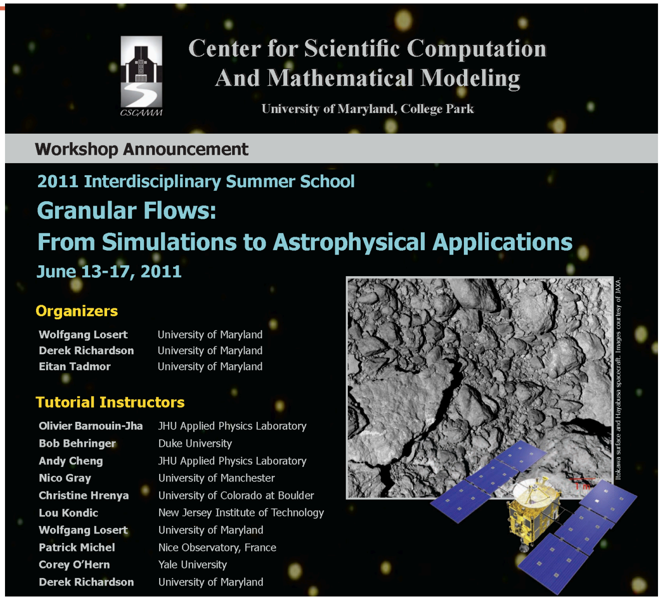
GRANULAR DYNAMICS ON ASTEROIDS

Derek C. Richardson
University of Maryland

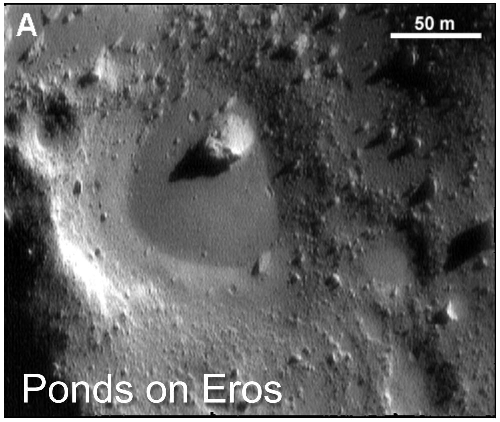


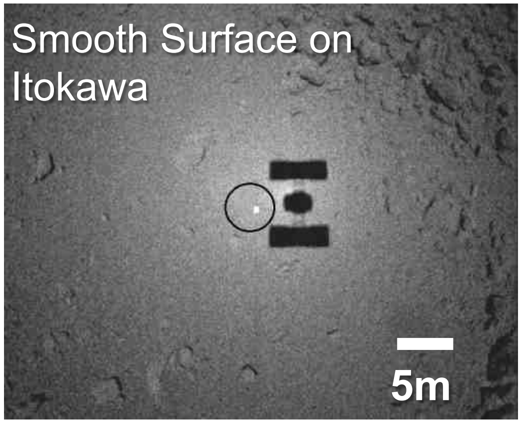
Overview

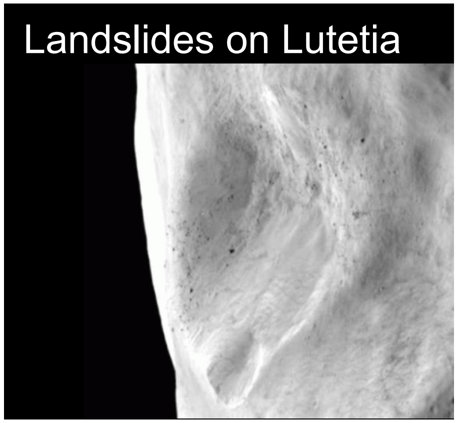
- Main topic: simulating granular dynamics with aim of applying method to asteroid surfaces.
  - HSDEM approach.
  - Test cases: model atmosphere, vibrating plate, tumbler, avalanche.
  - Modeling cohesion.
  - SSDEM approach.
  - Preliminary results.

Richardson et al. 2011, Icarus 212, 427.
http://www.astro.umd.edu/~dcr/reprints.html

Why investigate granular material?







- Surfaces of planets and small bodies in our solar system are often covered by a layer of granular material.
- Understanding dynamics of granular material under varying gravitational conditions is important in order to:
  1. Interpret the surface geology of small bodies.
  2. Aid in the design of a successful sampling device or lander.

Numerical Approach

- Need to combine granular physics and complex forces.
- To do this, we use a modified version of PKDGRAV, a well-tested, high-performance *N*-body code.
- Original modifications aimed at planetesimal dynamics using self-gravitating smooth spheres.
  - This is a hard-sphere discrete element method (HSDEM).
  - Can this be used successfully to model granular dynamics?
  - Validate numerical approach by comparing with lab experiments.
  - HSDEM successful in dilute regime.
  - Need soft-sphere DEM (SSDEM) for dense, near-static regime.
- Goal: develop hybrid HS/SSDEM suitable for wide range of applications.

Granular Dynamics with HSDEM

- Typically have no interparticle forces: particles only feel collisions and uniform gravity field:

$$\ddot{\mathbf{r}}_i = -g\hat{\mathbf{z}}$$

- Could solve equations of motion analytically, but want to allow for complexity (e.g. self-gravity, cohesion, etc.).
- Leapfrog remains advantageous for collision prediction.
- Tree code and parallelism speed up neighbor searches.
- But, no resting contact forces: best in dilute regime.
- And, need walls! (particle confinement).

Walls

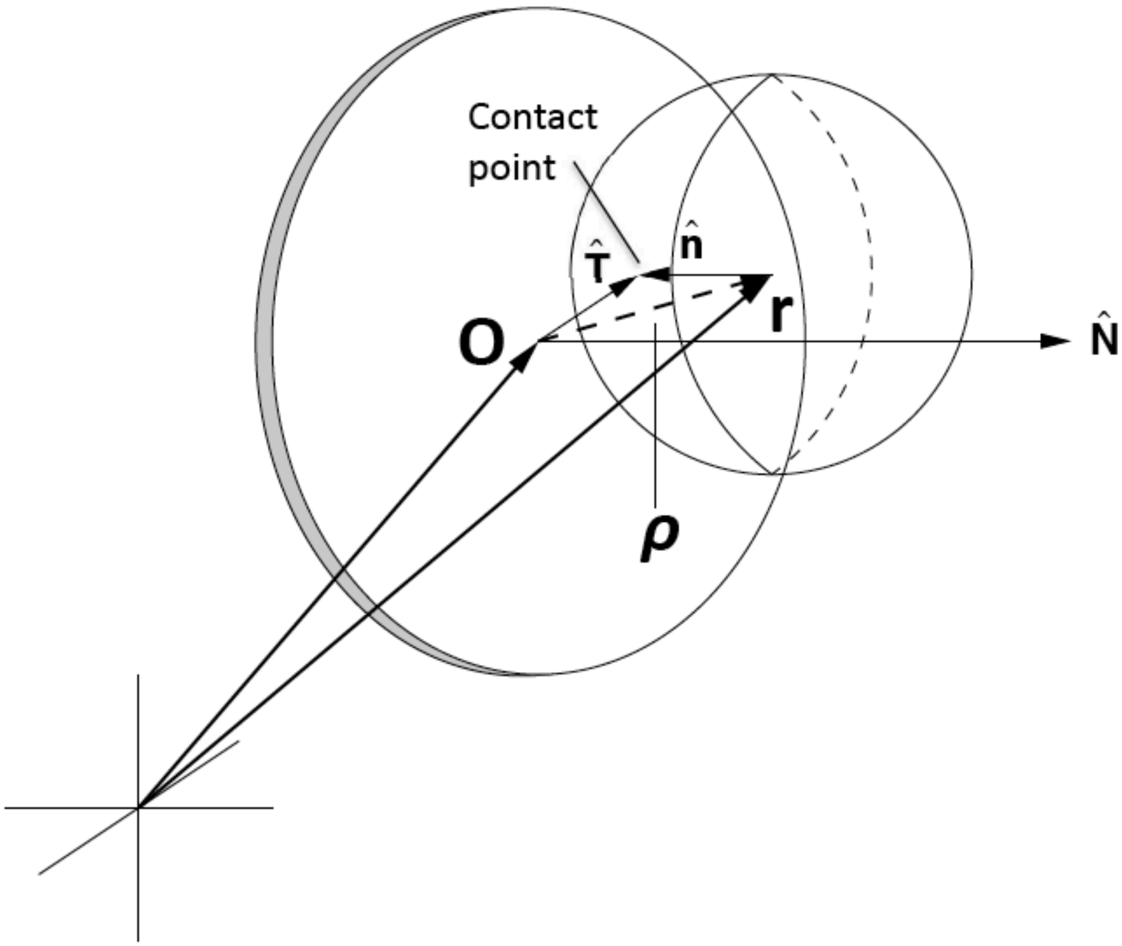
- Approach: combine wall "primitives" in arbitrary ways.
- Each wall has an origin and orientation.
- May also have translational velocity, oscillation amplitude and frequency (in orientation direction only), and rotation (around orientation axis, if symmetric).
- Each wall also has ε_n, ε_t, a drawing color, and configurable transparency.
  - Particles can stick to a wall ($\varepsilon_n = 0$), even if moving/rotating, or be destroyed by it ($\varepsilon_n < 0$).
  - NOTE: In HSDEM, surface "friction" (ε_t) is really an instantaneous alteration of particle's transverse motion and spin on contact.
- Walls have infinite mass (unaffected by particles).

Walls

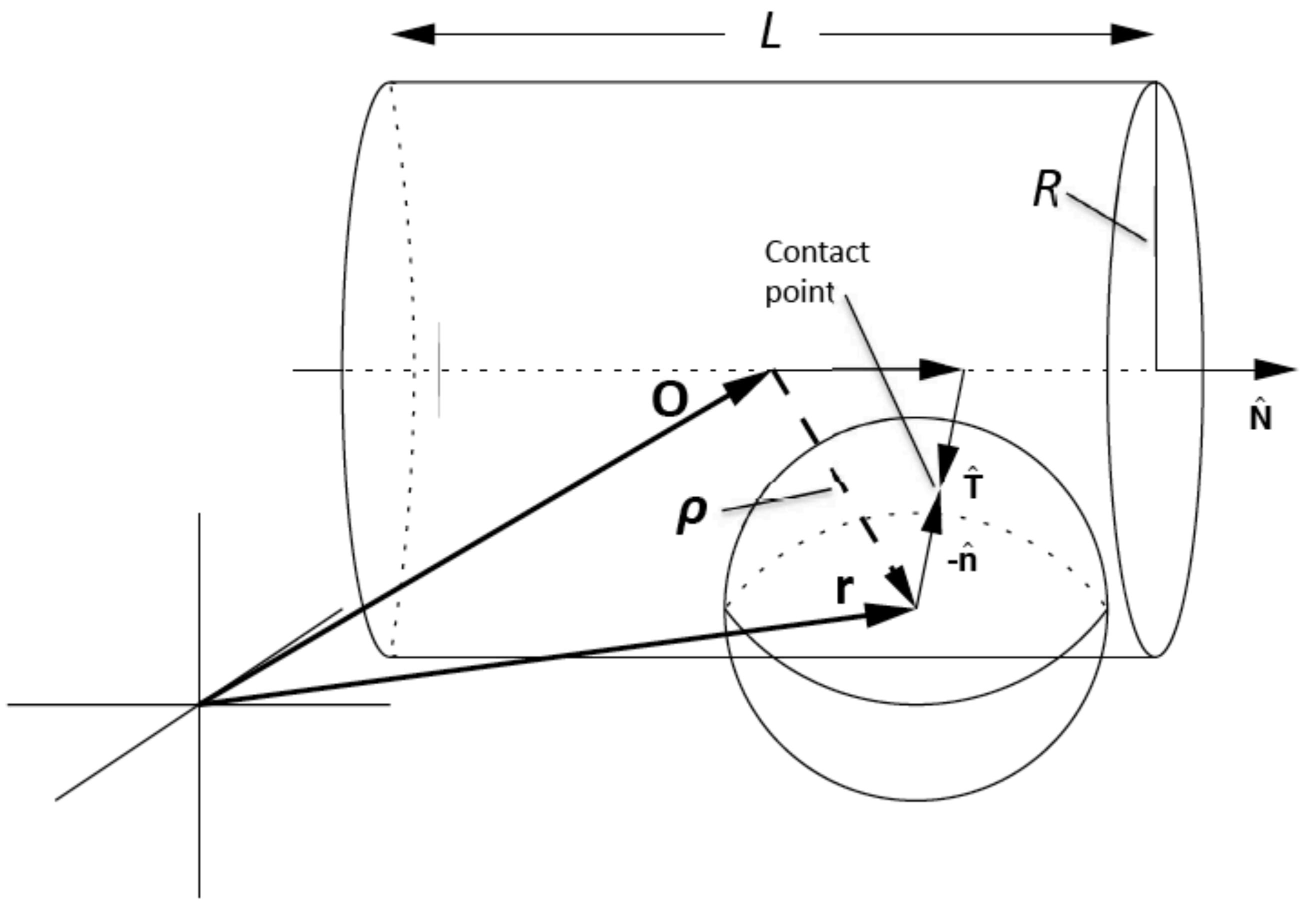
- Collision condition: $|\mathbf{r}_{impact} - \mathbf{c}| = s$, where **c** is the point of contact on the wall, which depends on the wall geometry.
- Following geometries supported:

Geometry	Unique Parameters	Degenerate Cases
Plane (infinite)	none	none
Triangle (finite)	*vectors to 2 vertices*	*point, line*
Rectangle (finite)	vectors to 2 vertices	point, line
Disk (finite)	radius, hole radius	point
Cylinder (infinite)	radius	line
Cylinder (finite)	radius, length, *taper*	point, line, ring
Spherical shell (finite)	*radius, opening angle*	*point*

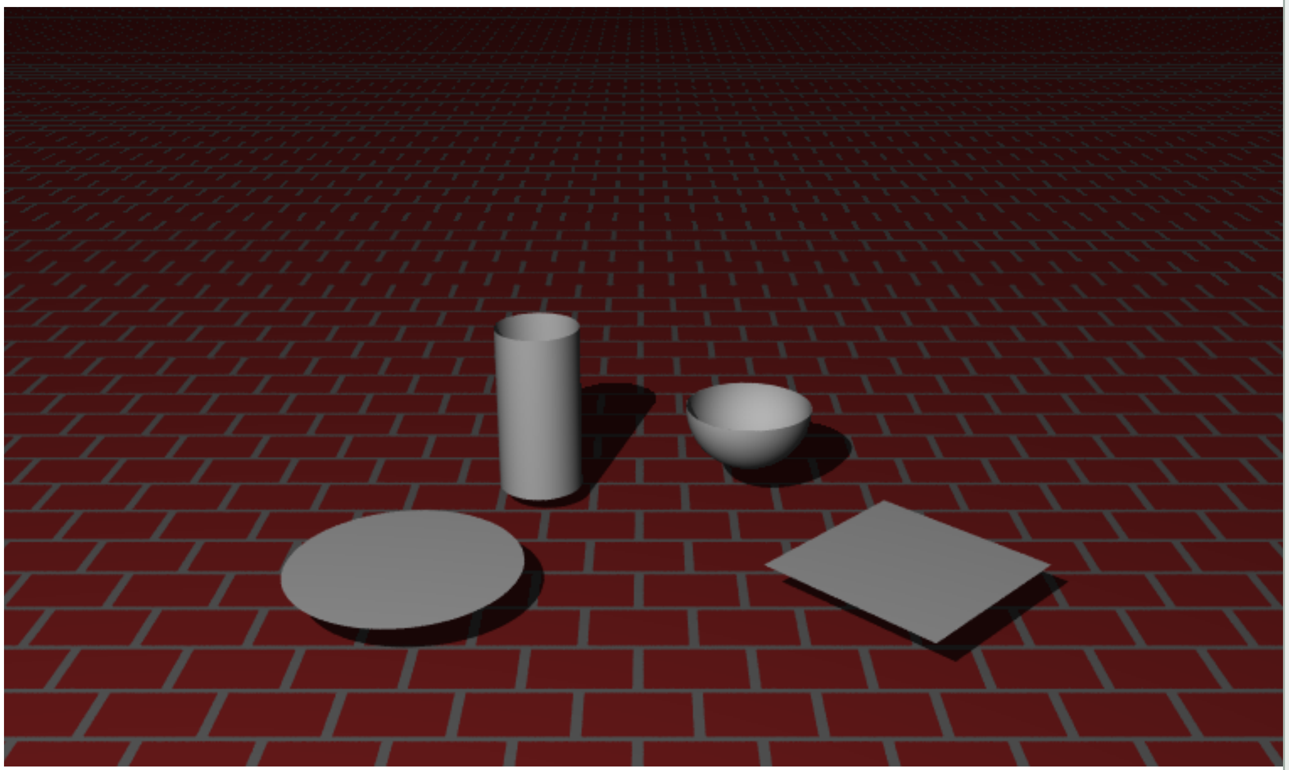
Plane/Disk Impact Geometry



Cylinder Impact Geometry



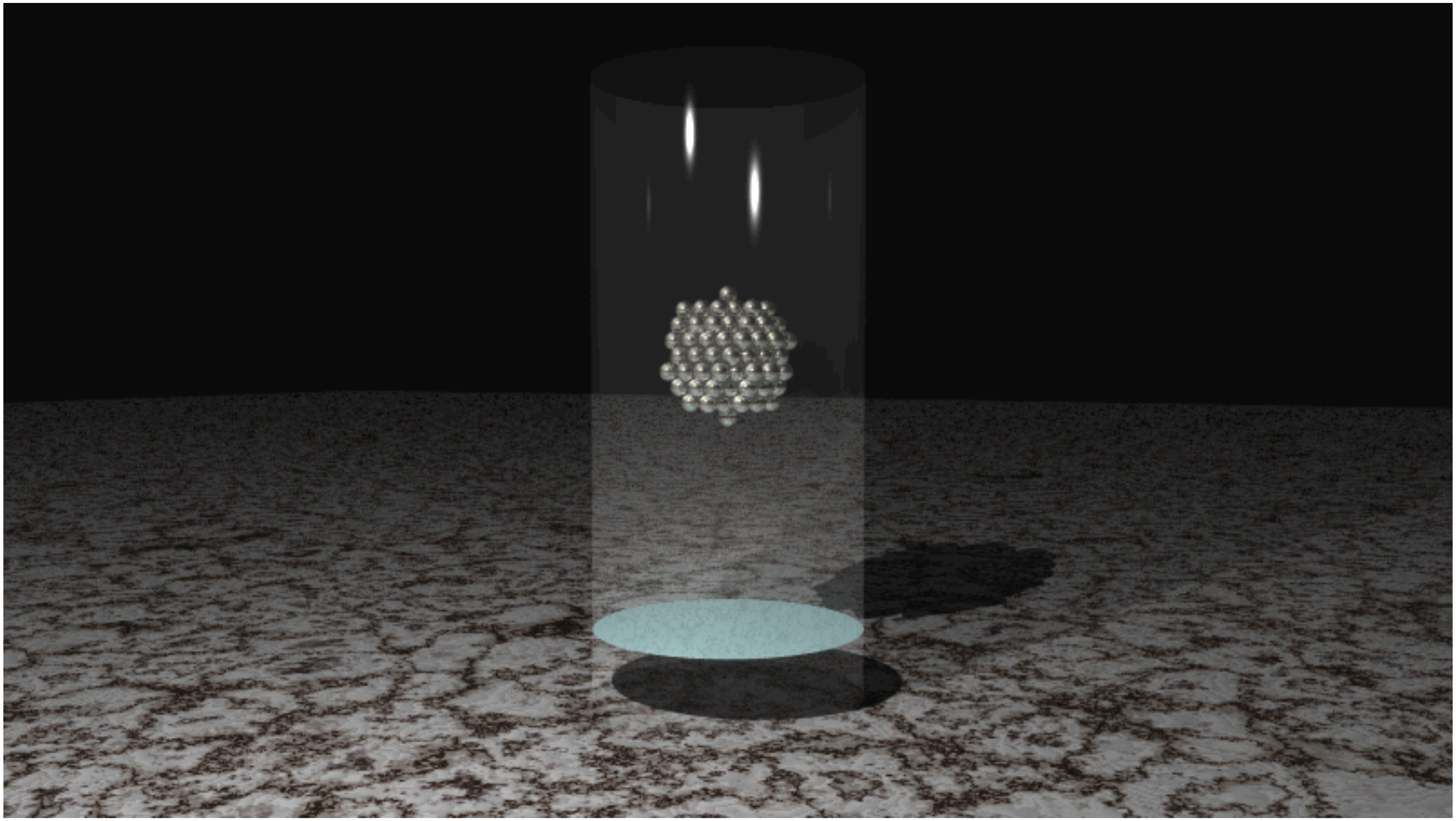
Example Configuration



Ray-traced with POV-Ray

```
wall type plane
  transparency 1

wall type disk
  origin -1 0 0.2
  orient 0 0 1
  radius 0.5

wall type cylinder-finite
  origin -0.5 1 0.5
  radius 0.2
  length 0.8

wall type shell
  origin 0.5 1 0.5
  radius 0.3
  open-angle 90

wall type rectangle
  origin 0.5 0 0.2
  vertex1 -0.6 0.6 0
  vertex2 0.6 0.6 0
```

Example



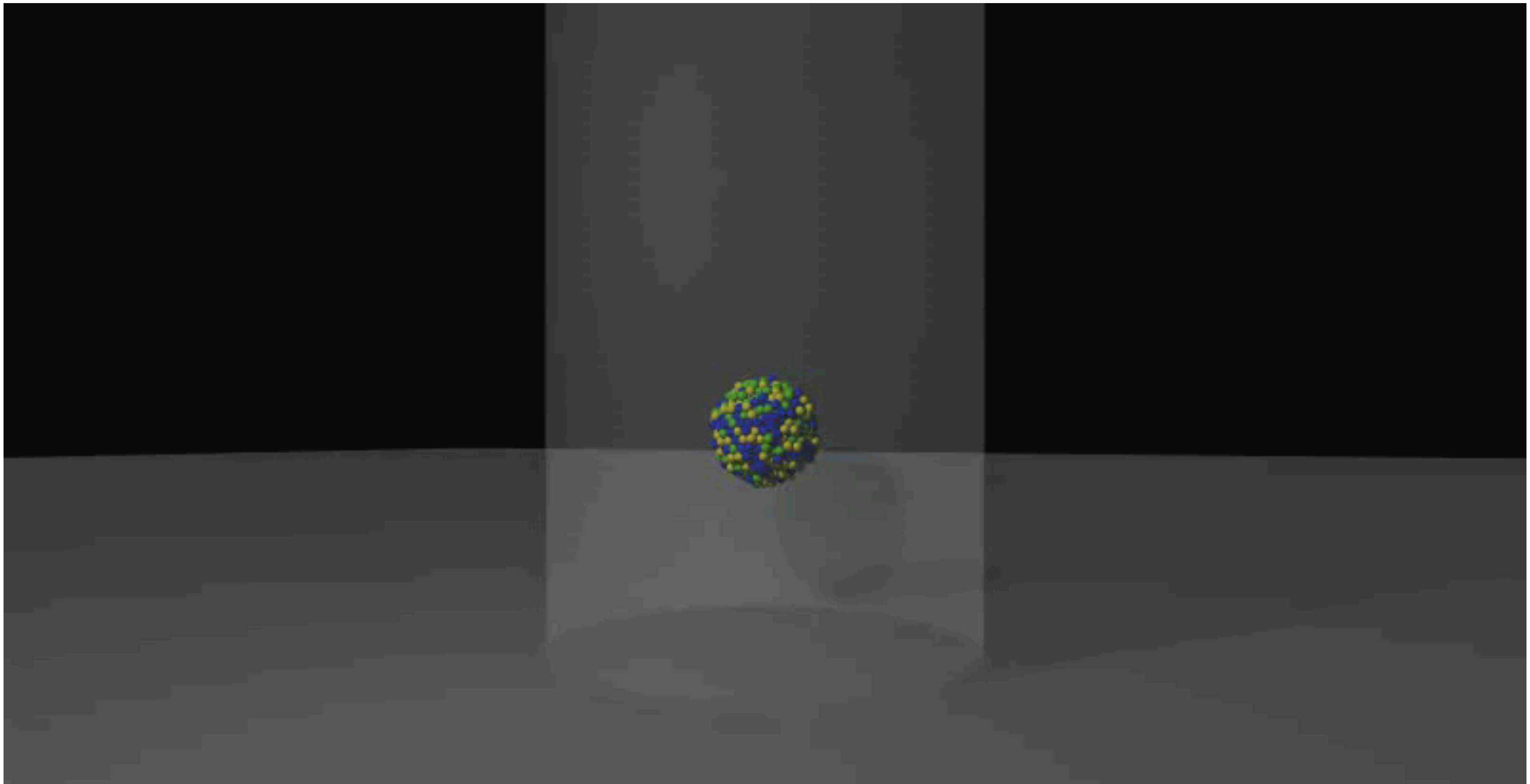
Test: Model Atmosphere

- Drop ~1000 particles in cylinder.
- NO dissipation (walls or particles).
- Particle masses 1, 3, 10 (all same radius).
- Expect energy equipartition, leading to a vertical probability distribution:

$$P_m(z) \propto \exp\left(-\frac{z}{h_m}\right),$$

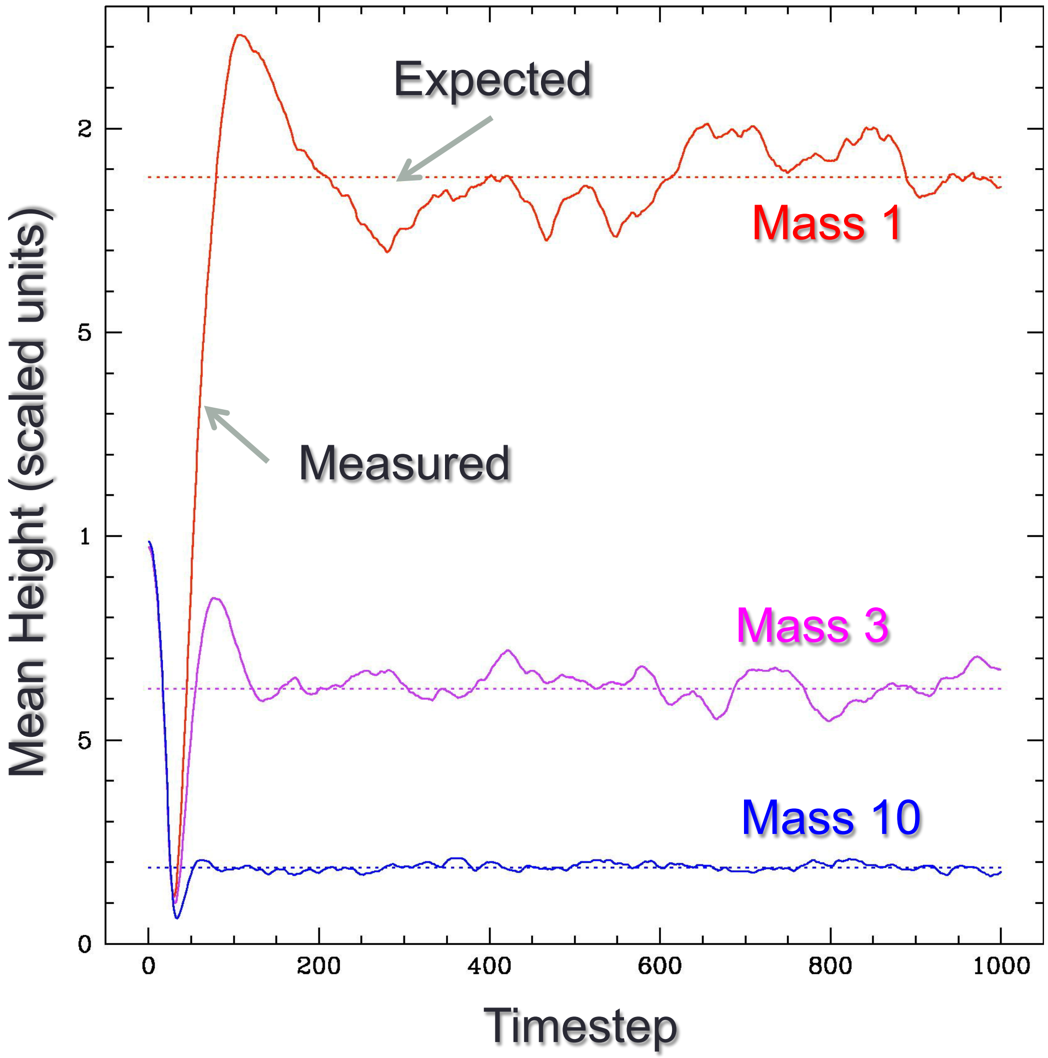
where h_m = (2/5) <*E*>/*mg*, and <*E*> = *E*/*N* is the mean particle energy (KE + PE).

Test: Model Atmosphere

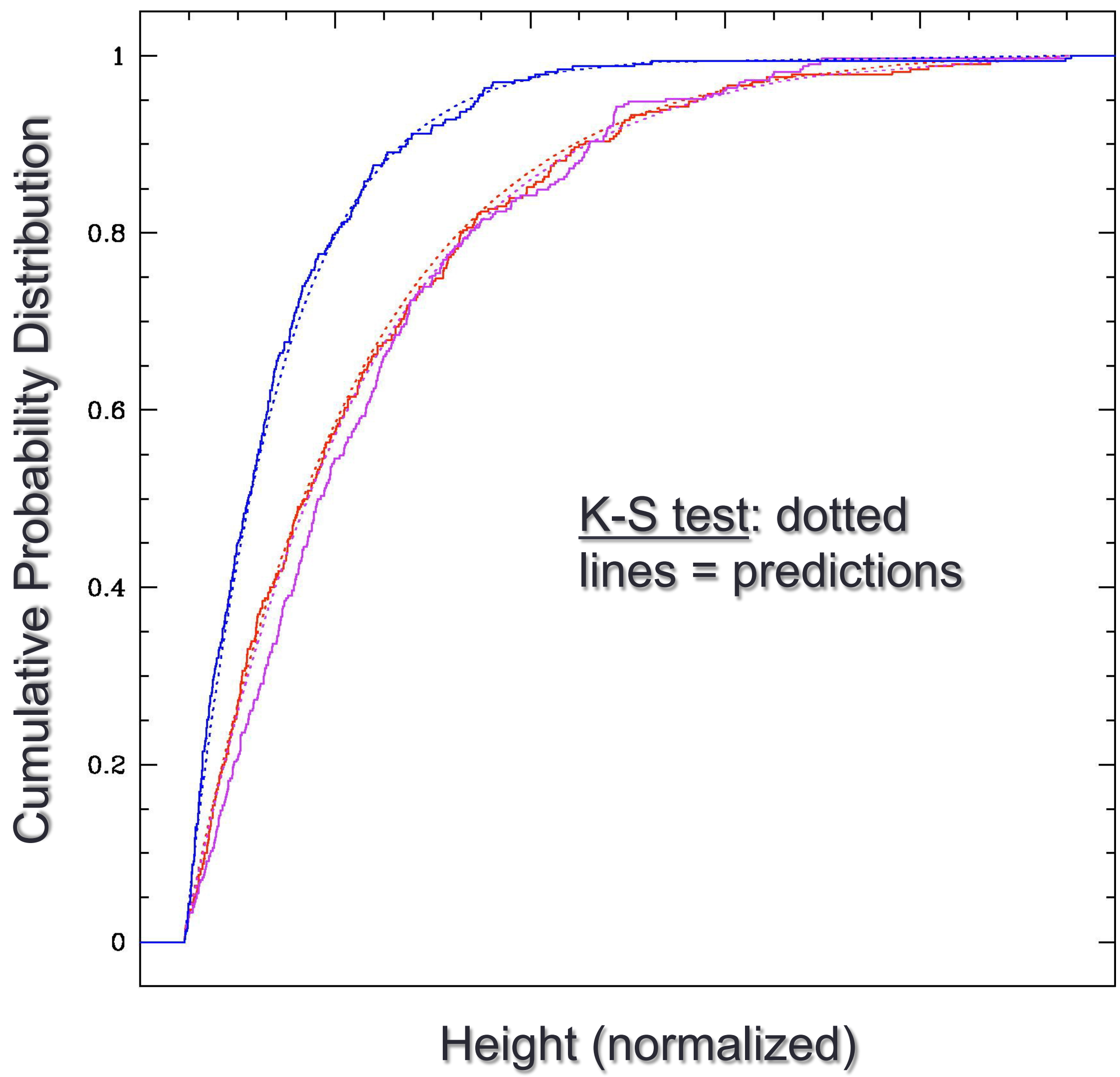


Green, blue, yellow = mass 1, 3, 10. Only bottom portion shown.

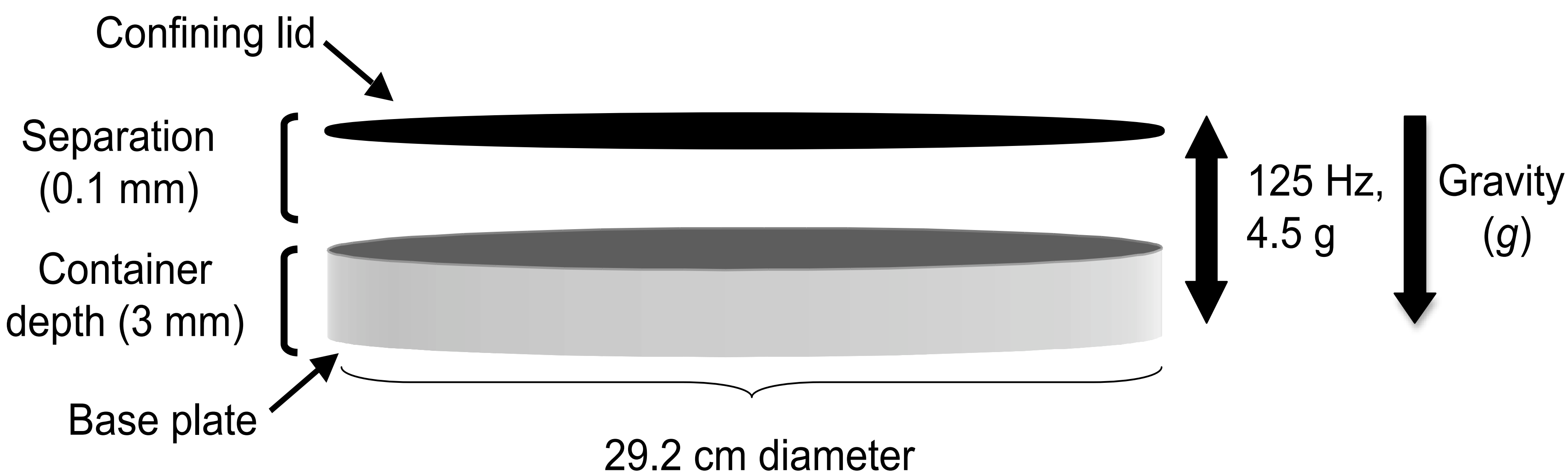
Test: Model Atmosphere



Test: Model Atmosphere



Test: Vibrating Plate (Murdoch et al. 2011, submitted)



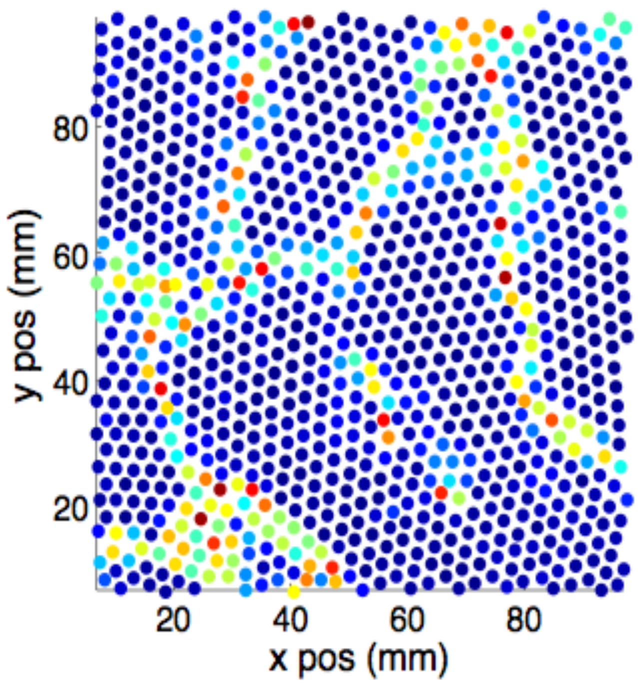
Berardi *et al*. 2010: vibrate densely packed layer of particles (3mm and 2mm) at nearly close packing (~85%).

Note: Figure not to scale.

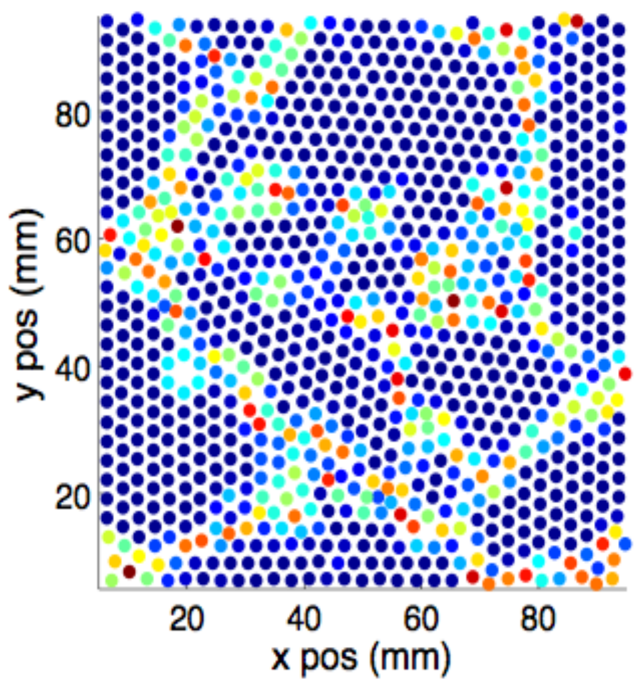
Grains, Boundaries, & Strings

We correctly model grains, grain boundaries, and "strings."

85% total coverage and 3% small particle additives.



Lab Experiment†



Numerical Simulation*

Purple: near hexagonal particle packing.

Red: more disordered packing (i.e. GB regions).

† Berardi *et al.*, 2010

* Murdoch *et al.*, 2011 (submitted)

Test: Tumbler

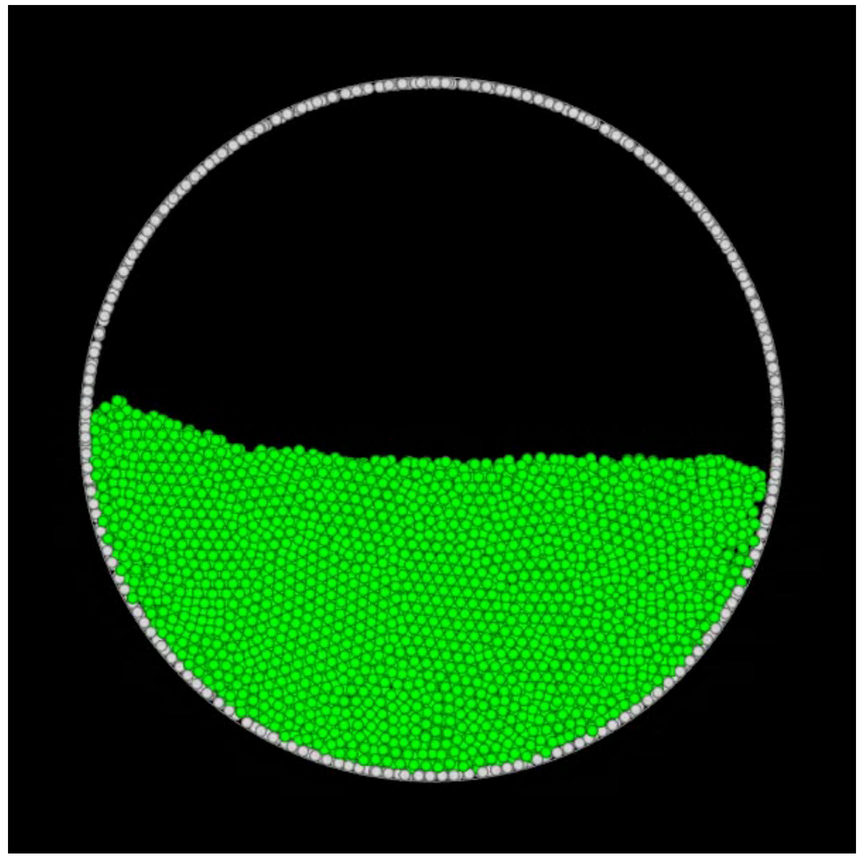
- Attempt to replicate experiments of Brucks et al. (2007).
- Idea: rotate short cylinder (radius R, half-filled with beads) at various rates. Measure dynamical angle of repose.
- Theory: response is a function of the "Froude" number

$$\mathrm{Fr} = \frac{\Omega^2 R}{g}.$$

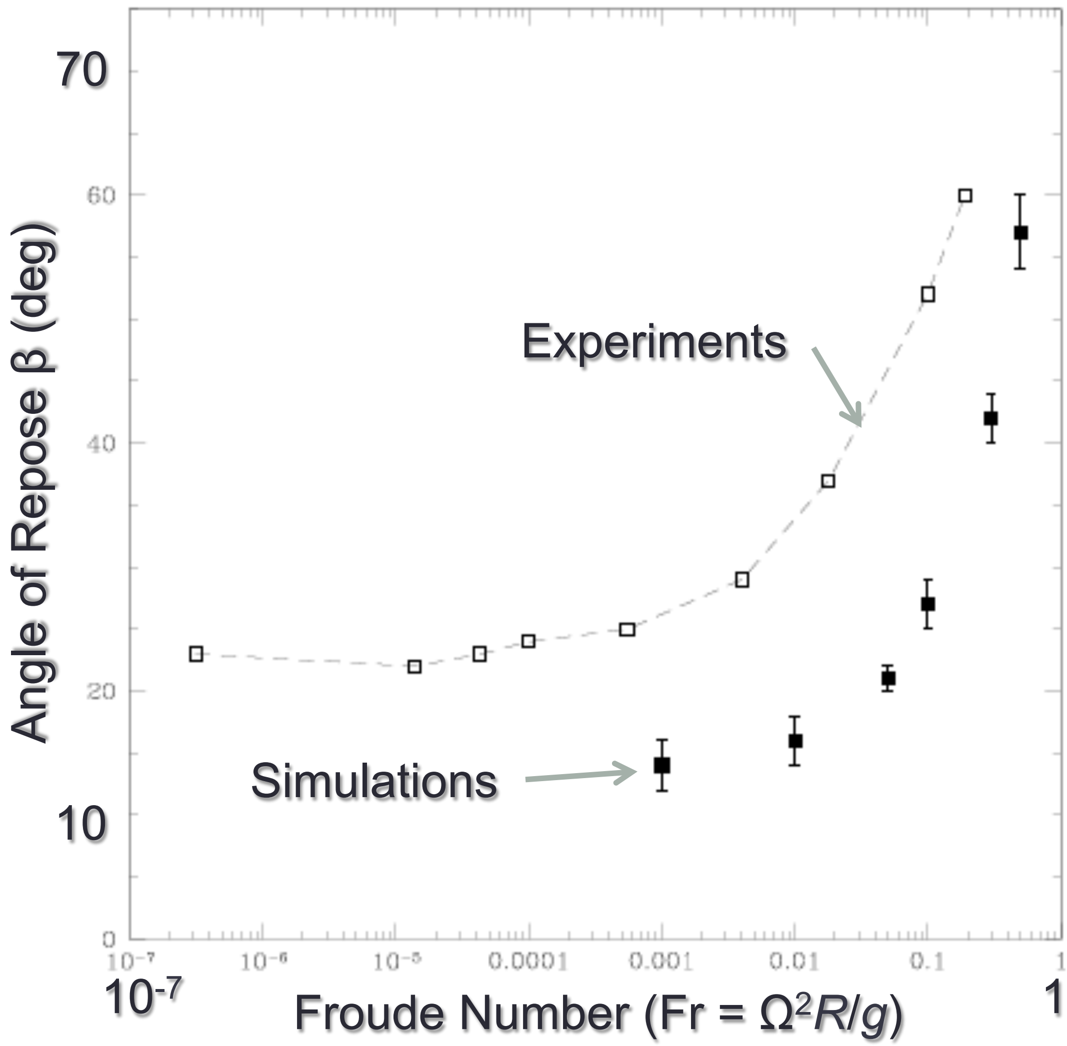
  - E.g. Fr = 1.0 ➔ centrifuging.

Test: Tumbler

- 3-D simulation (cylinder is about a dozen particle diameters long).
- Wall roughness provided by gluing particles to inner wall (experiments used coarse sandpaper).
- Movie: Fr = 0.5.

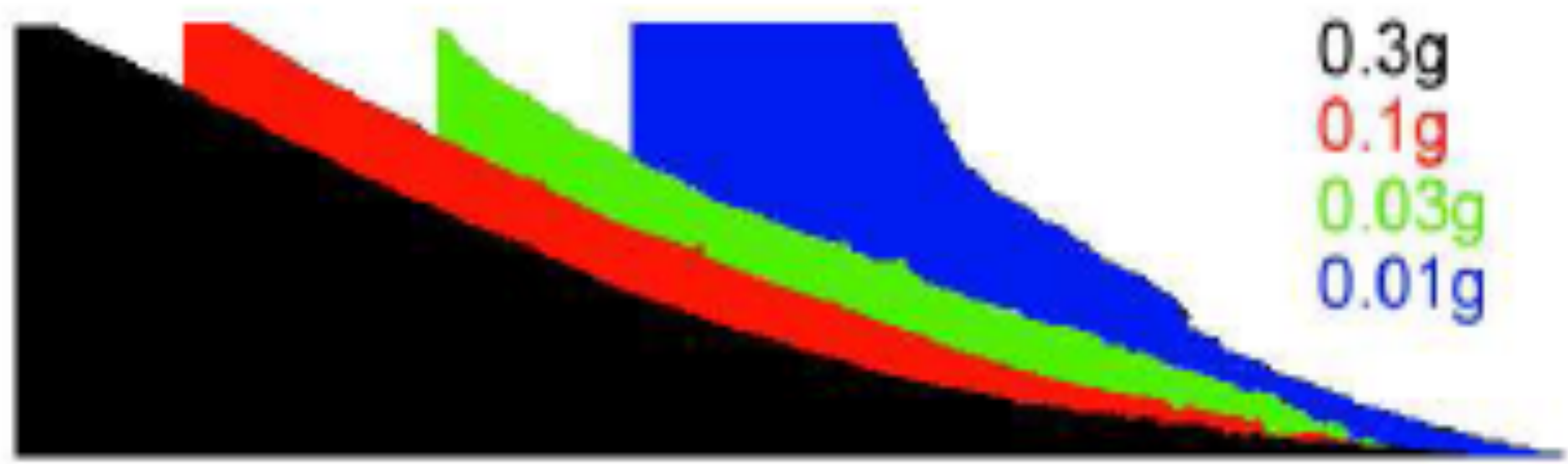


Test: Tumbler



Avalanche: Experiment

Different morphologies at end of drop-tower flight (after 4.7 s):



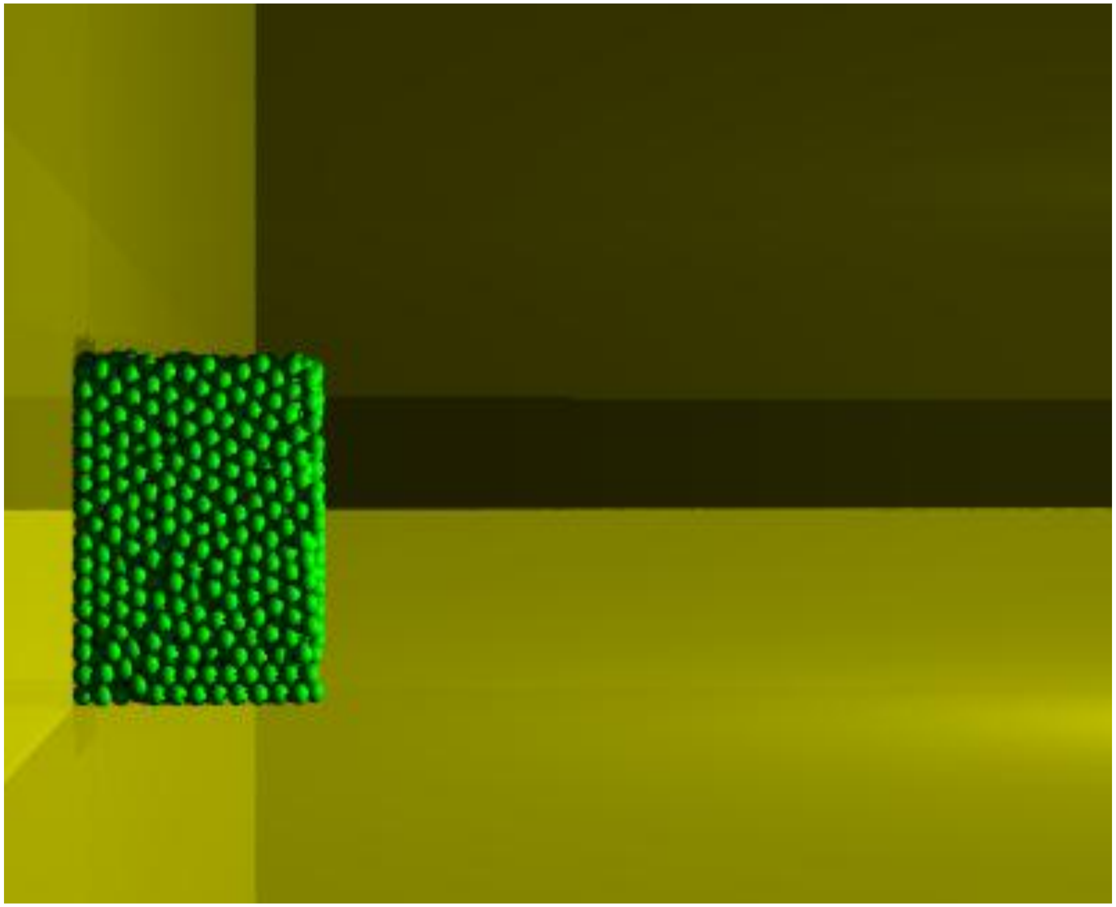
Hofmeister *et al.* 2009

- Above were experiments with GLASS beads (size: 0.1–0.2 mm).
- Avalanches are SHORTER with decreasing gravity ➔ cohesion.

Avalanche: Simulation

Towards Center of Rotation

0.1g



~2000 particles

Gravity = 0.1 g

Full parameter space still being explored

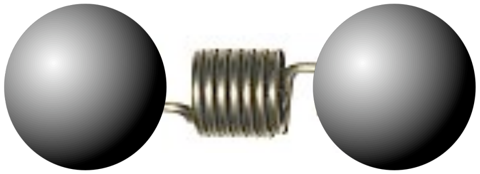
1g until Drop

HSDEM Successes and Failures

- HSDEM works well in hot, dilute "gas" regime, less well in cold, dense regime.
  - E.g. Dynamic repose angles too low in tumbler experiments.
- What is missing is "stickiness" and "true" surface friction.

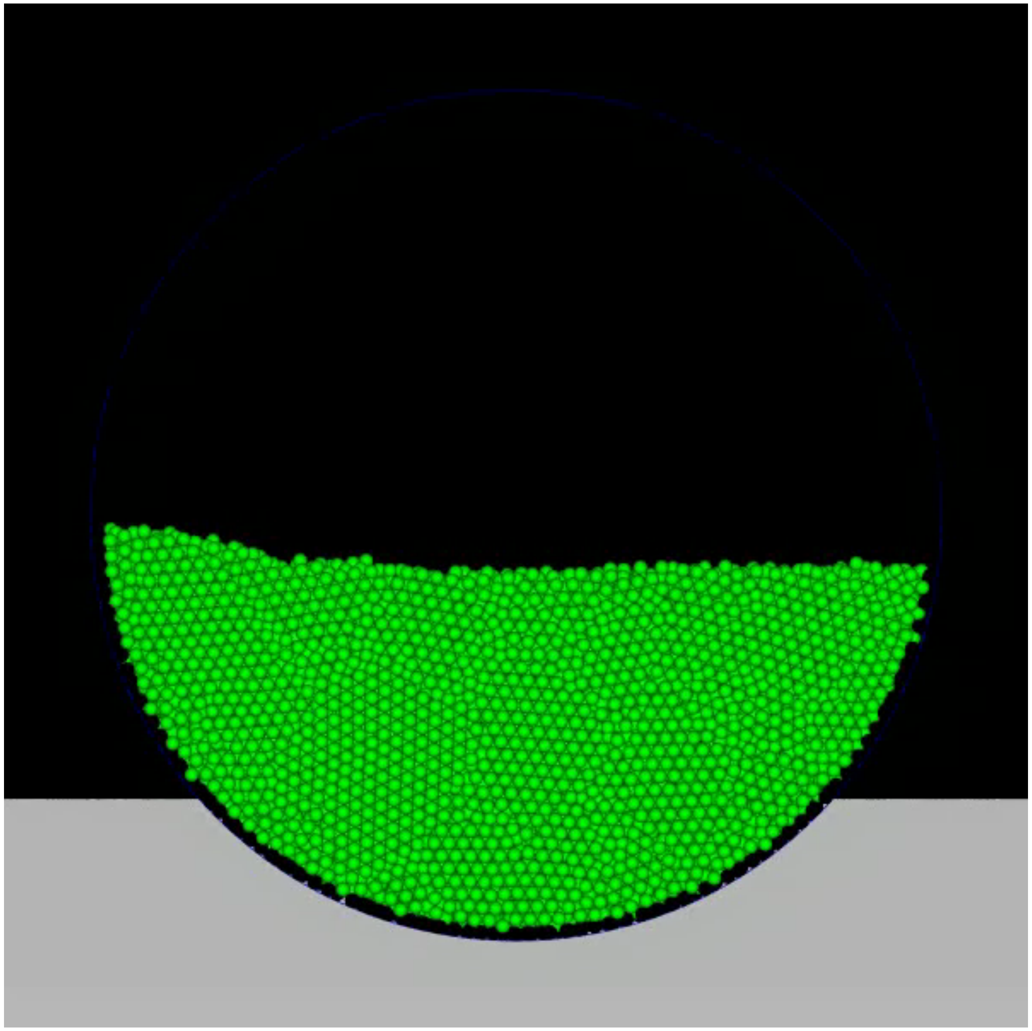
Modeling Weak Cohesion

- Add simple Hooke's law restoring force between nearby particles.



- Deform elastically up to maximum strain (spring rigidity set by Young's modulus).
- Other force laws can be implemented, e.g. van der Waals.

Weak Cohesion in Granular Fluids



Soft-sphere Discrete Element Method (SSDEM): *Stephen Schwartz*

- Cf. Cundall and Strack 1979; Cleary 1998.
- Allow (spherical) particles to penetrate.
  - Resulting forces depend on relative velocities, spins, and material properties of particles.
- Use neighbor finder to find overlaps in ~$O(N \log N)$ time. Also works in parallel.
- Strategy: let $x = s_p + s_n - |\mathbf{r}_p - \mathbf{r}_n|$. Overlap means $x > 0$.

Normal Restoring Force

- Overlapping particles feel a normal restoring force:

$$\mathbf{F}_{N,\mathrm{rest}} = -(k_N x)\hat{\mathbf{n}}, \quad \hat{\mathbf{n}} \equiv (\mathbf{r}_p - \mathbf{r}_n)/\left|\mathbf{r}_p - \mathbf{r}_n\right|.$$

  - Here k_N is a constant that can be tuned to control the amount of penetration.
  - This example uses Hooke's law (linear in x); other forms easily included.

Tangential Restoring Force

- Overlapping particles also feel a tangential restoring force:

$$\mathbf{F}_{T,\mathrm{rest}} = k_T \mathbf{S}.$$

  - Here **S** is the vector giving the tangential projection of the spring from the equilibrium contact point to the current contact point.
  - The tangential direction comes from the total relative velocity at the contact point:

$\mathbf{u}_N$

$$\hat{\mathbf{t}} = \mathbf{u}_T / |\mathbf{u}_T|, \text{ where } \mathbf{u}_T \equiv \mathbf{u} - (\mathbf{u}\cdot\hat{\mathbf{n}})\hat{\mathbf{n}}, \text{ and}$$

$$\mathbf{u} = \mathbf{v}_p - \mathbf{v}_n + l_n(\hat{\mathbf{n}}\times\boldsymbol{\omega}_n) - l_p(\hat{\mathbf{n}}\times\boldsymbol{\omega}_p).$$

Moment arms from particle centers to effective contact point.

Kinetic Friction (Damping)

- Use "dashpot" model:

$$\mathbf{F}_{N,\mathrm{damp}} = C_N \mathbf{u}_N,$$
$$\mathbf{F}_{T,\mathrm{damp}} = C_T \mathbf{u}_T,$$

  - Here C_N and C_T are material constants. If the desired coefficient of restitution is ε_N, have:

$$C_N = -2(\ln \varepsilon_N)\sqrt{\frac{k_N \mu}{\pi^2 + (\ln \varepsilon_N)^2}},$$

where μ = reduced mass = $m_p m_n / (m_p + m_n)$.

Static Friction

- Maximum supportable tangential force at contact point:

$$\mathbf{F}_{T,\max} = \left(\mu_s \left|\mathbf{F}_N\right|\right)\left(\mathbf{S}/\left|\mathbf{S}\right|\right),$$

  where μ_s is the coefficient of static friction and $\mathbf{F}_N = \mathbf{F}_{N,\text{rest}} + \mathbf{F}_{N,\text{damp}}$.
- If $|\mathbf{F}_T| > |\mathbf{F}_{T,\max}|$, **S** is set to zero (other strategies possible); here $\mathbf{F}_T = \mathbf{F}_{T,\text{rest}} + \mathbf{F}_{T,\text{damp}}$ affects spins and velocities of particles, conserving total angular momentum.

Rolling Friction

- Induced torque due to rotational friction:

$$\mathbf{M}_{\text{roll}} = \mu_r \frac{\mathbf{F}_N \times \mathbf{v}_{\text{rot}}}{\left|\mathbf{v}_{\text{rot}}\right|},$$

where μ_r is the coefficient of rolling friction and

$$\mathbf{v}_{\text{rot}} \equiv l_n(\hat{\mathbf{n}} \times \boldsymbol{\omega}_n) - l_p(\hat{\mathbf{n}} \times \boldsymbol{\omega}_p).$$

General SSDEM Equations

- Putting it all together,

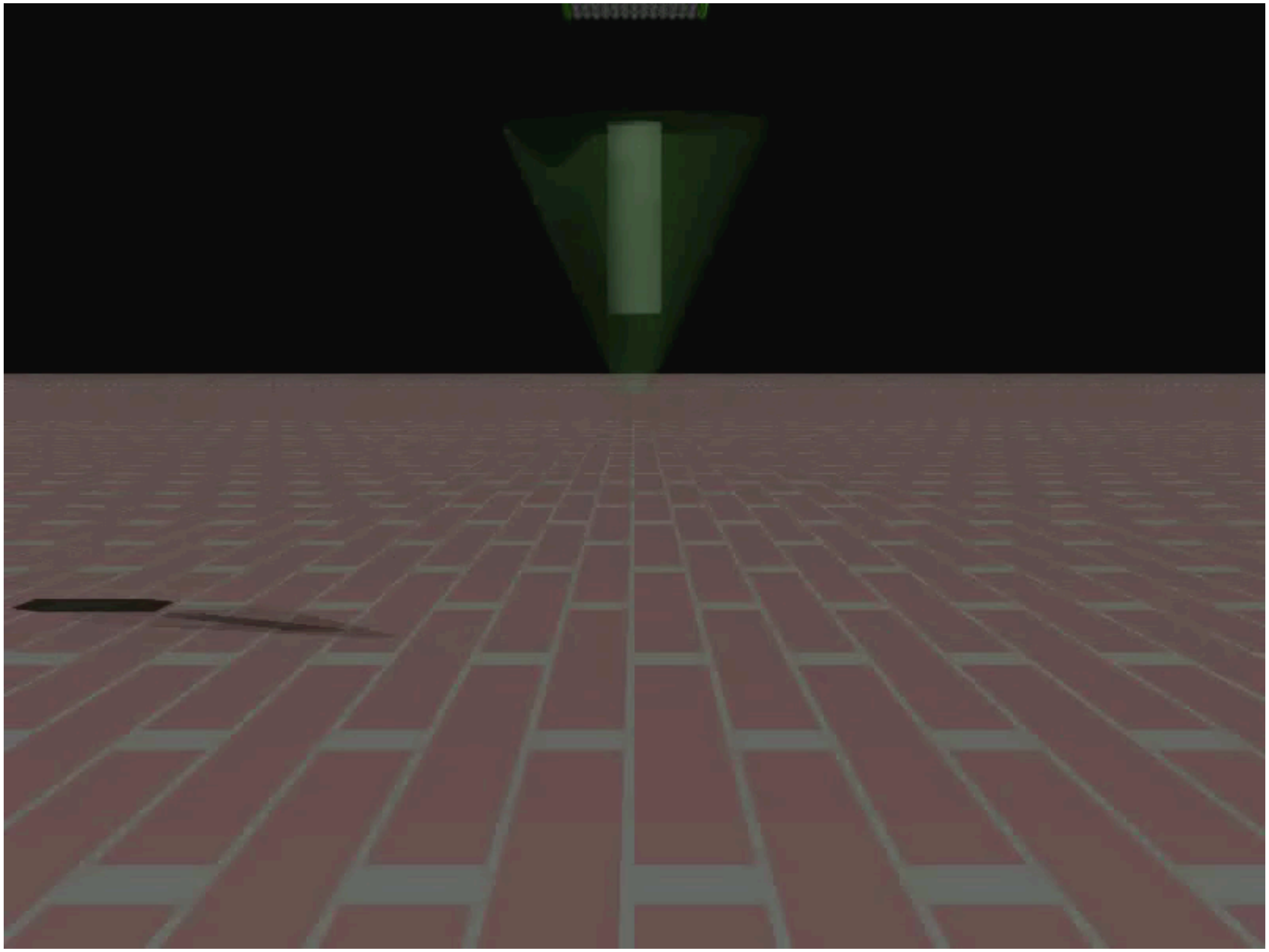
$$\mathbf{F}_p = \mathbf{F}_N + \mathbf{F}_T,$$
$$\mathbf{M}_p = l_p(\hat{\mathbf{n}} \times \mathbf{F}_T) + \mathbf{M}_{\text{roll}}.$$

- Similar expressions hold for the neighbor particle, by momentum conservation.
- We are currently implementing twisting friction as well, in order to damp relative spin around the contact normal.

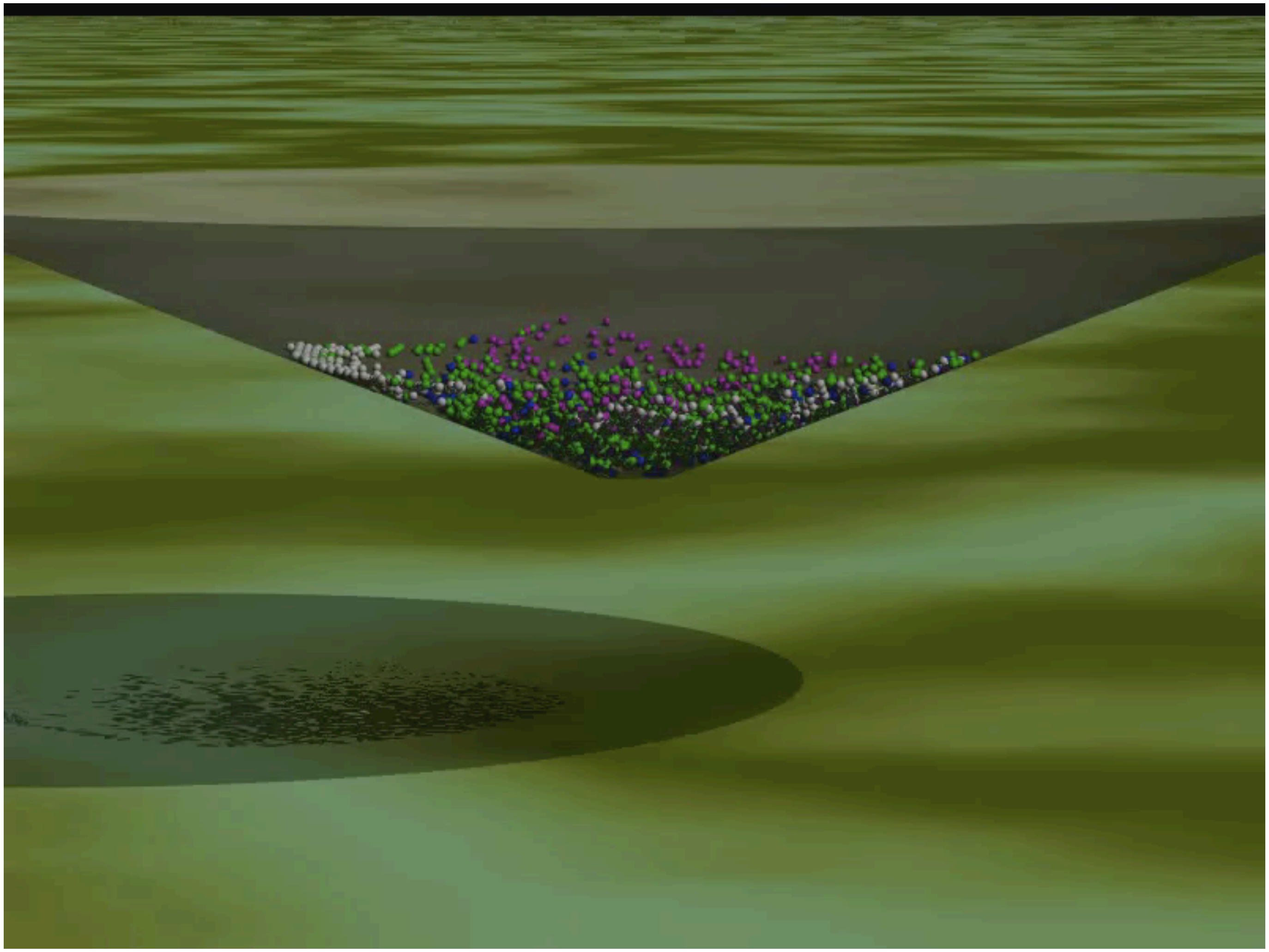
SSDEM with Walls

- Big advantage of SSDEM: do not need to *predict* particle-particle and particle-wall collisions: just detect the *overlap*.
- This comes with a price: timestep *h* must be small enough to ensure the overlap is detected.
- But, can handle more complicated geometries (e.g. cone).

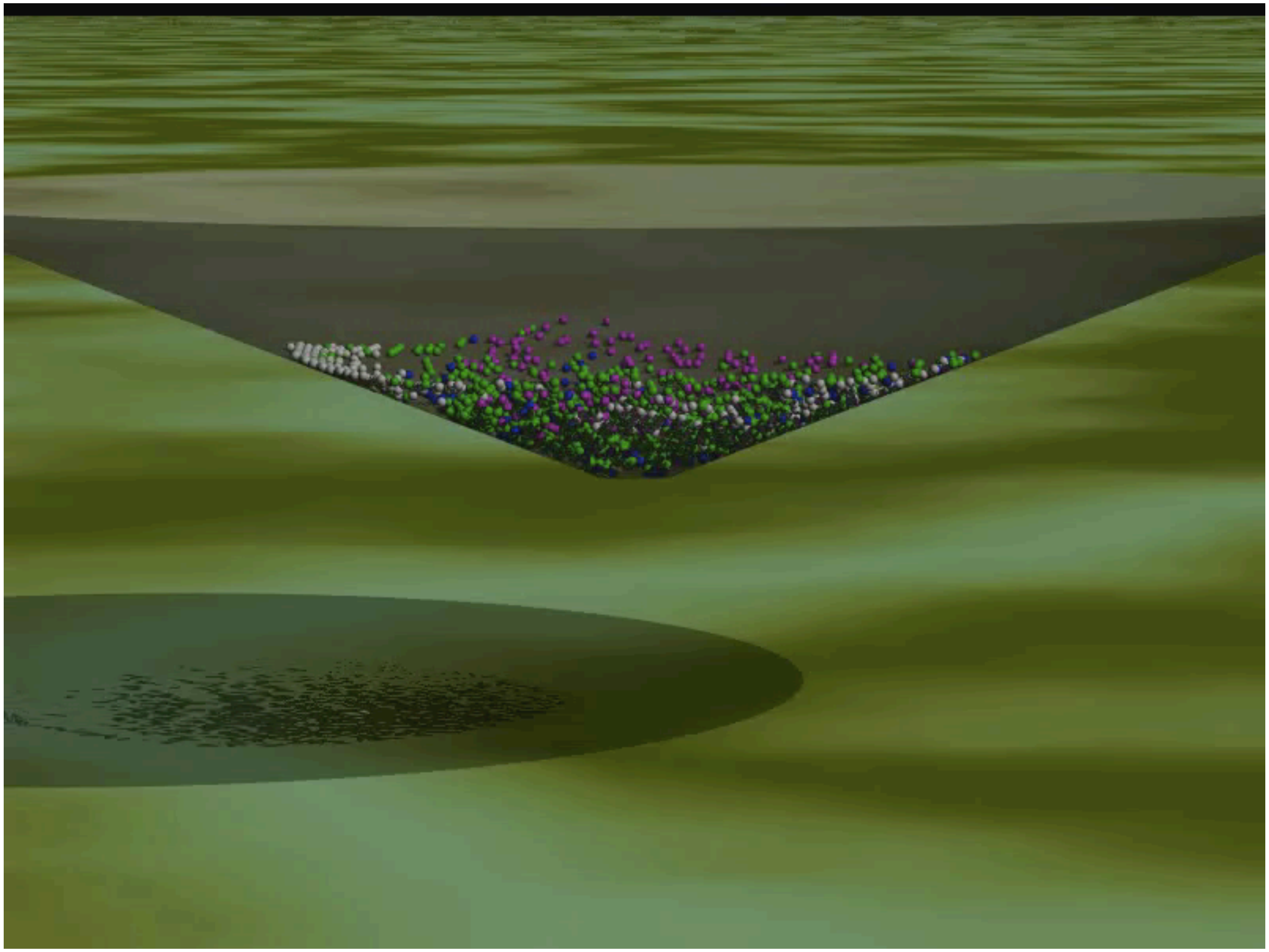
Example: Forcing Particles in a Funnel



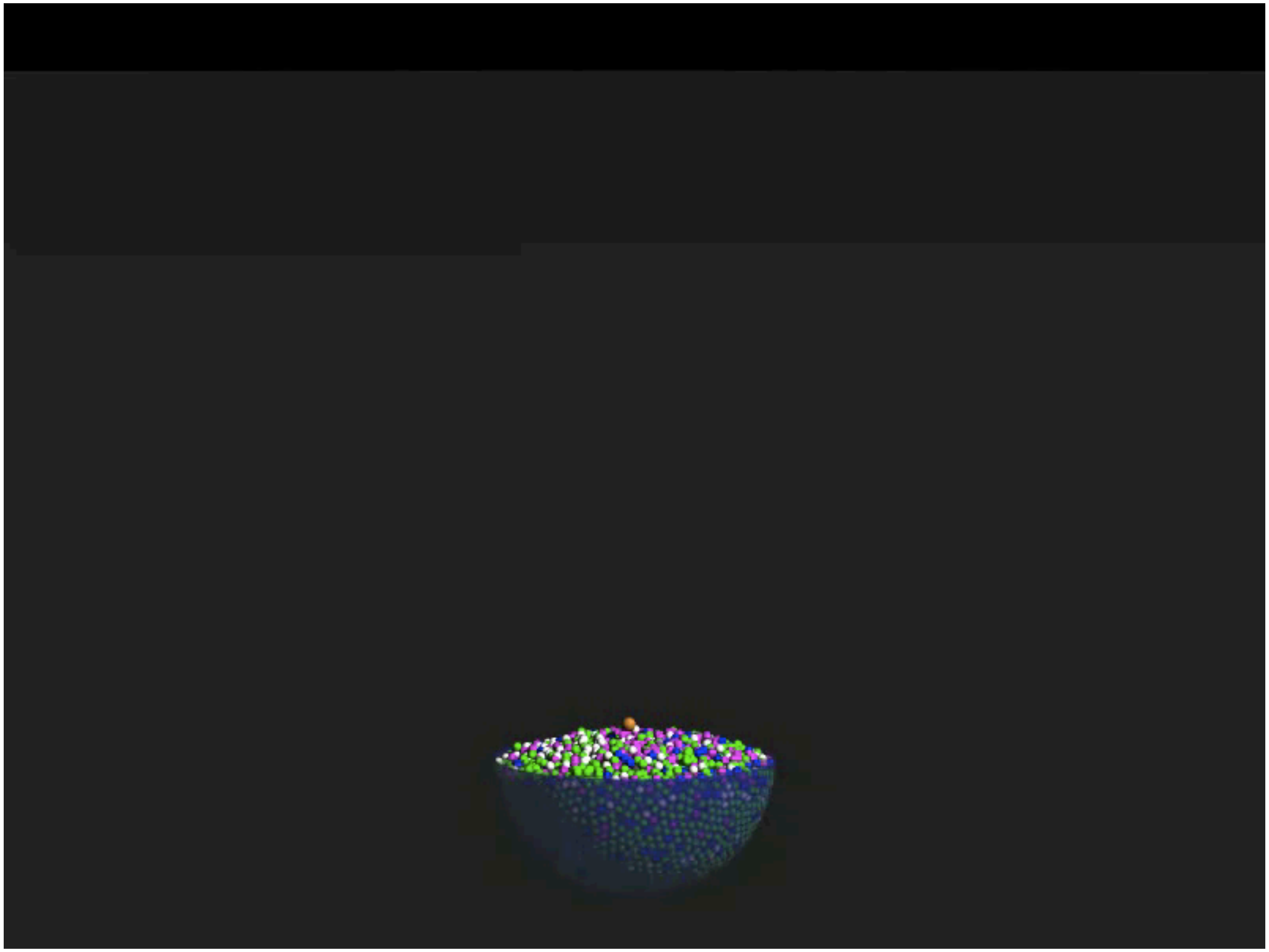
Example: Sandpile



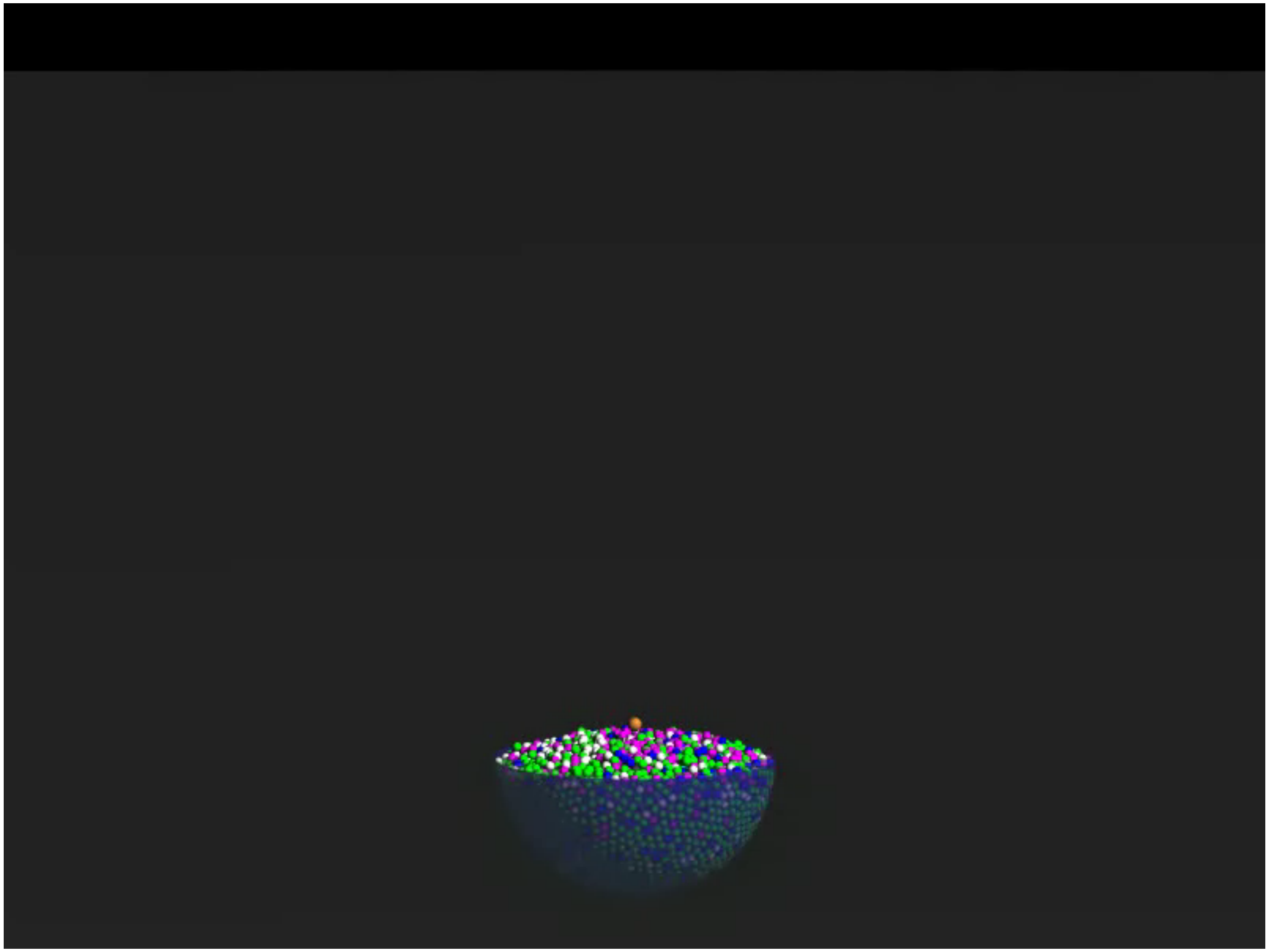
Example: Sandpile (No Friction)



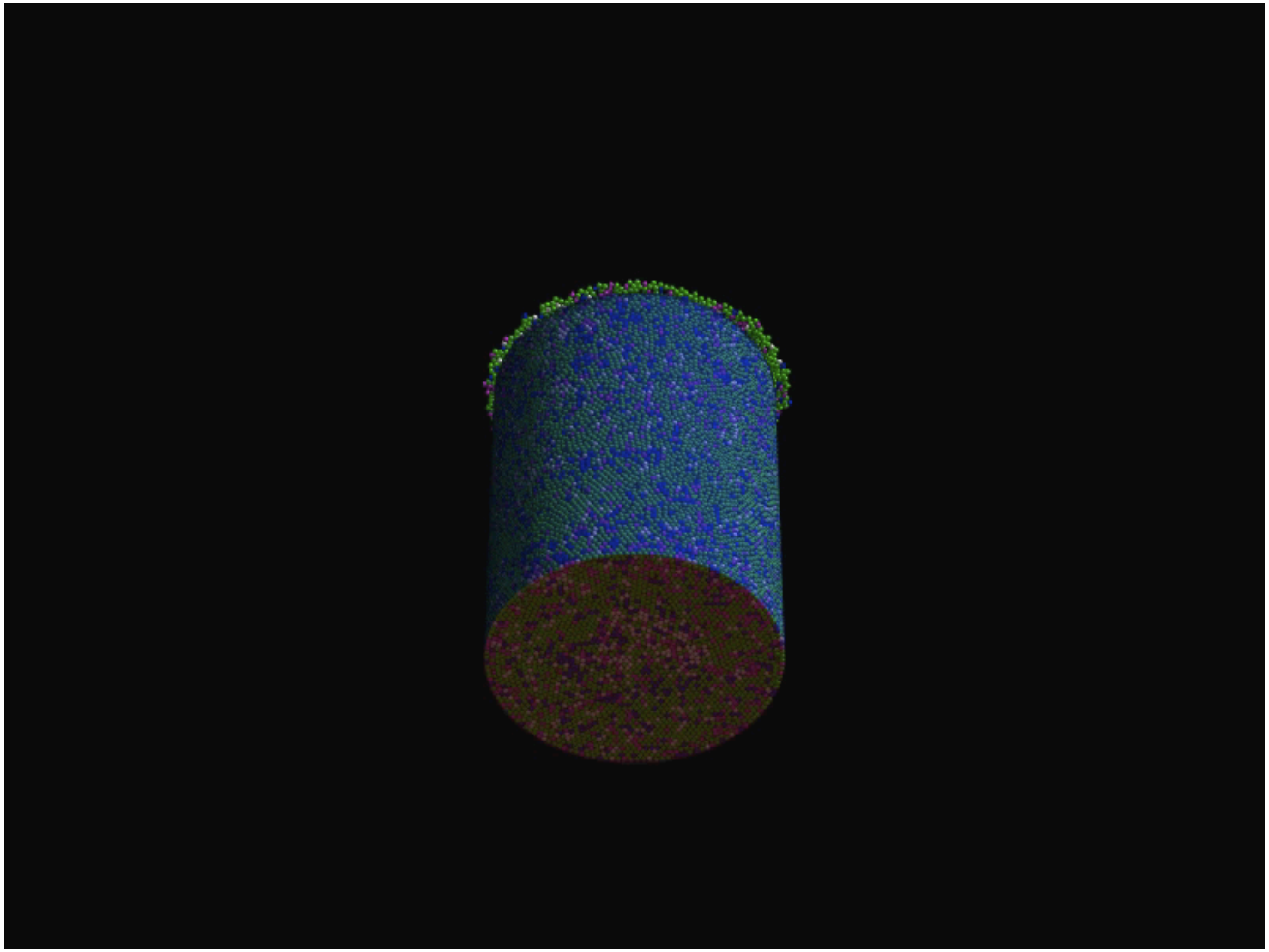
Example: Cratering



Example: Cratering (Low-energy)



Example: Hopper (*N* = 155,000)



Example: SSDEM + Springs

Summary and Future Directions

- We have adapted the *N*-body code PKDGRAV to allow for exploration of problems in granular dynamics.
- Hard-sphere DEM works well in the diffuse regime.
- Soft-sphere DEM provides more realistic friction in the dense regime.
- Goal is to construct flexible, general, efficient, accurate, hybrid HS/SSDEM for simulating wide variety of problems.