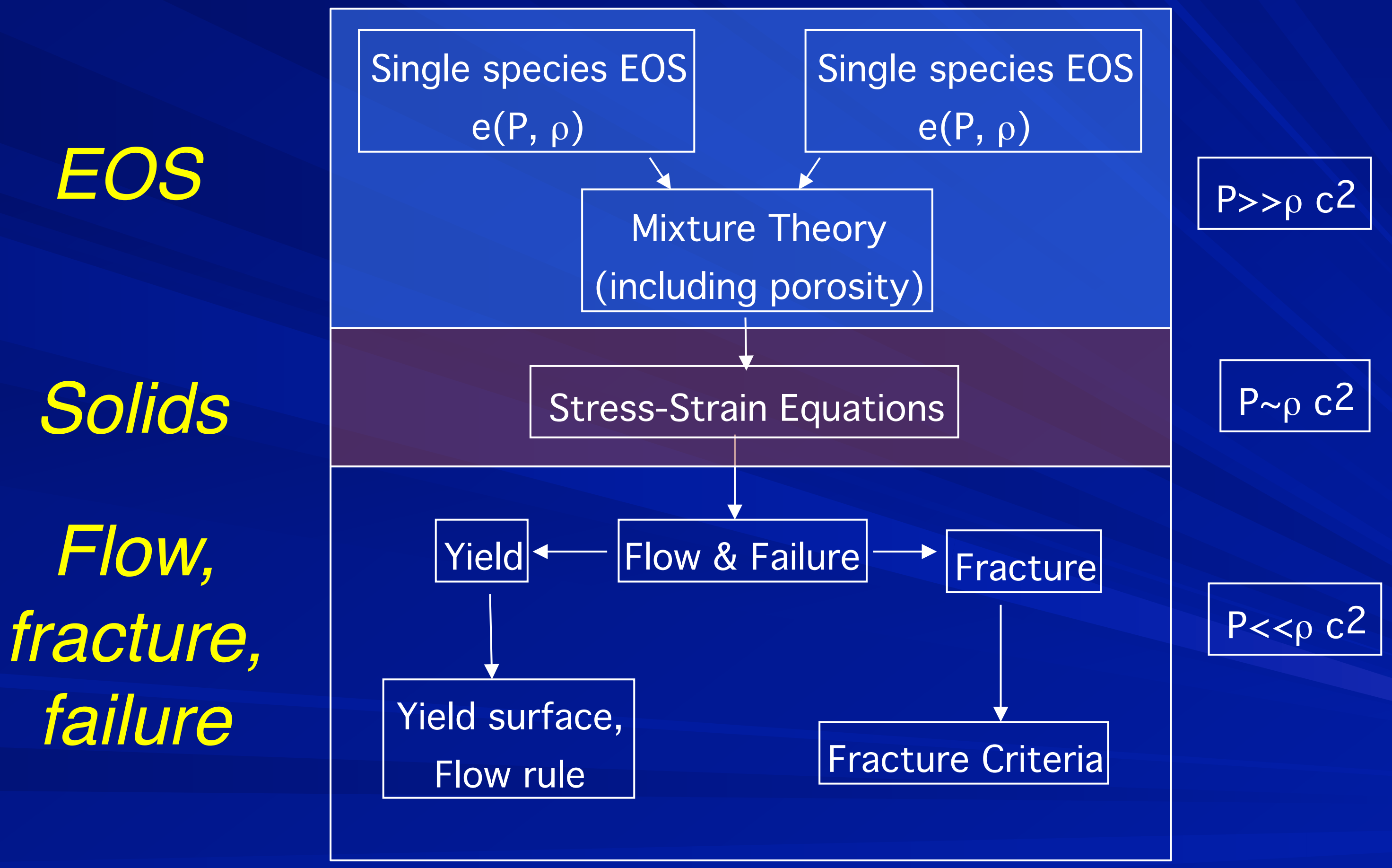
- **Eulerian Hydrocodes based on grid-method**
- **Lagrangian Hydrocodes based on the 3D Smooth Particle Hydrodynamic (SPH) method:**
  - **To simulate non-porous solids, standard SPH was extended to include a strength and fracture model (Benz & Asphaug 1994)**
  - **Recently, porosity models were included based on different relations between state variables (Jutzi et al. 2008, Wunneman et al. 2006, Speith et al. ??).**

Numerical Simulations of the fragmentation phase

- Solve conservation equations (using your favorite numerical method)
  - mass conservation
  - momentum conservation
  - energy conservation
- Define material properties
  - equation of state
  - elasticity/plasticity model
  - damage model
  - NEW: model of microporosity
  - ...
- Testing and testing
  - analytical solutions
  - laboratory experiments
  - code comparisons
  - observations/measurements in situ
  - ...

laboratory experiments
+
characteristics of small bodies

Material Behavior: Three regimes



Equations

1) momentum conservation

$$\frac{d v_i}{d t} = \frac{1}{\rho}\frac{\partial \sigma_{ij}}{\partial x_i} + \frac{\partial \phi}{\partial x_i}$$

stress tensor   self-gravity

with the stress tensor:

$$\sigma_{ij} = -P\delta_{ij} + S_{ij}$$

pressure   deviatoric stresses

2) mass conservation

$$\frac{d\rho}{d t} = -\rho\frac{\partial v_i}{\partial x_i}$$

3) energy conservation

$$\frac{du}{dt} = -\frac{P}{\rho}\frac{\partial v_i}{\partial x_i} + \frac{1}{\rho}S_{ij}\dot{\epsilon}_{ij}$$

PdV term   elastic energy

with the strain rate tensor:

$$\dot{\epsilon}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$$

Equations

4) elasticity: Hooke's law

$$\Delta\frac{l}{l}=\epsilon=\frac{\sigma}{E}$$ *E*: Young's modulus

$$\frac{dS_{ij}}{dt} = 2\mu\left(\dot{\epsilon}_{ij}-\frac{1}{3}\delta_{ij}\dot{\epsilon}_{kk}\right) + S_{ik}R_{jk} + S_{jk}R_{ik}$$

deformation terms rotation terms

with the rotation rate tensor: $R_{ij}=\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right)$

Equations

5) stress limiters

- von Mises (plasticity)

$$\sqrt{3}\,\bar{\sigma} - Y_0 = 0$$

Y_0: Yield strength

$$\bar{\sigma} = \sqrt{\frac{S_{ij} S_{ij}}{2}}$$

equivalent stress

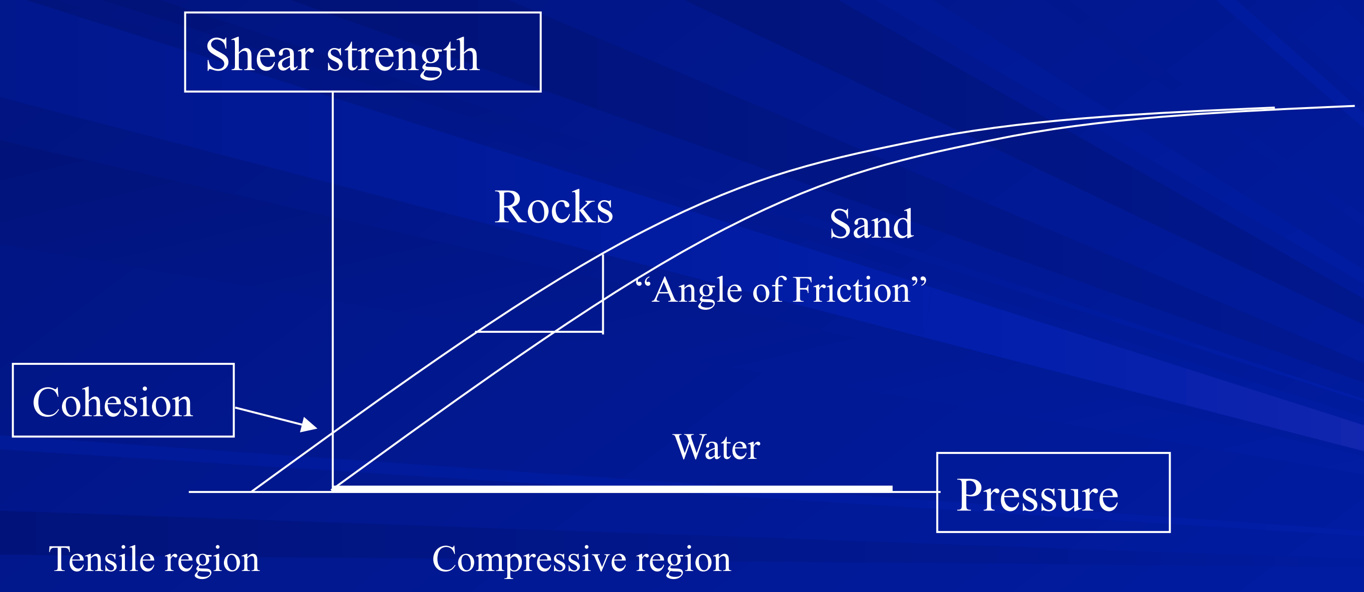
6) equation of state: $P = f(\rho, u, \alpha, x, \ldots)$ with

α: porosity
x: chemical composition

- multi-material
- multi-phase description

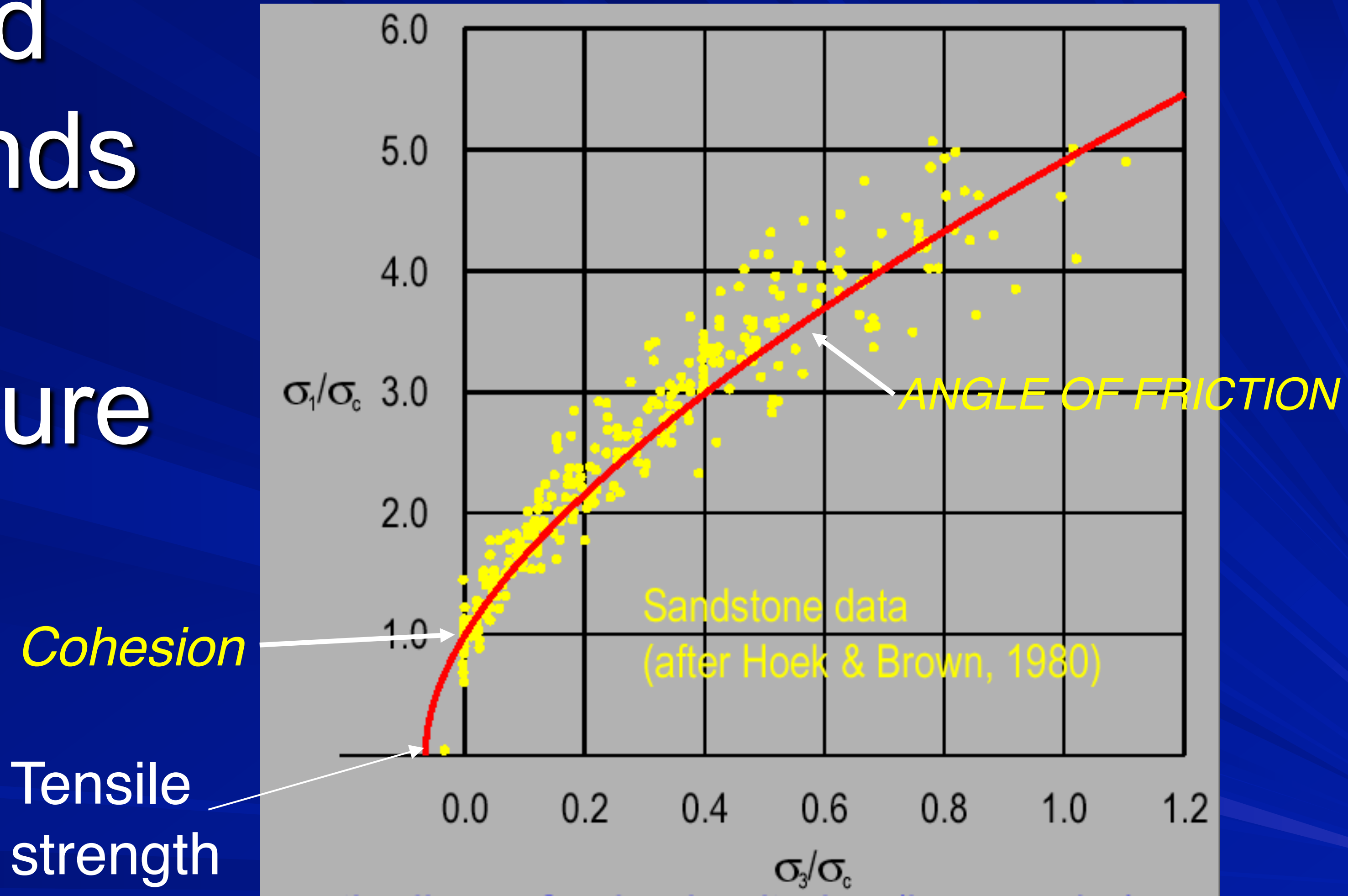
Strength:

- The Mohr-Coulomb (or Drucker-Prager) model:



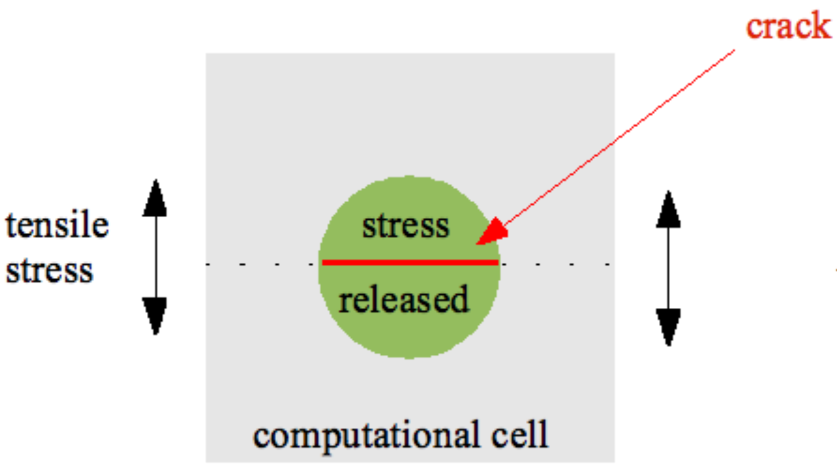
Some real data

Yield depends on pressure



Damage

→ **feed back on dynamics: Damage D**



$$D(t) = \frac{V(t)}{V} = \frac{\frac{4}{3}\pi\left[c_c\left(t-t_0\right)\right]^3}{V}$$

t_0: activation time

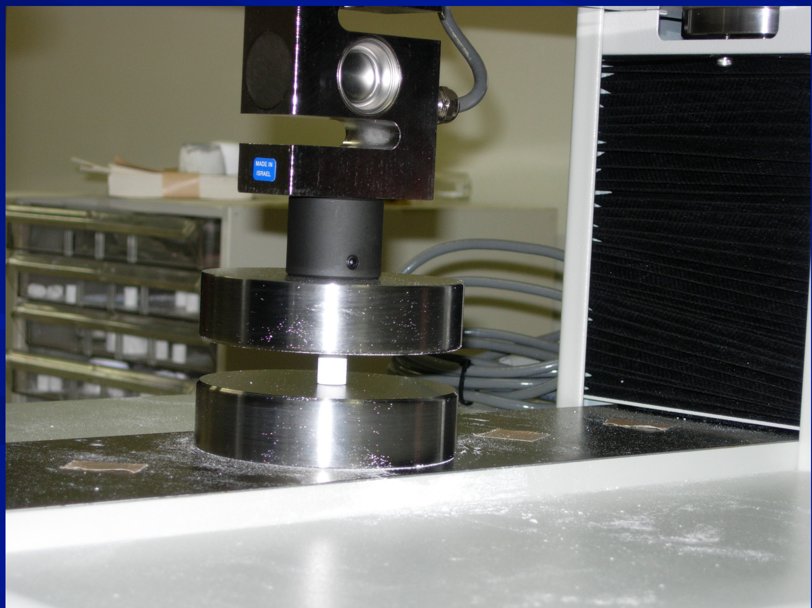
$$0 \leq D \leq 1$$

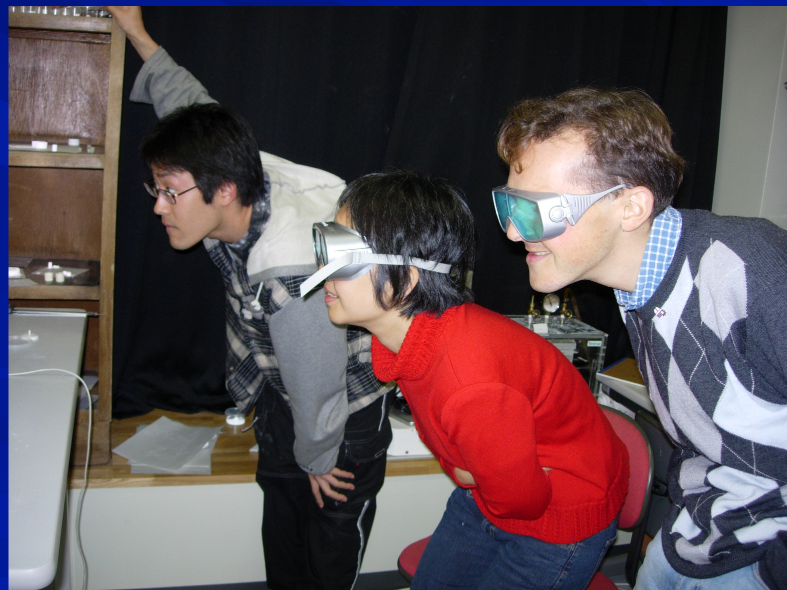
undamaged
full shear and strength

totally damaged
no shear and no strength

Strength

- A rock has each of:
  - Tensile strength
  - Shear strength (cohesion) ~same as tensile
  - Compressive strength ~5-7* tensile





The "F" words: Flow, Fracture and Failure

- Models for these fall into three groups:
  - *"Degraded Stiffness", no explicit flow or fracture.*
  - *"Flow" including plasticity and damage, used to model microscopic voids and cracks leading to an inability to resist stress.*
  - *"Fracture", involving actual macroscopic cracks and voids which are tracked, leading to an inability to resist stress.*

The Grady-Kipp Model

Special nature

- It is a Tensile Brittle Fracture Mechanism
  - For fragmentation in mining
- One-Dimensional Model
- Synthesized for constant strain rate histories only
- Governed by Crack Distributions (Weibull) and growth
- Implies rate and size-dependent strength

But Attractive Physics

There exists an initial distribution of incipient flaws in the target

- Weibull distribution:

$$N(\varepsilon) = k\,\varepsilon^{m}$$

where:

N = density number of flaws activating at or below the strain ε

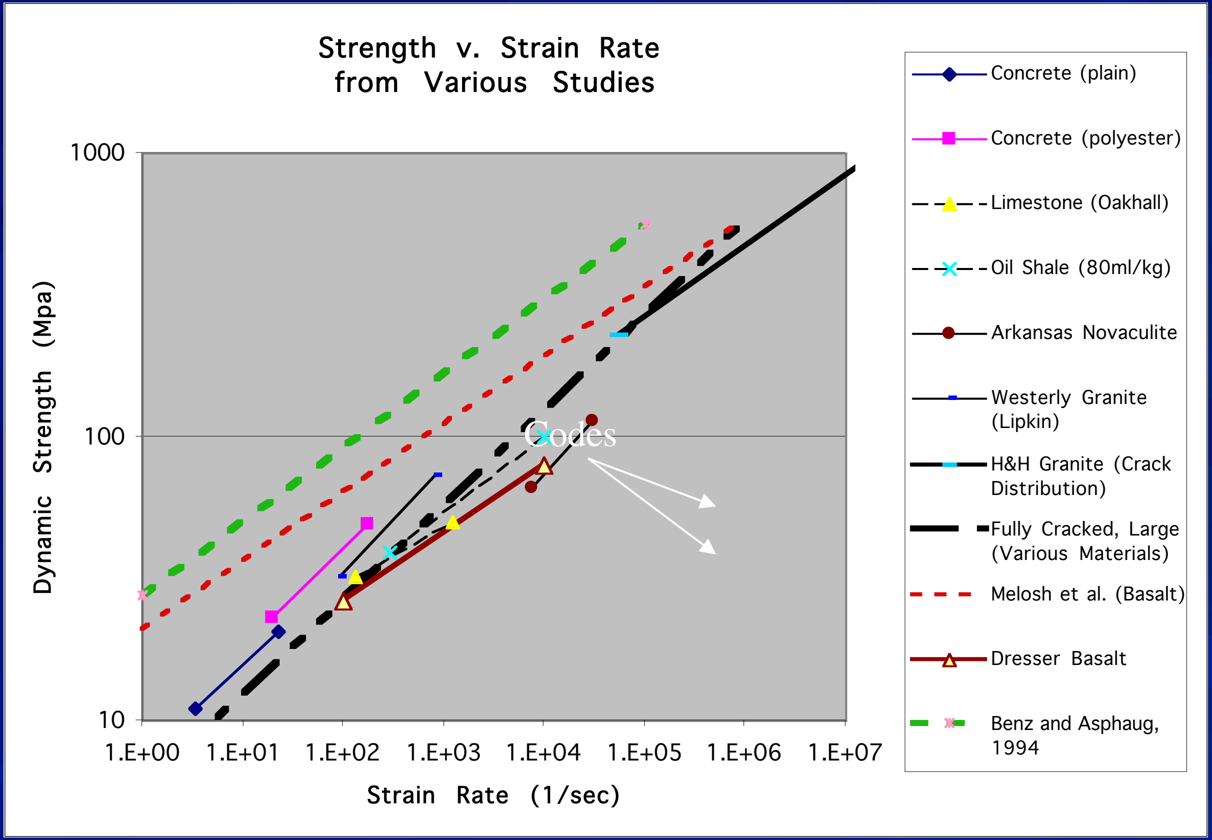
k, m: Weibull parameters (large m= more homogeneous material)

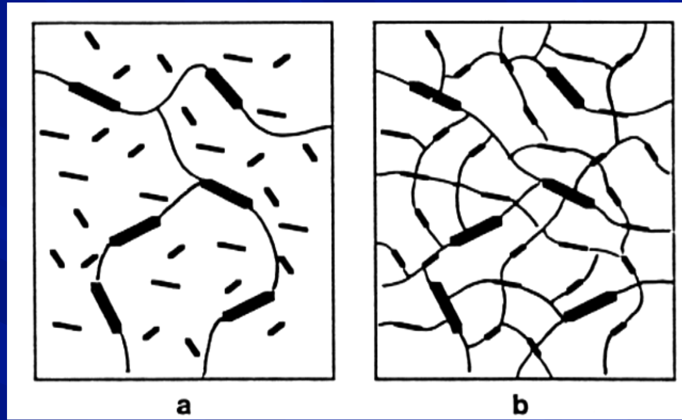
$$\varepsilon_{min} = (1/\,kV)^{-m}$$

Larger targets (volume V) activate largest crack at lower strain

$\Rightarrow$ Larger targets are weaker

Tensile fracture depends strongly on strain rate



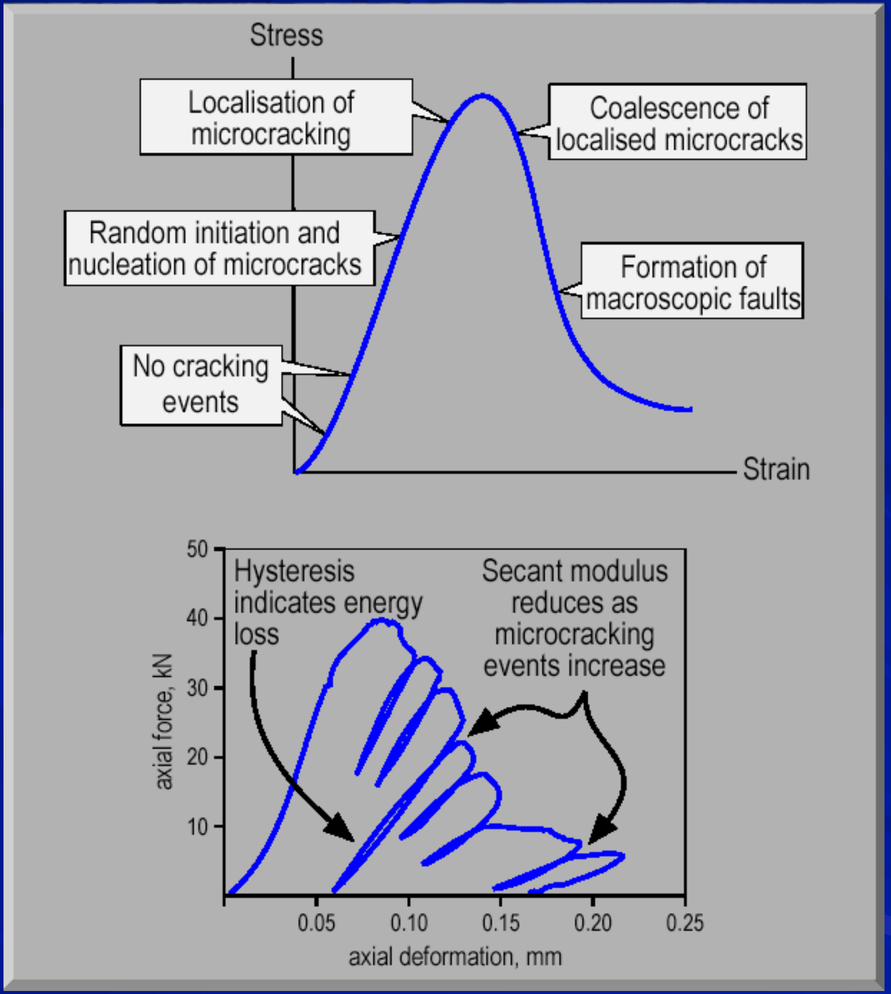


Low strain rate

High strain rate

(From Asphaug)

Damage and degradation leading to ultimate failure occur at some limiting strain



A Grady Kipp Implementation in 3D

•Damage is isotropic, so that when a crack is formed in one directions, all directions lose stiffness

•As damage accumulates, the stiffness in both tension and in shear decrease, eventually to zero.

•Therefore, material failed by the outgoing shock behaves as water.

•*Calibrated to disruption test, by adjusting the strength (Weibull) parameters*

Fragmentation phase: principles

Equation of state
$P=f(E,\rho)$

Model of brittle Failure

Stress tensor

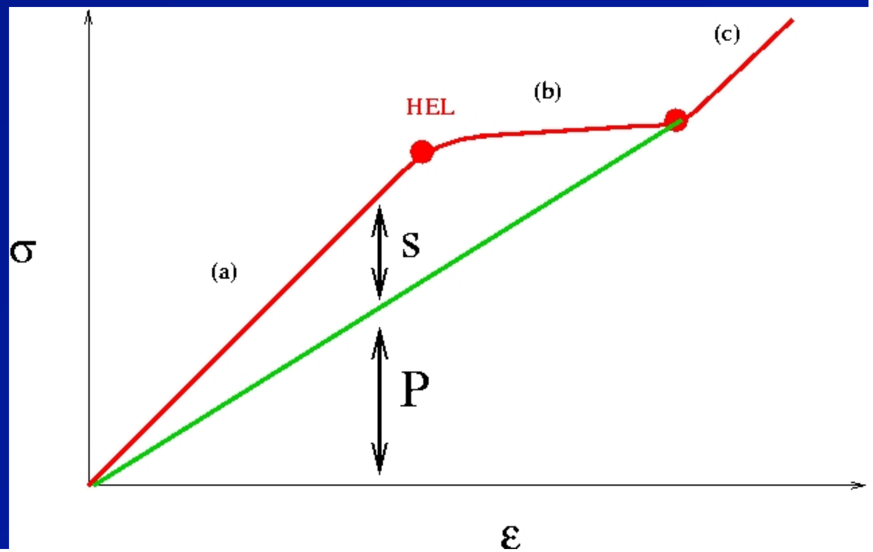
$$\sigma_{\alpha\beta} = -P\,\delta_{\alpha\beta} + S_{\alpha\beta}$$

$$S_{\alpha\beta} = \mu(\varepsilon_{\alpha\beta} - 1/3\,\varepsilon_{\gamma\gamma}\delta_{\alpha\beta})$$

Yielding criterion:
$S_{\alpha\beta} \rightarrow f\,S_{\alpha\beta}$

Conservation equations

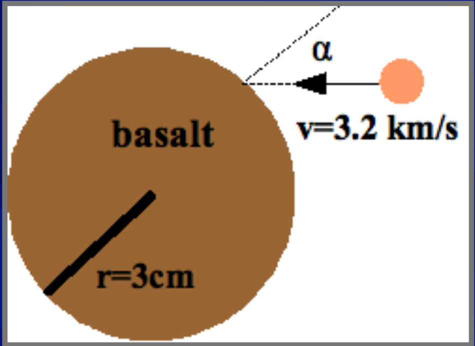
SPH techniques



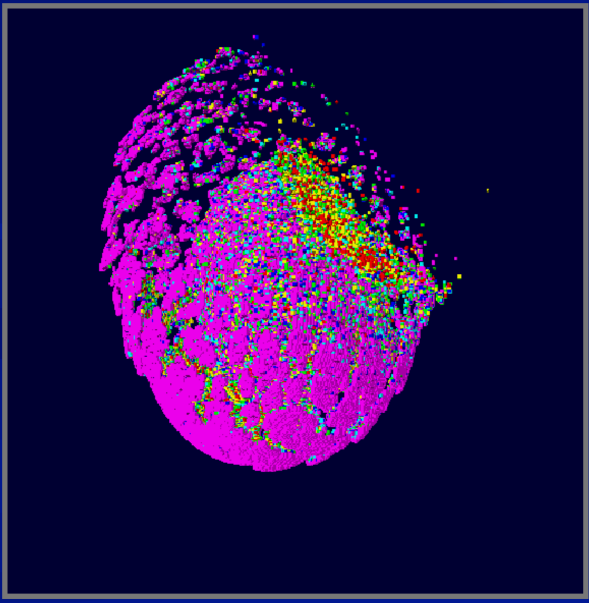
Validation with impact experiments on basalt

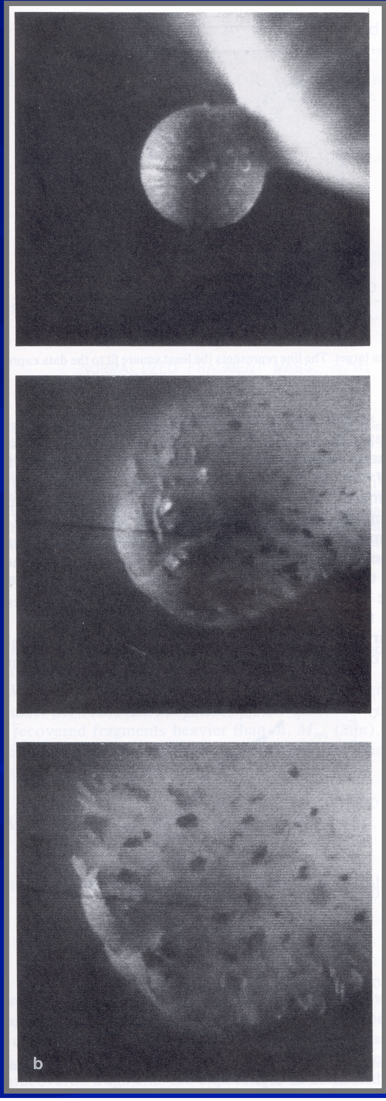
→ SPH simulations using 3.5×10^{6} particles

Nakamura & Fujiwara 93



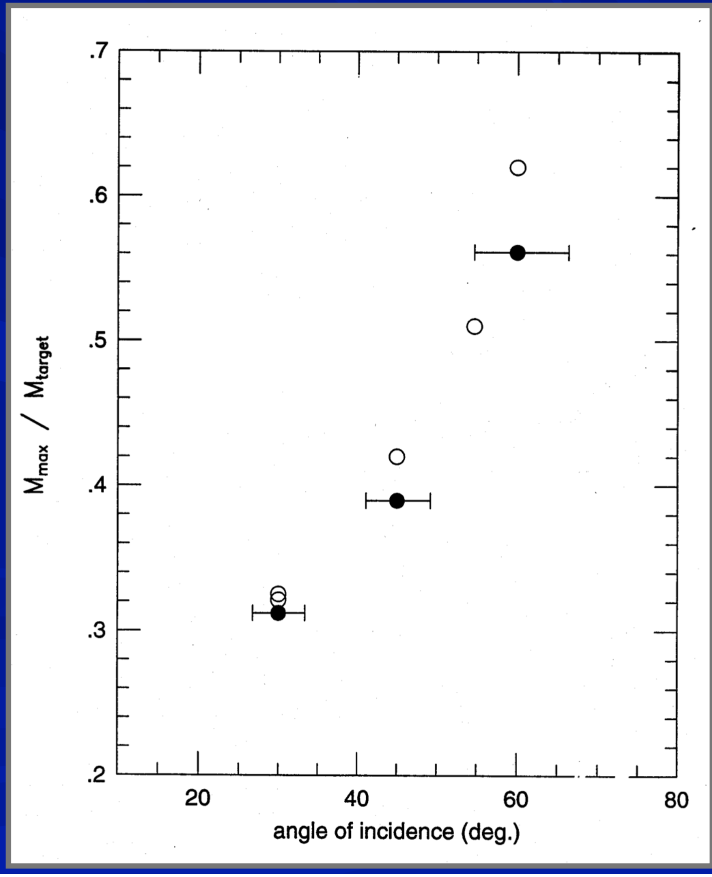
dust removed

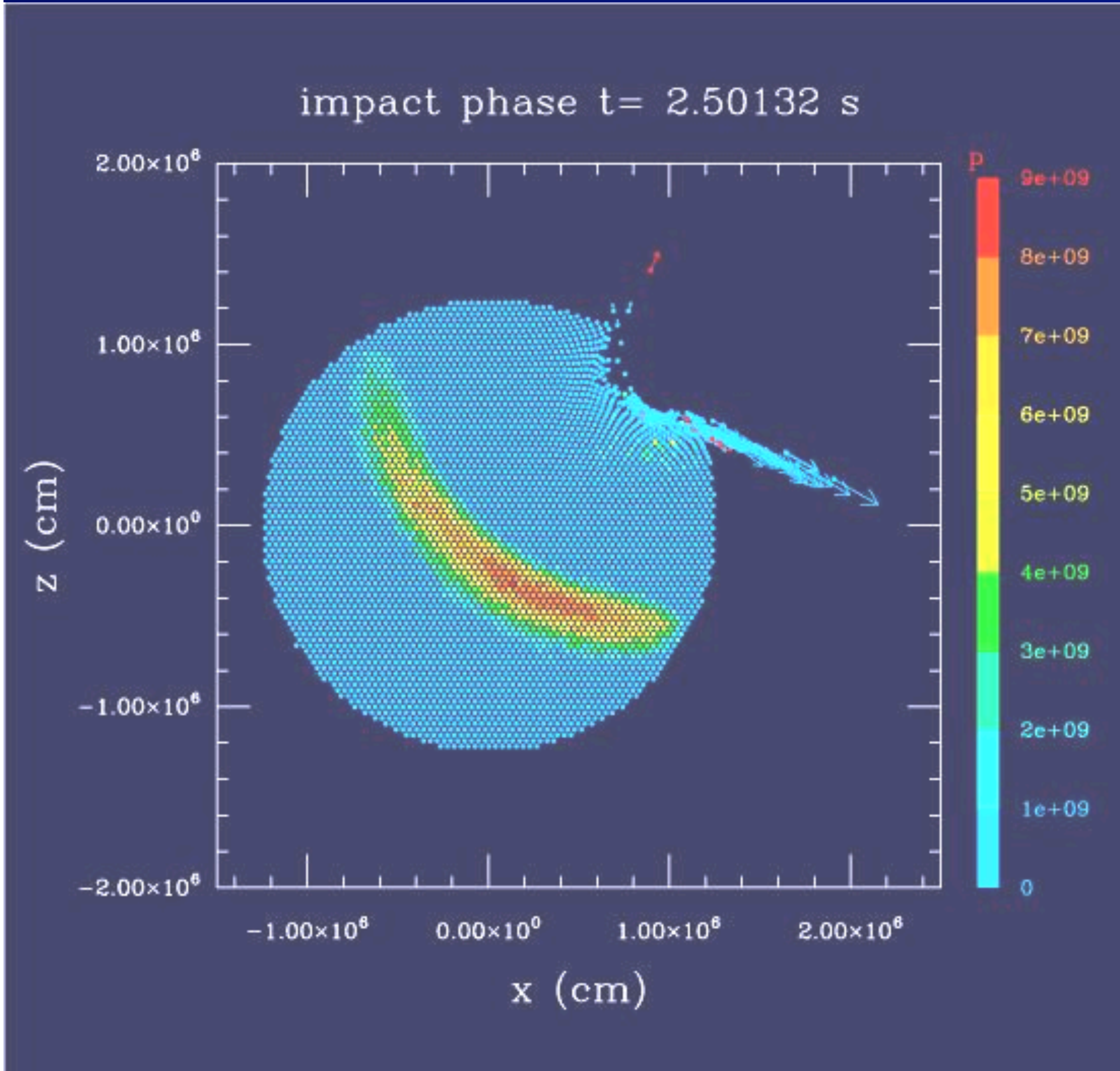




Benz & Asphaug 1994
High-res. Runs by M. Jutzi

largest fragment as a function of impact angle





Fragmentation Phase

Shock wave Propagation

Impact velocity: 5 km/s

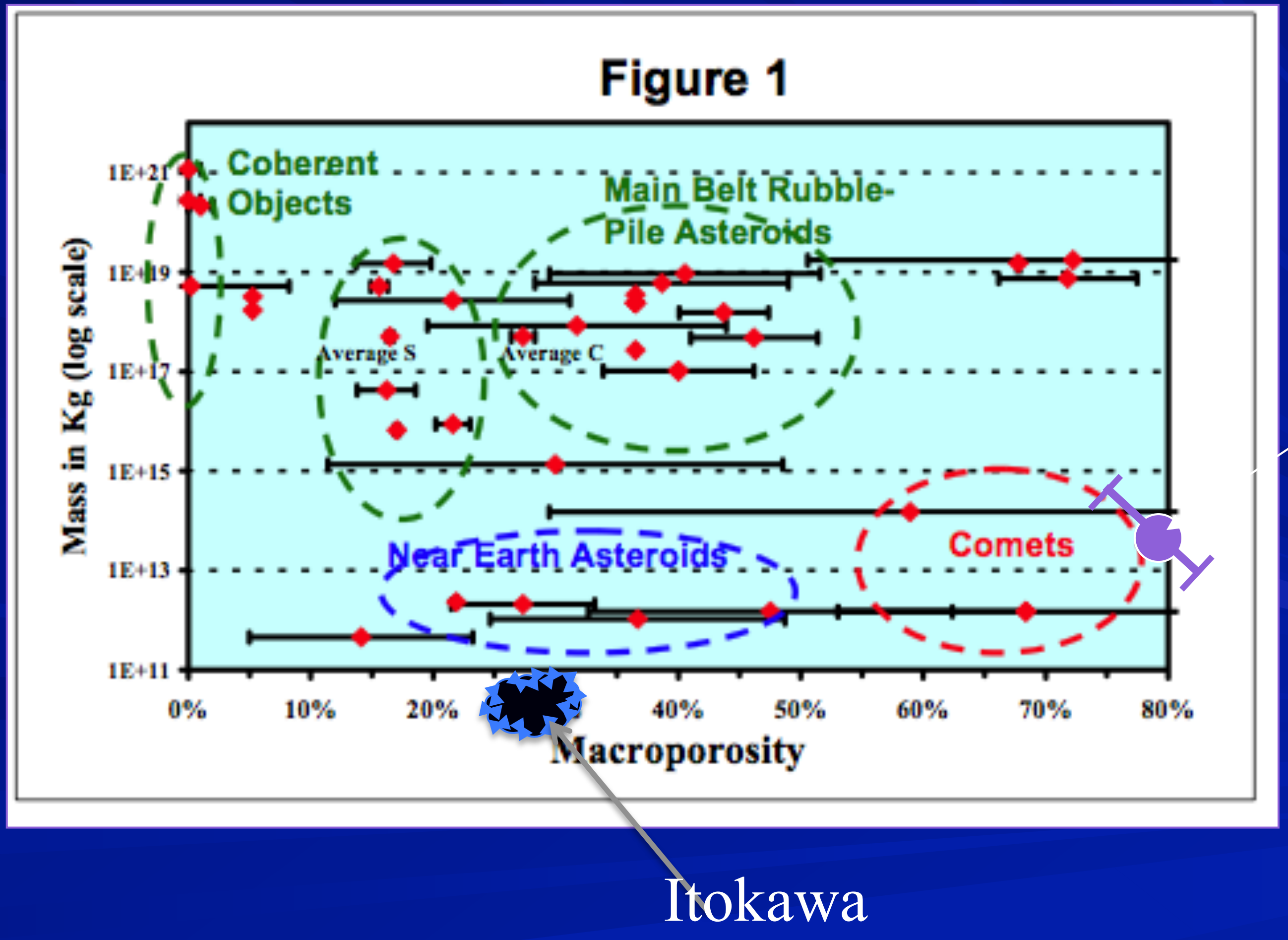
Impact angle: 45°

P. Michel & W. Benz

Why porosity is important

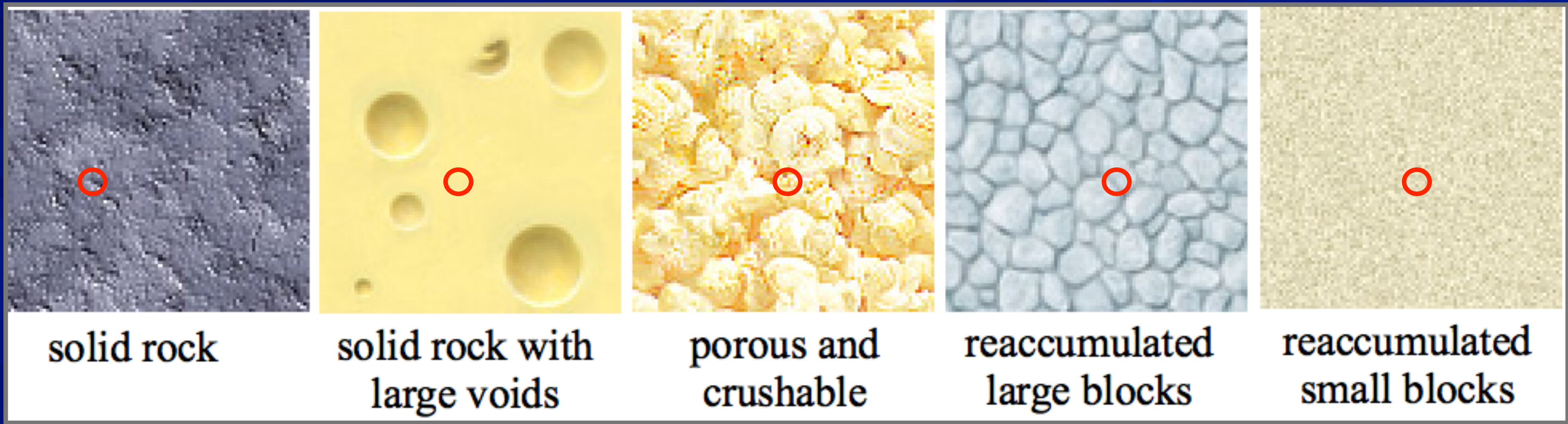
Many (most?) asteroids and comets are porous..

Ref: Consolmagno, Britt



Internal structure

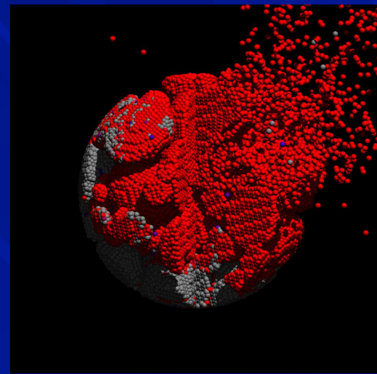
○ size of computational element



→ the internal structure will determine the ability to survive an impact
→the structure within some depth will determine
-size and geometry of crater
-amount of ejected mater
-velocity of ejected matter

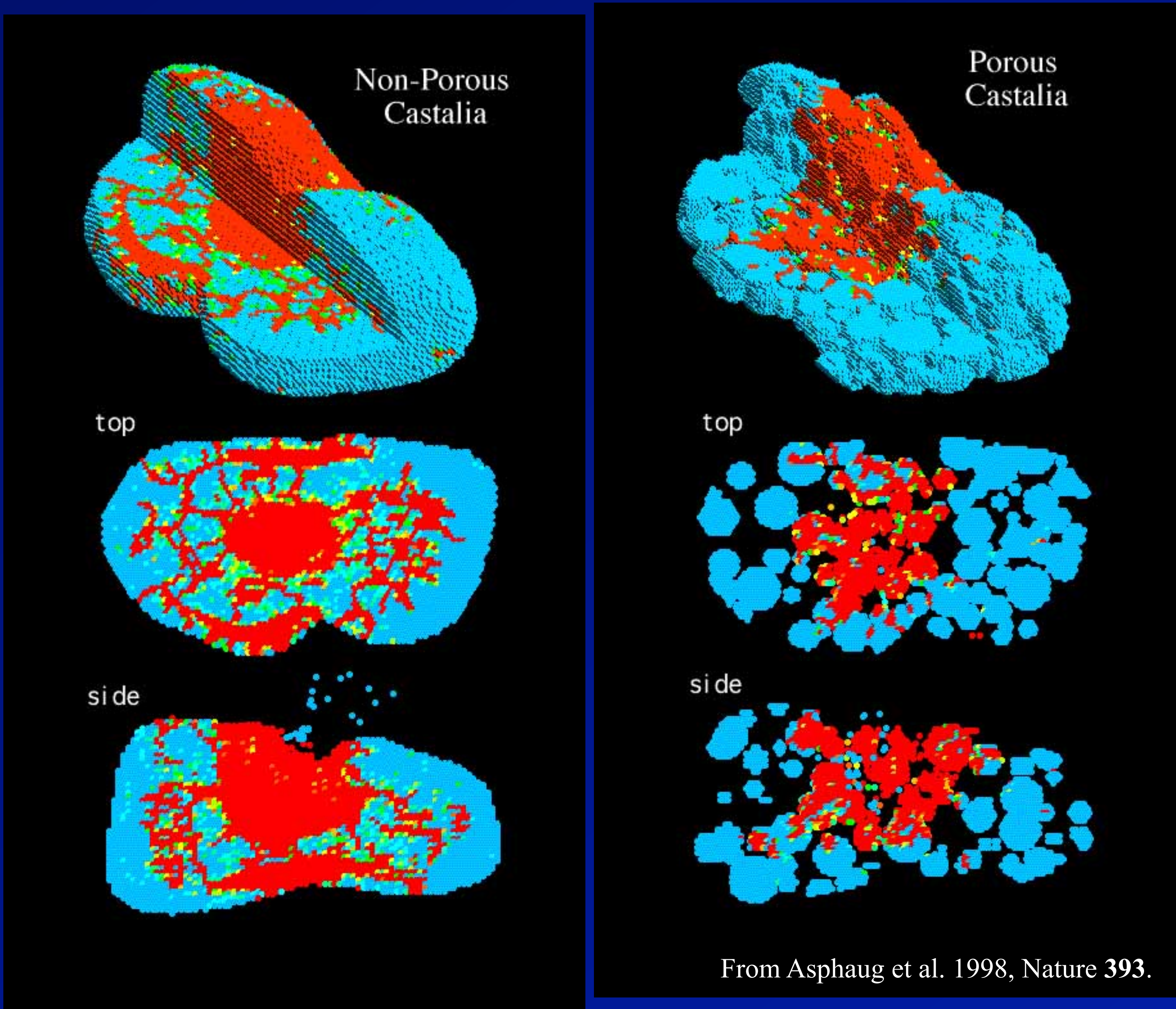
→ momentum transfer

Modeling porous material



Two types of porosity:

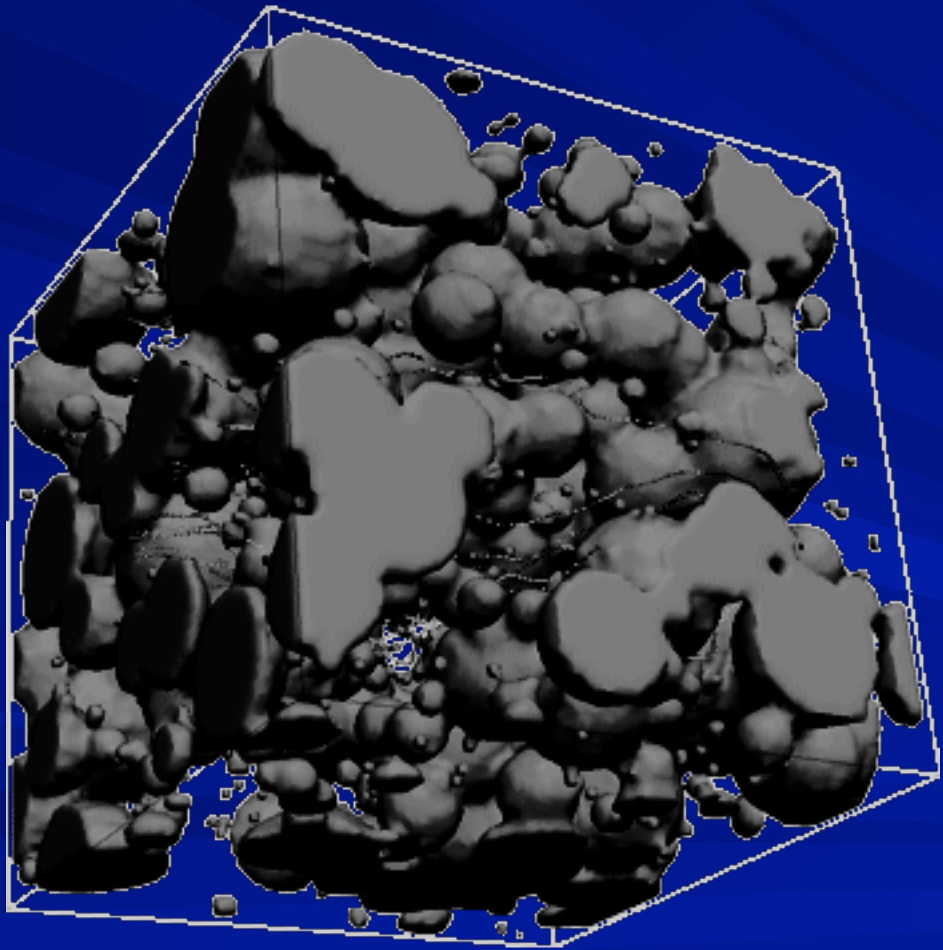
- Macroscopic scale:
  - Void sizes can be modeled explicitly
  - Rock components are not porous and there fragmentation is driven by classic model of brittle failure of non-porous material
- Microscopic scale:
  - Void/pore sizes are smaller than the thickness of the shock front
  - Void/pore sizes are smaller than the numerical resolution
  - Fragmentation modeled using the so-called P-α (Herrmann 1968) or ε-α or ρ-α model
- ➔ assumes uniform and homogeneous porosity...



From Asphaug et al. 1998, Nature **393**.

Macro porous

Porosity:



Volume of voids: Vv

Volume of matter: Vs

Total: V=Vs+Vv

Void ratio (Porosity): ϕ=Vv/V

Solid ratio: β=Vs/V=1- ϕ

Distension: α=V/Vs=1/ (1-ϕ)

Mass of solids: m_s

Density of mixture: $\rho = m_s/V$

Density of solid: $\rho_s = m_s/Vs$

Distension: $\alpha = \rho_s/\rho$

Porosity: $\phi = 1 - 1/\alpha$

Modeling porous material

Type of porosity:

- macroscopic scale: modeled explicitly using the classical model of brittle fail.
- microscopic scale: modeled using the so-called P-α model (Herrmann 1968)

➔ assumes uniform and homogeneous porosity...

Definition:

→ porosity: $$\phi = \frac{V - V_S}{V} \rightarrow \frac{V_V}{V}$$

→ distension: $$\alpha = \frac{\rho_s}{\rho} \qquad 1 \leq \alpha \leq \alpha_0$$

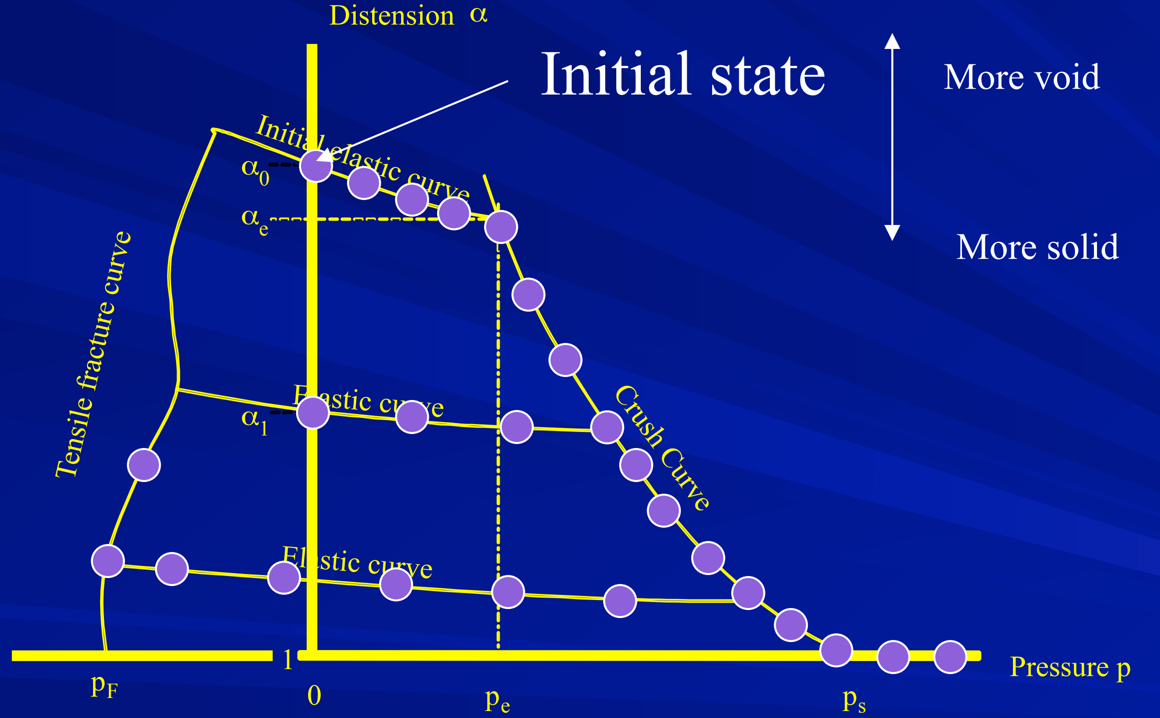
→ $$\phi = 1 - \frac{1}{\alpha}$$

With
V_V : Volume of voids
V_S : Volume of matrix
V : total volume
ϱ_s : density of matrix
ϱ : bulk density

Distention is defined as a function of pressure: α = α(P); but it can also be defined as a function of density or strain

Concepts

How a porous material responds to loading…Distension as a function of pressure: A p-α description



Modeling porous material

Distention is used to modify the following equations:

→ equation of state: $P \longrightarrow \frac{1}{\alpha} P(\alpha\rho, u, ...) = \frac{1}{\alpha} P(\rho_s, u, ...)$

→ deviatoric stresses: $S_{ij} \longrightarrow S_{ij}(\ldots, \alpha)$

→ fracture model: $D \longrightarrow D(\ldots, \alpha)$

Time evolution of distention:

$$\dot{\alpha}(t) = \frac{\dot{u}\left(\frac{\partial P_s}{\partial u}\right) + \alpha\dot{\rho}\left(\frac{\partial P_s}{\partial \rho_s}\right)}{\alpha + \frac{d\alpha}{dP}\left[P - \rho\left(\frac{\partial P_s}{\partial \rho_s}\right)\right]} \cdot \frac{d\alpha}{dP}$$

Damage and porosity

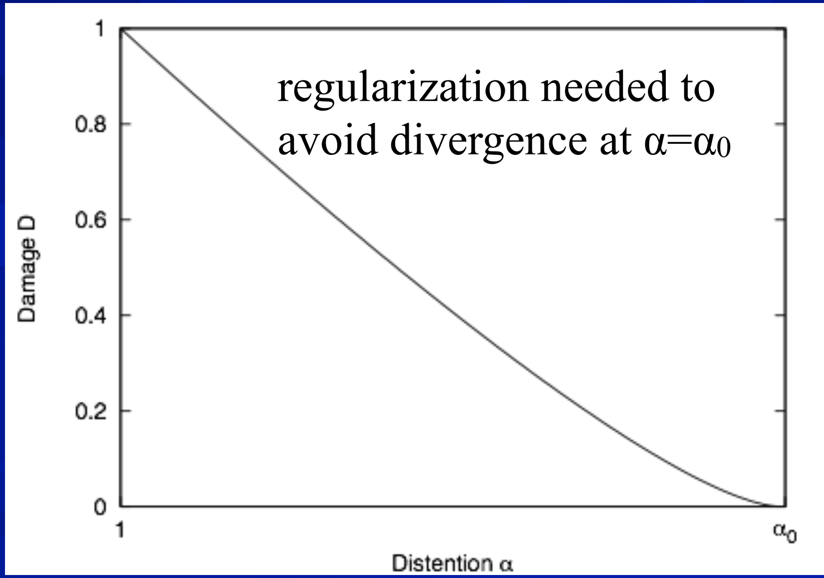
As the pores are crushed, the material is slowly turned into sand (at the scale of the numerical resolution element).

Since both damage D and distension α are volume ratios, we can relate the two by (linear relation)

$$D = 1 - \frac{(\alpha - 1)}{(\alpha_0 - 1)}$$

Time evolution:

$$\frac{dD^{1/3}}{dt} = -\frac{1}{3}\left[1 - \frac{\alpha - 1}{\alpha_0 - 1}\right]^{-\frac{2}{3}} \frac{1}{\alpha_0 - 1}\frac{d\alpha}{dt}$$

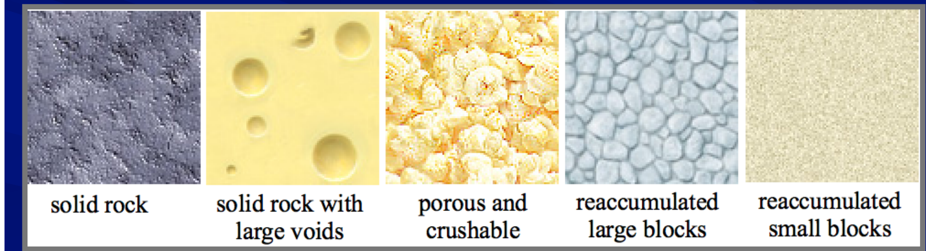


total damage = tension damage *(Weibull flaws)* + ***compression damage*** *(breaking pores)*

First simulations of an impact experiment on a porous target (pumice)

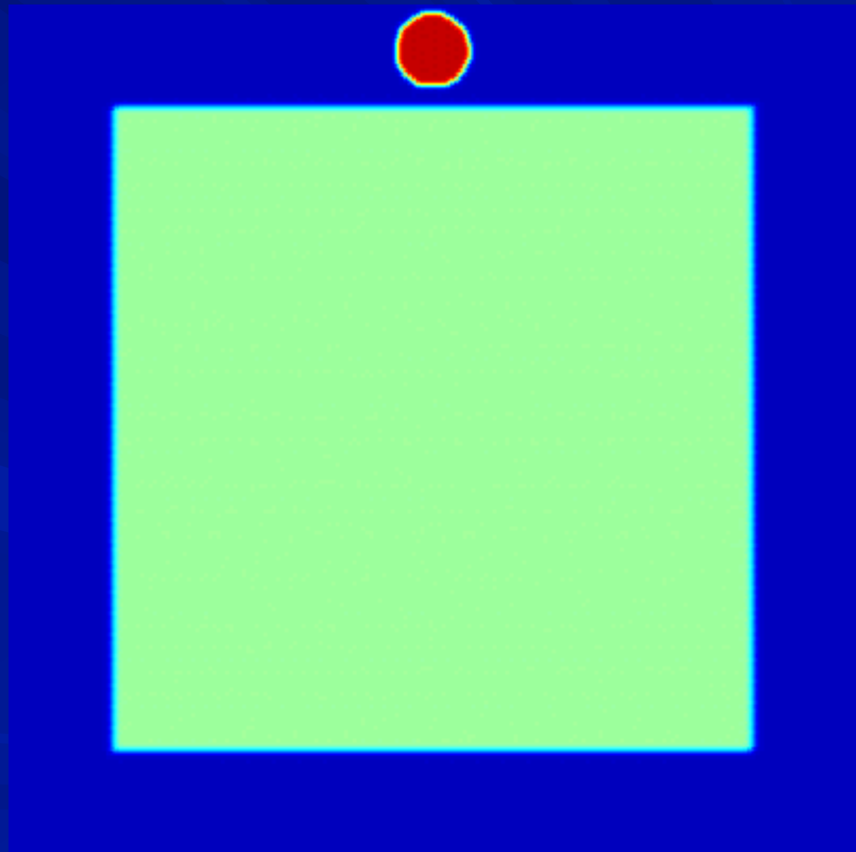
Jutzi, Michel, Hiraoka, Nakamura, Benz, 2009, Icarus 201

Différent kinds of porosity



Damage propagation (red) from the numerical simulation with porosity model

Initial material properties are those measured for the real target

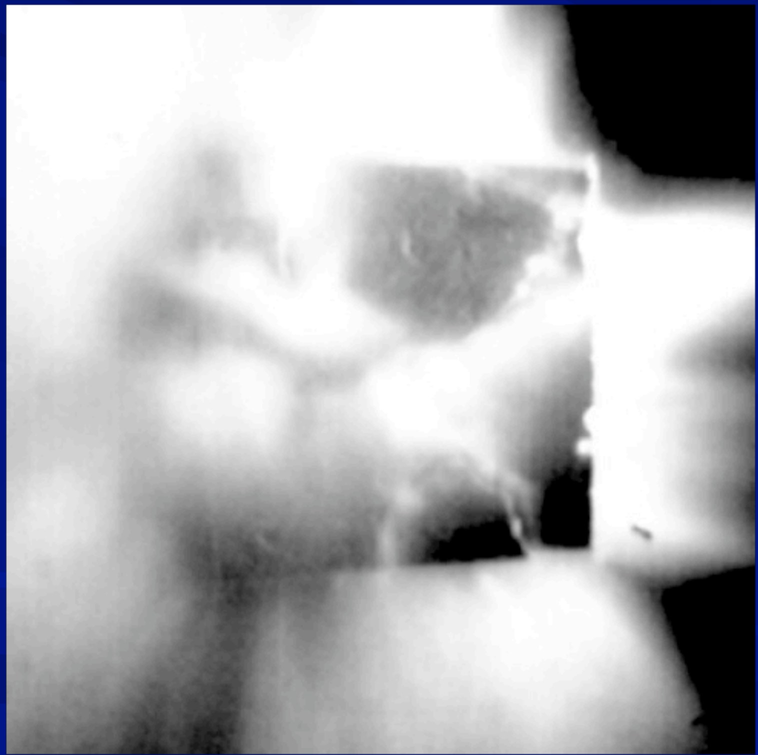


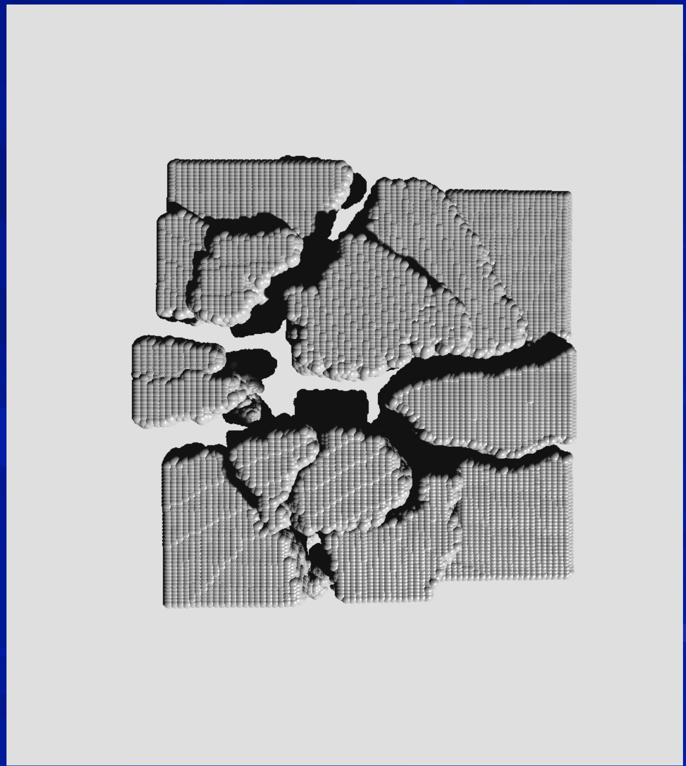
Impact speed: 3 km/s

Confrontation simulations/experiments

Jutzi, Michel, Hiraoka, Nakamura, Benz, 2009, Icarus 201

Experiment T = 1.5 ms Simulation

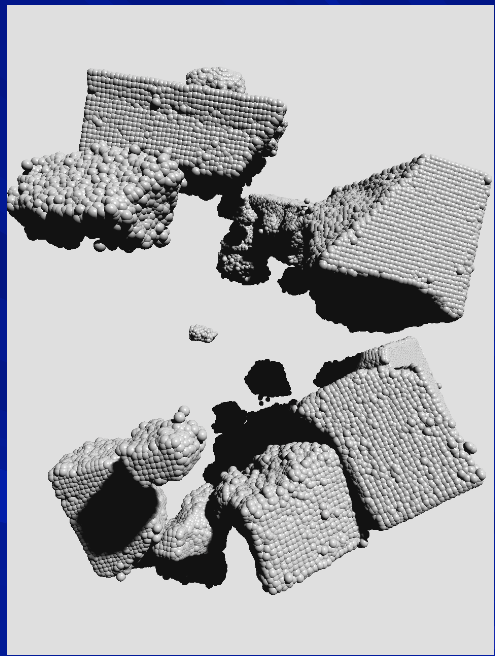




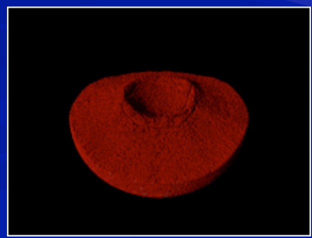
First validations of a model of fragmentation of porous body

Confrontation simulation/experiment

Experiment $T = 8$ ms Simulation



First application at large scale: formation of the crater on the asteroid Stein (Rosetta image)



Jutzi, Michel, Benz 2010. A&A 509, L2

Simulating an asteroid disruption

Requires:

1. To compute the fragmentation phase (hydrocode):

Hydrodynamical equations + model of brittle failure

$\Rightarrow$ **Propagation of the shock wave and of cracks into the target**

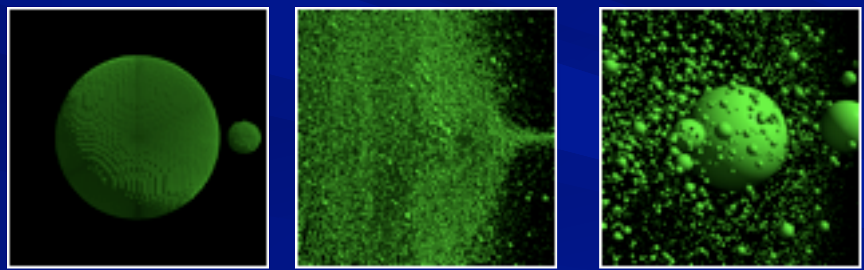
2. To compute the gravitational phase between the generated fragments (parallel N-body Code)

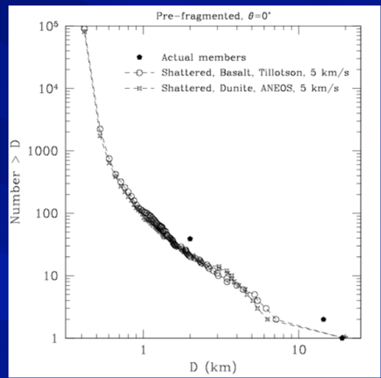
First results: Michel et al. (2001), Science Vol. 294, pp 1696-1700.

Internal structure of small bodies: Characterisation and role

Our simulations of asteroid disruptions reproduced for the first time asteroid families and suggest that objects > km are gravitational agregates (rubble piles)

Michel et al., *Science* 294 (2001)



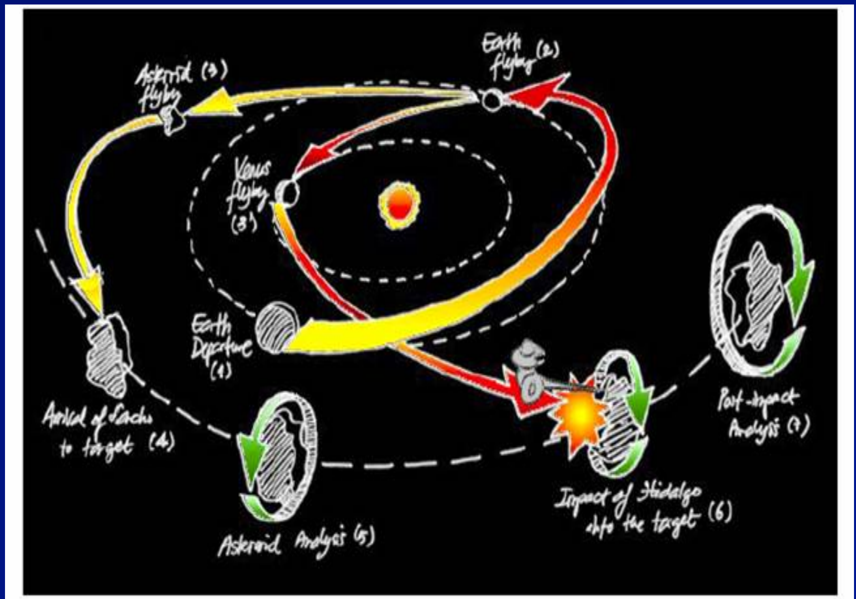


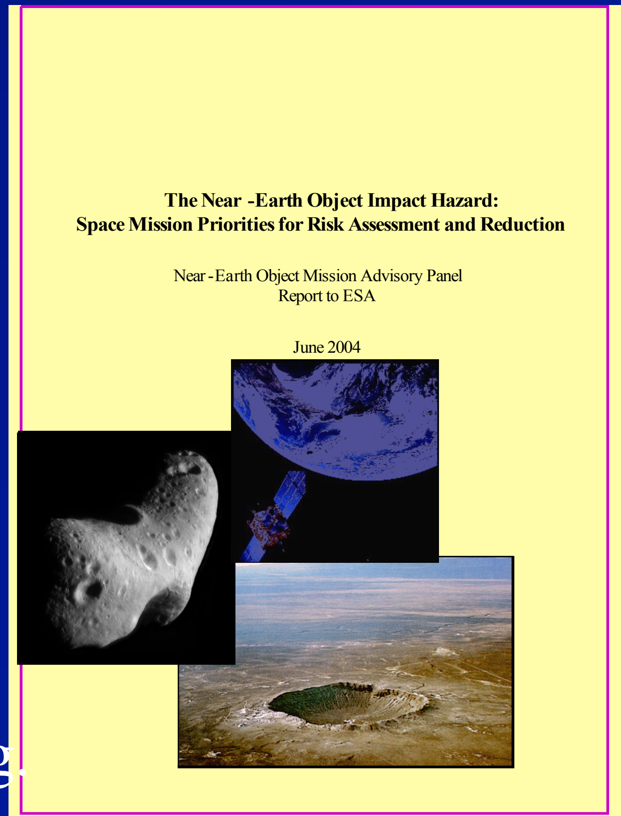
Disruption outcomes and impact energies greatly depend on the initial internal structure of the impacted body

Michel et al., *Nature* 421 (2003)

Surface and internal properties: crucial information for hazard mitigation

- **Example: Mission Don Quijote: phase A studies at ESA (final presentation: 17-18 Avril 2007)**




The Near -Earth Object Impact Hazard:
Space Mission Priorities for Risk Assessment and Reduction

Near-Earth Object Mission Advisory Panel
Report to ESA

June 2004

The momentum transfer efficiency highly depends on the (sub)surface properties (e.g. porosity, regolith properties)

Current difficulties in modeling

Mass ratio:

$$\frac{M_p}{M_t} \simeq 4.4 \times 10^{-10}$$

Max. number of SPH particles:

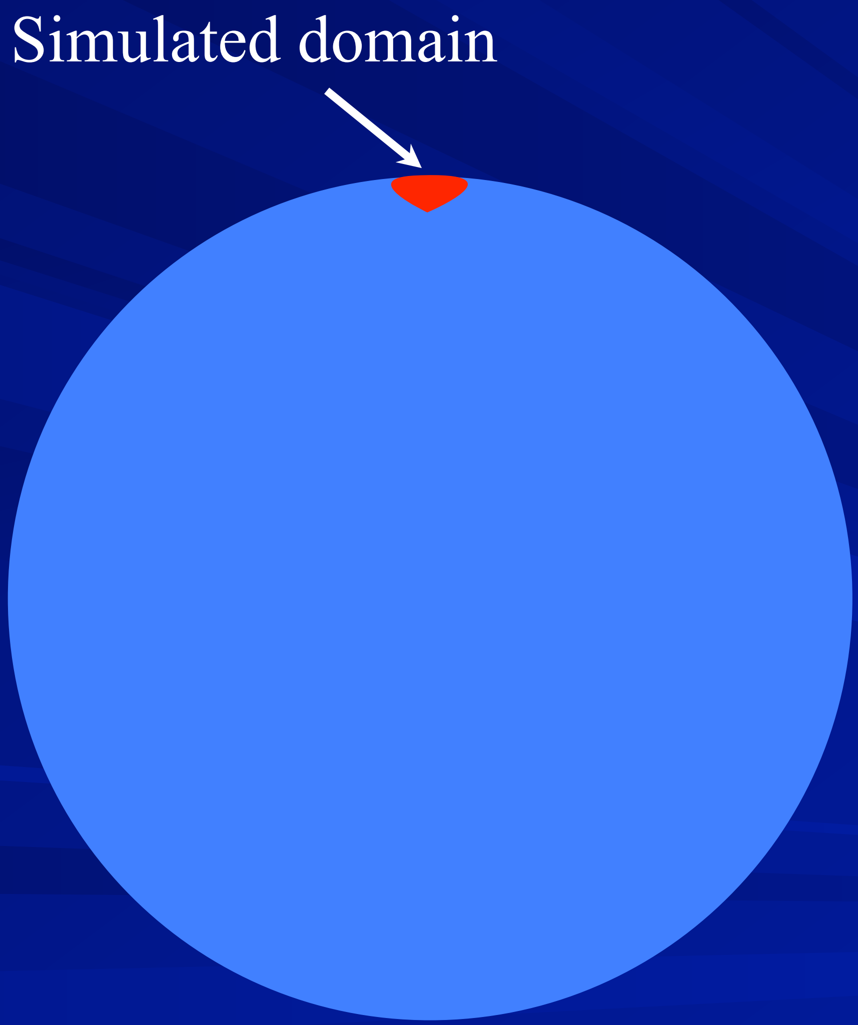
$$N \simeq 10^7$$

One SPH particle

$$\sim 225 \times M_p$$

➔ **We cannot simulate the whole asteroid**

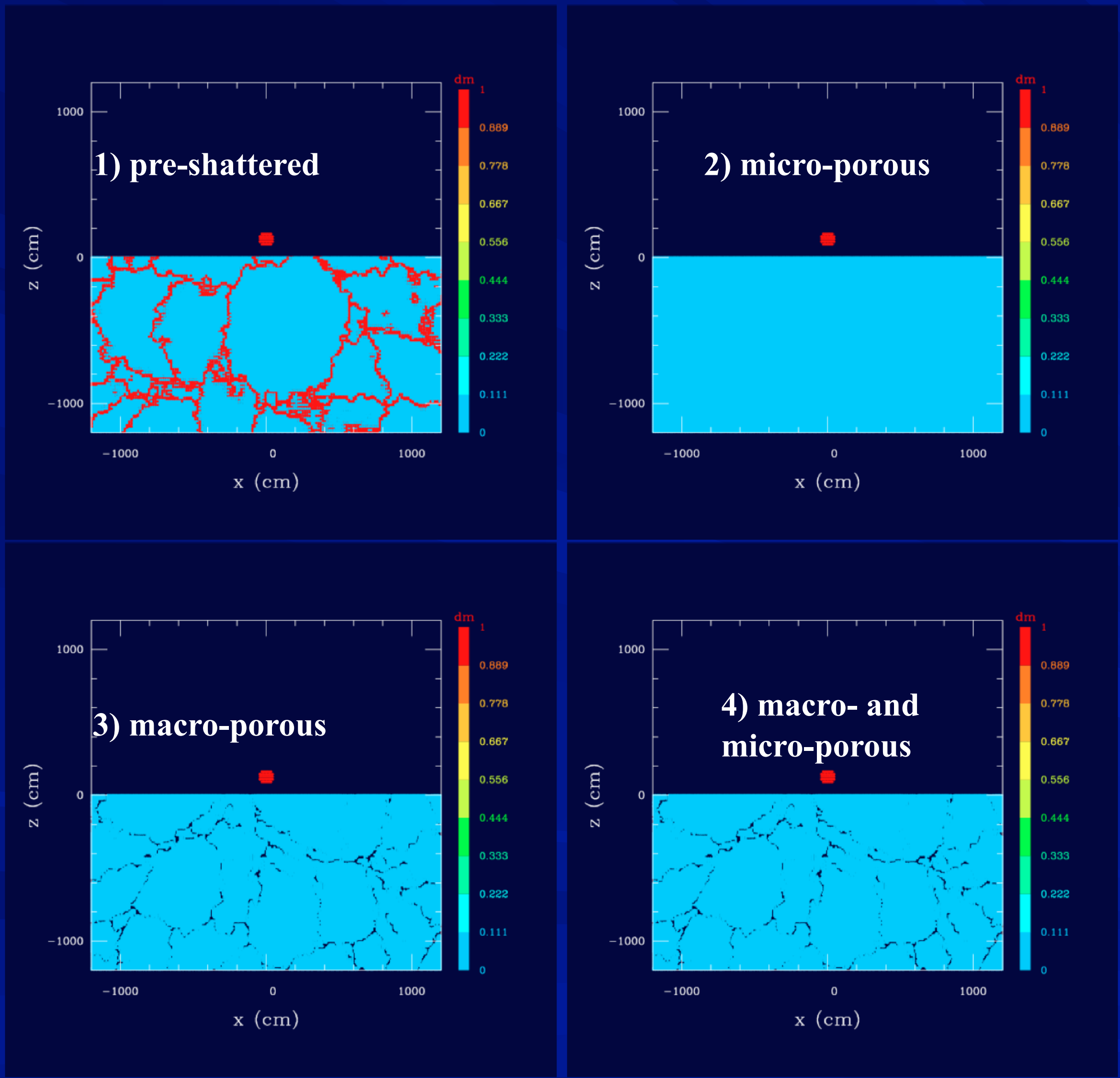
Current difficulties in modeling



The size of the simulated domain (half-sphere) should be larger than the size of the damaged region

Global effects can not be studied easily

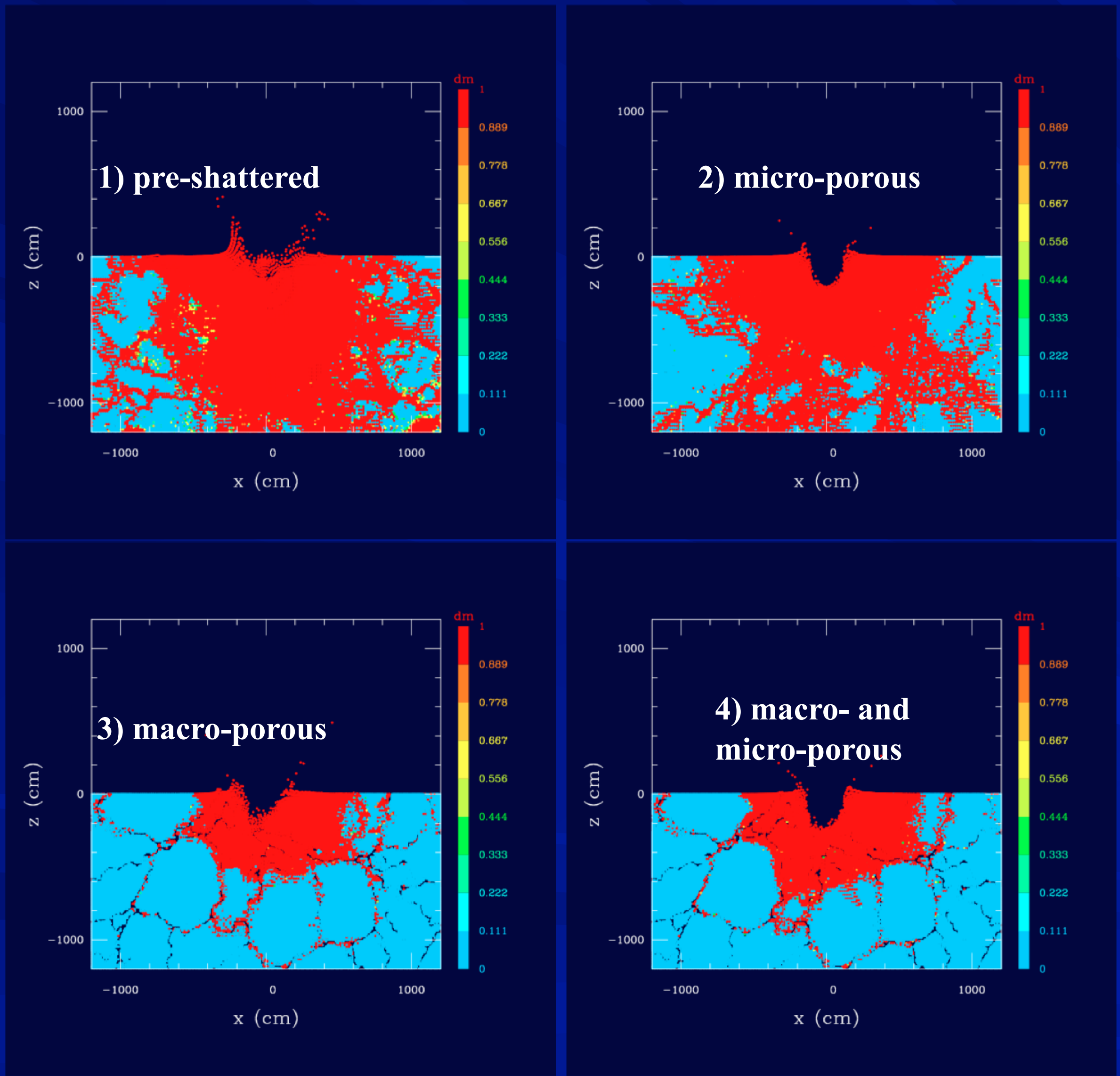
Initial conditions (target structures)



Target:

- half-sphere of 34 m diameter
- 4.4 10^6 SPH particles
- spatial resolution ~ 15 cm

Results: damage

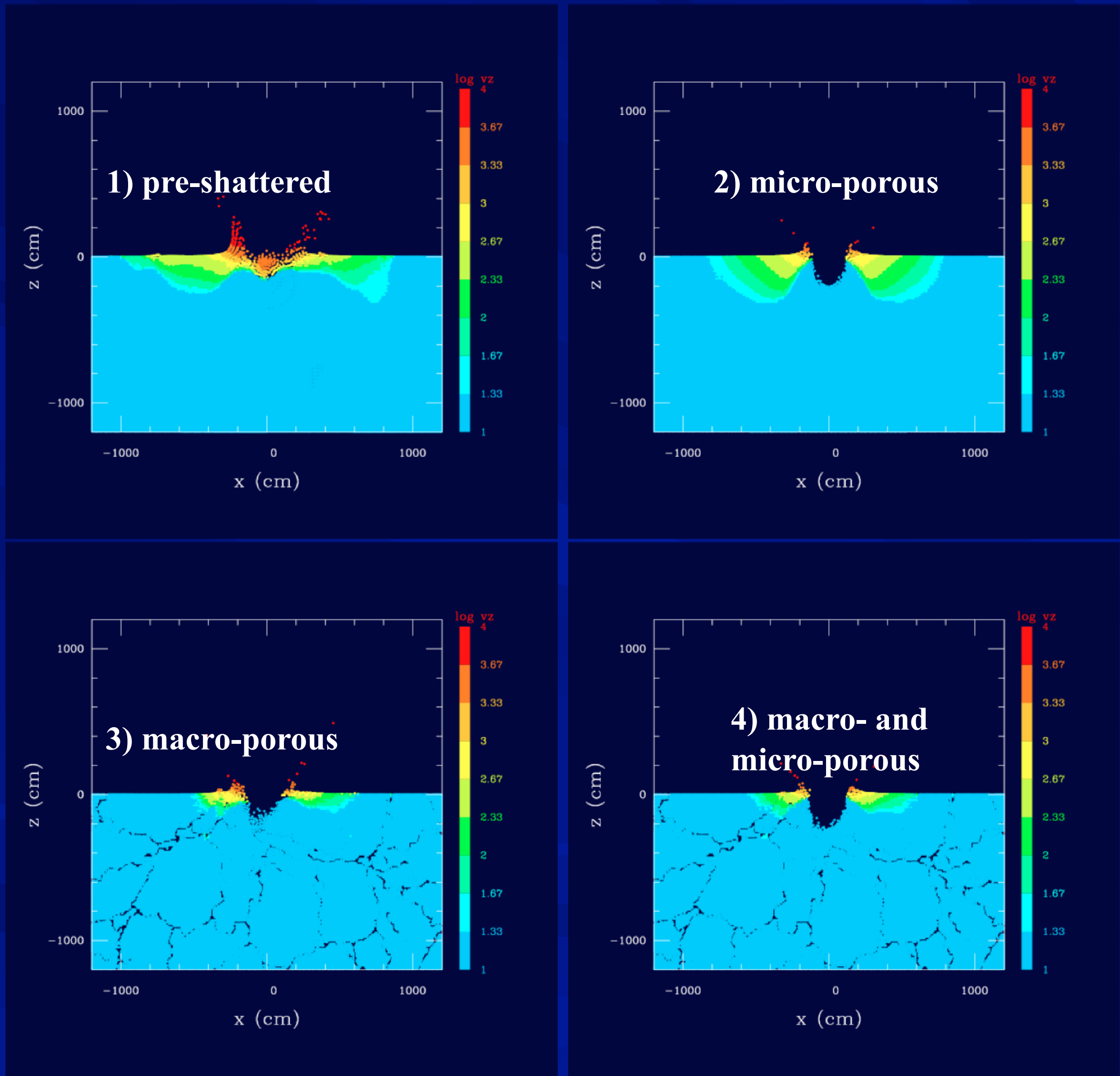


Simulations after 20 ms

Red: fully damaged material

Simulations and plots made by M. Jutzi

Results: velocity

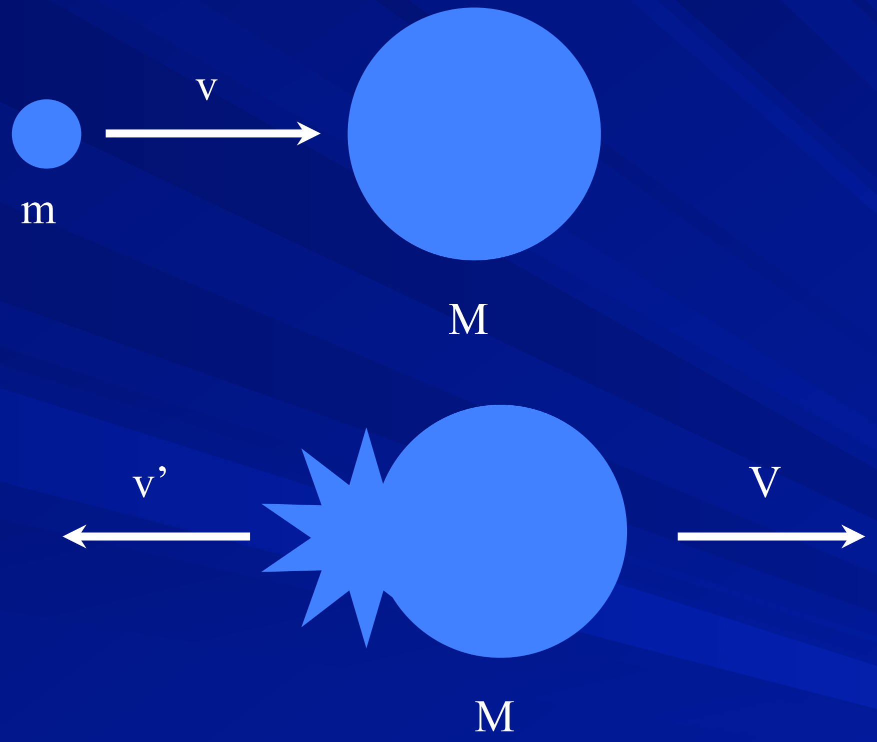


Simulations after 20 ms

Colors: vertical velocity 0.1 to 10^3 m/s (log-scale)

Simulations and plots made by M. Jutzi

Momentum transfer



$$\vec{P}_{target} = \vec{P}_{projectile} + \vec{P}_{ejecta} > \vec{P}_{projectile}$$

Momentum transfer

- Normalized with the momentum of the projectile:

$$P_{target} = 1 + P_{ejecta} \equiv \beta \geq 1$$

- Change of the target velocity

$$\Delta V = \frac{P_{target}}{M_{target}} = \beta \times \frac{P_{projectile}}{M_{target}}$$

Momentum transfer

- Momentum multiplication factor

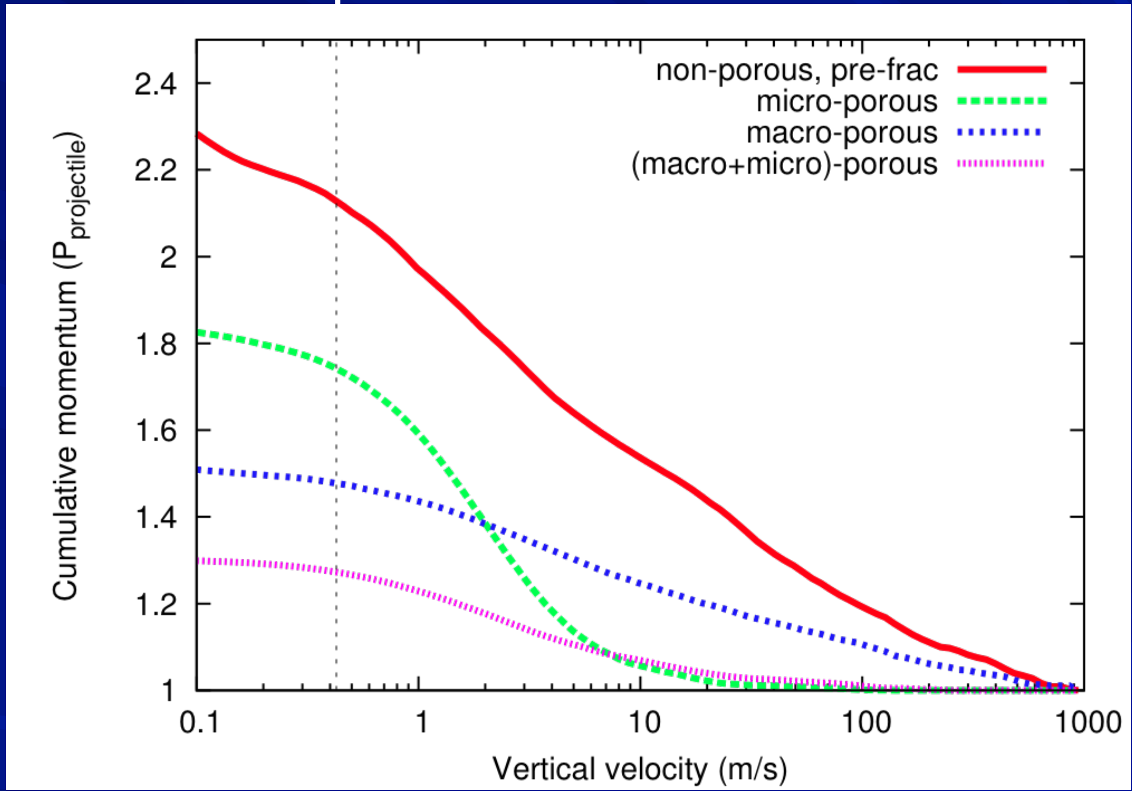
$$\beta = ?$$

- Target structure
- Material characteristics
- Impact velocity
- Target size etc.

from scaling laws: $$\beta \sim \left(\frac{\rho U^2}{Y}\right)^{(3\mu-1)/2}$$

Cumulative momentum distribution

Escape velocity



⇒ Momentum multiplication factor β

Velocity change (of a 1 km asteroid)

ΔV is given by $\beta \times \frac{mU}{M}$

Simulation	β	ΔV (μm/s)
1	2.13	2.8
2	1.74	2.3
3	1.48	2.0
4	1.27	1.7

1: pre-shattered
2: micro-porous
3: macro-porous
4: macro- and micro-porous

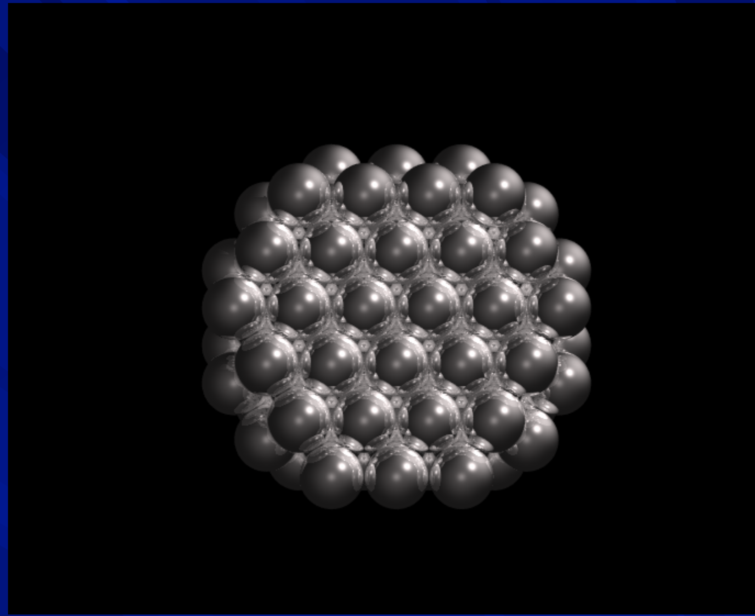
Laboratory Impact Disruption
Granular material with cohesion

Target

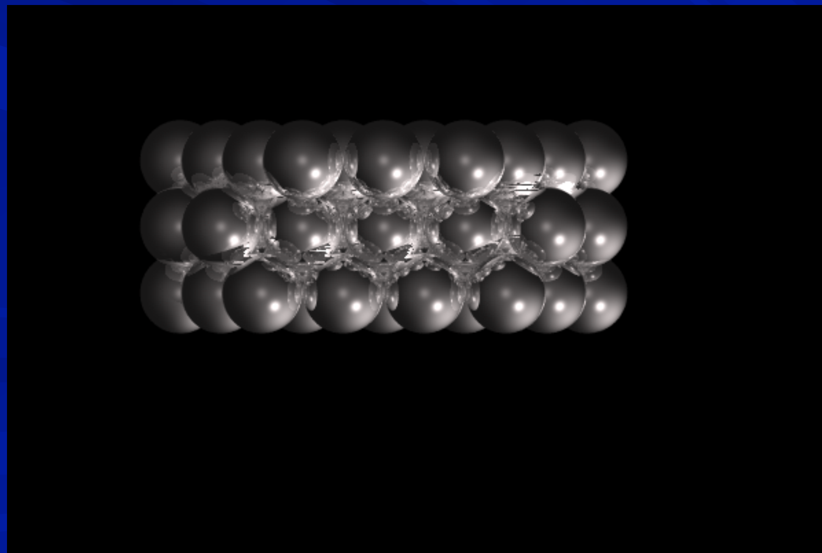
- Glass beads arranged in three layers to form a disk.
- “Sintered” in oven.
- Bond strength controlled by cooking duration.
- 90 beads total (each are $^{3}/_{16}$ inches across, 2.5 g/cc).

Initial Impact Trials

- Projectile is single glass bead $^{1}/_{8}$ inches in diameter.
- Shot from gas gun at 277 m/sec.
- Impacts near center of target at a 45° angle.



Numerical Model, S. Schwartz



Building a Computational Model

N-body code (pkdgrav) is used to simulate forces between particles:

- Gravity
- Collisions
- Strength
  - **Elastic Deformation equivalent to Hooke's Law** (springs)

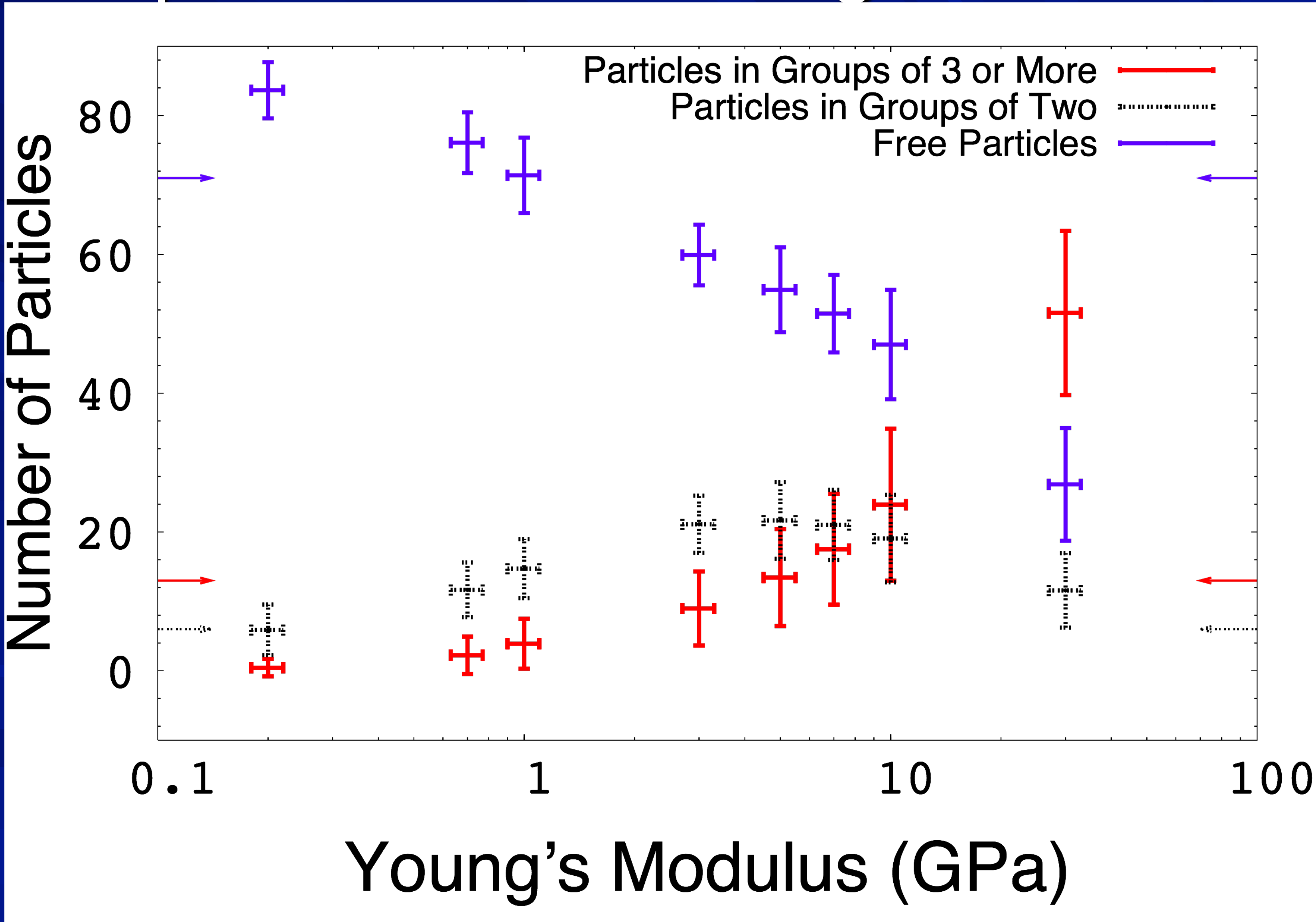
Elastic (Springs) Model

- Neighboring particles "connected" by springs.
- Each spring is defined by:
  - An equilibrium separation (length at zero strain)
  - A Young's modulus
  - A maximum stress/strain beyond which spring breaks
  - A damping term

Building the Target in Stages

- STEP ONE: Placement of bottom layer and outside middle layer atop a wall.
- STEP TWO: Adjust to avoid overlaps and attach springs, drop in remaining beads that will comprise the rest of middle layer by introducing uniform gravity (self-gravity is off).
- STEP THREE: Introduce (translucent) wall that pushes middle layer into configuration.
- STEP FOUR: Drop top layer on top.

Dependence on Young's Modulus



Thank you!

Porous versus non-porous!