New high-order, high-frequency methods in computational electromagnetism

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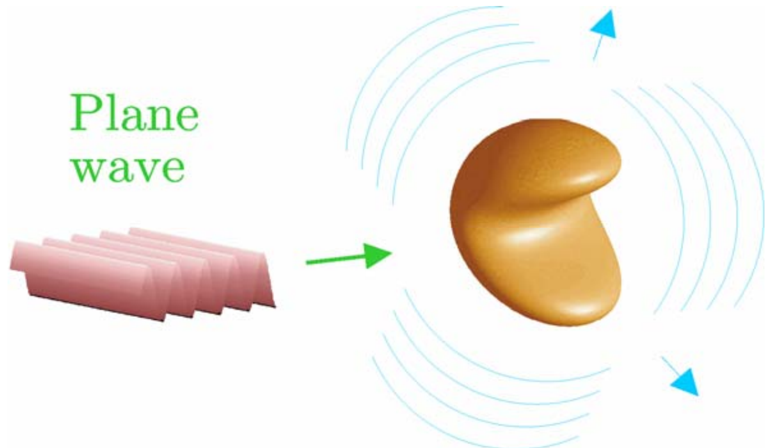
Governing Equations



$$\Delta\psi(\mathbf{r}) + k^2\psi(\mathbf{r}) = 0$$

$$\nabla \times E = i\omega\mu H$$

$$\nabla \times H = -i\omega\varepsilon H$$

$$\frac{1}{2}\varphi(\mathbf{r}) + (K\varphi)(\mathbf{r}) - i\gamma\,(S\varphi)(\mathbf{r}) = \psi^i(\mathbf{r}), \qquad \mathbf{r} \in \partial D$$

$$\Phi(\mathbf{r}, \mathbf{r}') = e^{ik|\mathbf{r}-\mathbf{r}'|}/4\pi\,|\mathbf{r}-\mathbf{r}'|$$

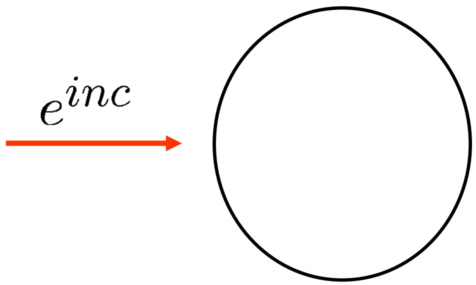
$$(K\varphi)(\mathbf{r}) = \int_{\partial D} \varphi(\mathbf{r}')\,\frac{\partial}{\partial\boldsymbol{\nu}(\mathbf{r}')}\Phi(\mathbf{r}, \mathbf{r}')\,dS(\mathbf{r}')$$

$$(S\varphi)(\mathbf{r}) = \int_{\partial D} \Phi(\mathbf{r}, \mathbf{r}')\varphi(\mathbf{r}')dS(\mathbf{r}')$$

Topics

- *High-frequency, high-order, O(1) integral solvers*
  - *Single scattering ………(Bruno, Geuzaine and Monro, [2002-05])*
  - *Multiple scattering ……………(Bruno and Reitich, [2002-05])*
  - *Volumetric……………………………(Bruno and Chaubell, in progress)*
- *Fast surface solvers ……(Bruno, Kunyansky and Paffenroth, [2001-05])*
  - *Regular-surface, singular-kernel integration*
  - *Acceleration*
  - *Singular surfaces <u>and</u> kernels*
- *Volumetric scattering*
  - *Large Volumes ……………………….(Bruno and Hyde, [2004-05])*
- *High order surface representation………(Bruno & Pohlman, [2004-05])*

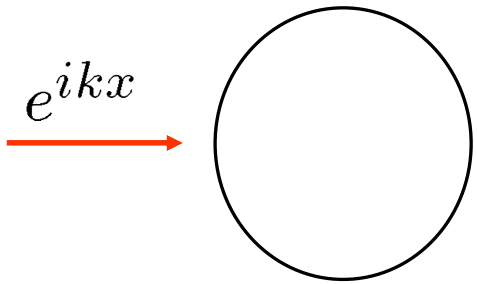
Simplest scattering integral equation example



$$\int_S \Phi(x,x')\mu(x')dx' = e^{ikx}$$

$$\Phi(x,x') = \begin{cases} H_0^1(k|x-x'|) & \text{in two dimensions} \\ e^{ik|x-x'|}/|x-x'| & \text{in three dimensions} \end{cases}$$

High Frequencies:
Phase extraction



$$\int_S H_0^1(k|x-x'|)\mu(x')dx' = f_{slow}(x)e^{ikx}$$

Ansatz: $\mu(x) = \mu_{slow}(x)e^{ikx}$

⇒

Highly oscillatory

$$\int_S \left[H_0^1(k|x-x'|)e^{ik(x'-x)}\right]\mu_{slow}(x')dx' = f_{slow}(x)$$

Previous Work

(Convex scatterers)

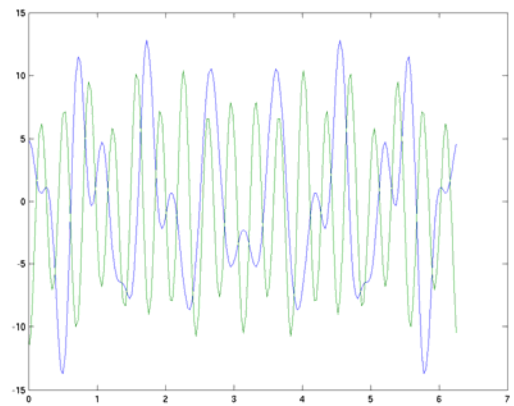
- *Melrose & Taylor, [1985], theoretical considerations*
- *Abboud, Nédélec & Zhou, [1994], $O(k^{2/3})$ operations*
- *Lagreuche and Bettess, [2000], $O(k^{2/3})$ operations*

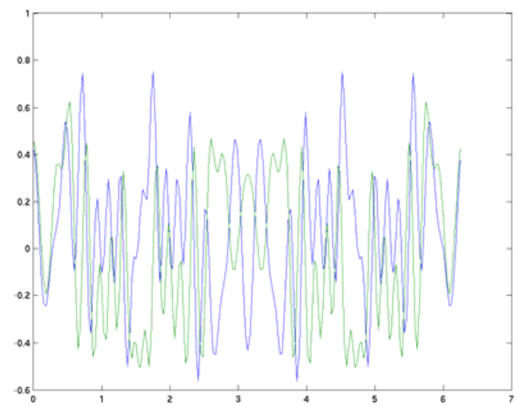
Present Approach

- *O(1) operations*
- *Convex and non-convex scatterers*
- *High-order*

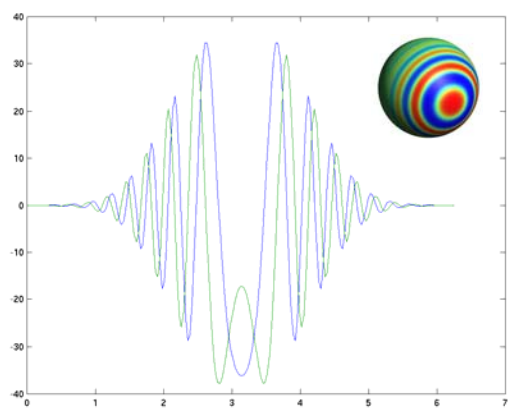
Is μ_{slow} actually slow?

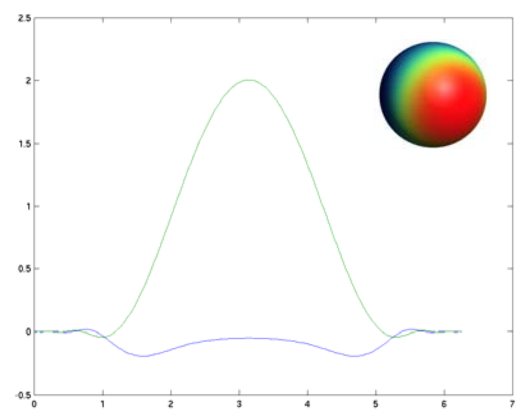
$$\frac{\varphi(\mathbf{r})}{2} = u^i(\mathbf{r}) - \int_{\partial D} \frac{\partial \Phi(\mathbf{r},\mathbf{r}')}{\partial \vec{\nu}_{\mathbf{r}'}} \varphi(\mathbf{r}')\, ds(\mathbf{r}') + i\gamma \int_{\partial D} \Phi(\mathbf{r},\mathbf{r}')\varphi(\mathbf{r}')\, ds(\mathbf{r}')$$





$$\frac{1}{2}\frac{\partial u}{\partial \vec{\nu}_{\mathbf{r}}}(\mathbf{r}) = \left(\frac{\partial u^i}{\partial \vec{\nu}_{\mathbf{r}}}(\mathbf{r}) + i\gamma u^i(\mathbf{r})\right) + \int_{\partial D} \frac{\partial \Phi(\mathbf{r},\mathbf{r}')}{\partial \vec{\nu}_{\mathbf{r}}} \frac{\partial u}{\partial \vec{\nu}_{\mathbf{r}'}}(\mathbf{r}')\, ds(\mathbf{r}') + i\gamma \int_{\partial D} \Phi(\mathbf{r},\mathbf{r}') \frac{\partial u}{\partial \vec{\nu}_{\mathbf{r}'}}(\mathbf{r}')\, ds(\mathbf{r}')$$





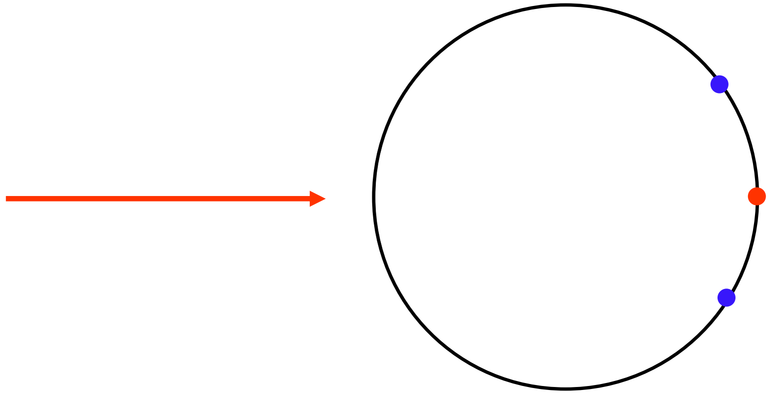
Key: Physical Density!

O(1)-methods for high-frequency scattering

Integration exercise

Highly oscillatory

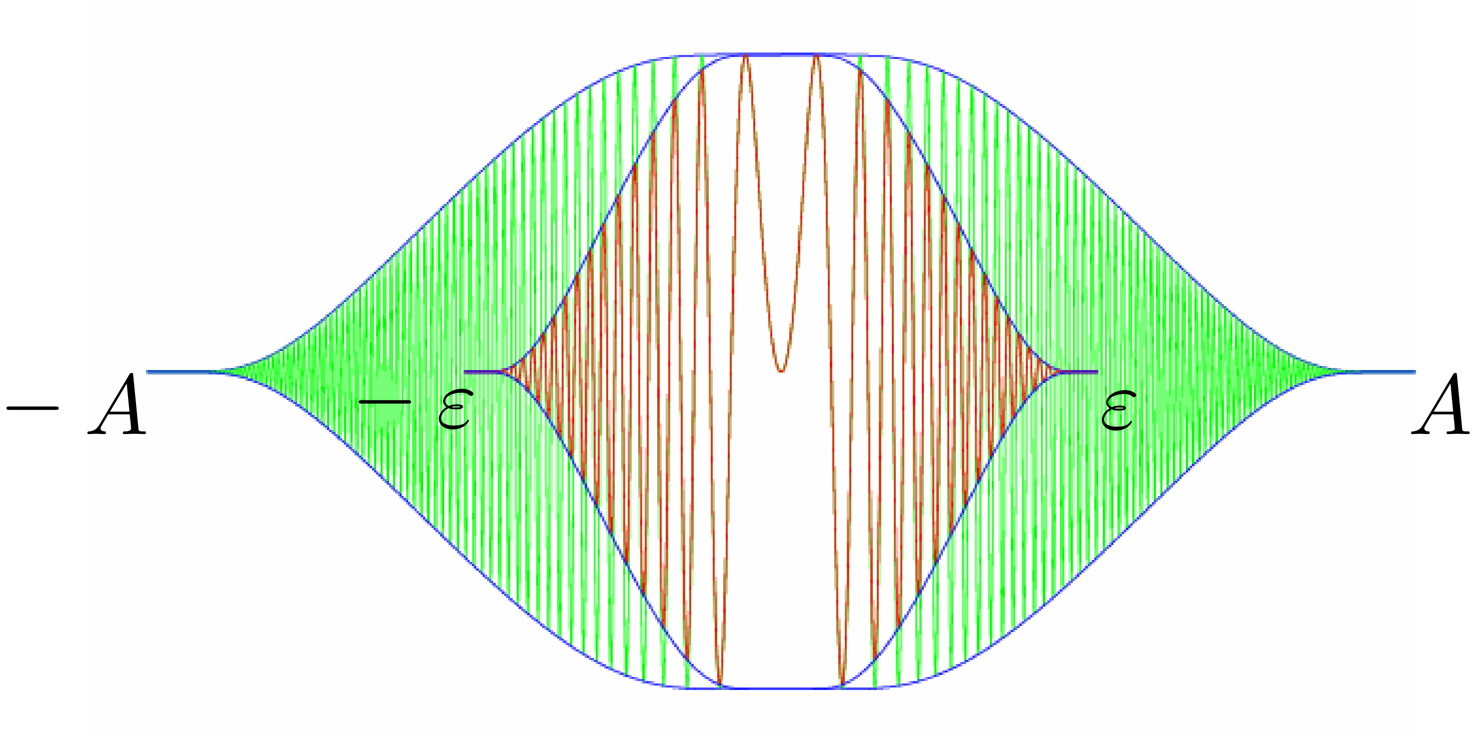
$$\int_S \left[H_0^1(\kappa|x - x'|)e^{i\kappa x'} \right] \cos(x')dx'$$



- Target Point
- Critical points (phase gradient = 0)

- Critical points?
- Asymptotically? Want convergence!!
- Idea: Why compute integral at other points?

Thus our proposed approach:
Localized Integration

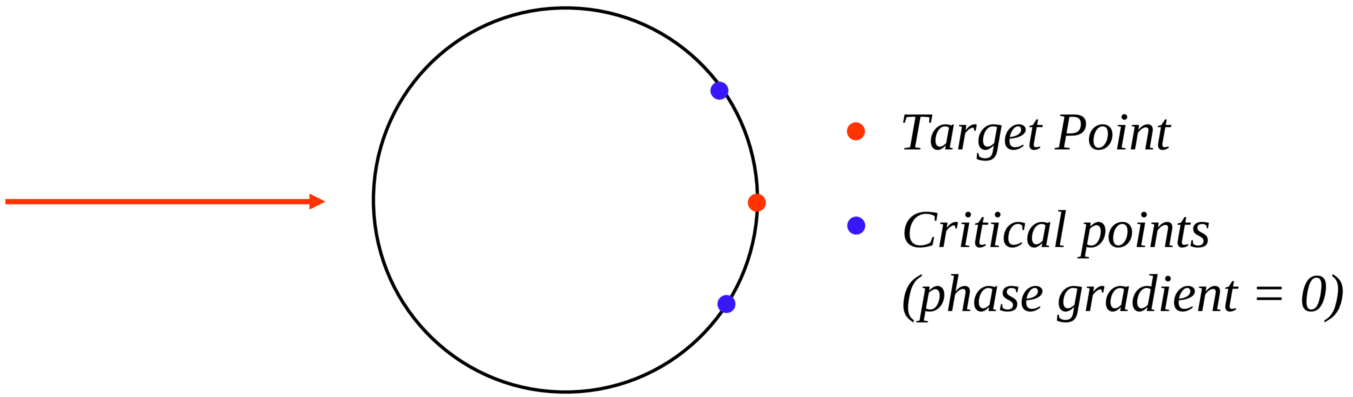


$$\int_{-A}^{A} f_A(x)e^{ikx^2} = \int_{-\varepsilon}^{\varepsilon} f_\varepsilon(x)e^{ikx^2} + \mathcal{O}((k\varepsilon^2)^{-n})$$

for all n!

Integration exercise

$$\int_S \left[H_0^1(\kappa|x - x'|)e^{i\kappa x'} \right] \cos(x')dx'$$



κ	N	ϵ	c	*Error*
1000	2100	1.0	0.5	1.5e-6
2000	2100	0.5	0.5	4.8e-8
4000	2100	0.25	0.5	1.2e-7
8000	2100	0.125	0.5	9.8e-7
16000	2100	0.0625	0.5	1.5e-6

Issues

- *Kernel Singularities*
- *Surface Representation*
- *Shadow Boundaries*
- *Creeping-Waves, Diffraction*
- *Multiple Scattering*
- *Three-dimensionality*
- *Corners, Edges*

… but first… trapezoidal rule!

Fourier Series and High-order Integration and the Trapezoidal Rule

$$\int_0^1 \sqrt{x}dx$$

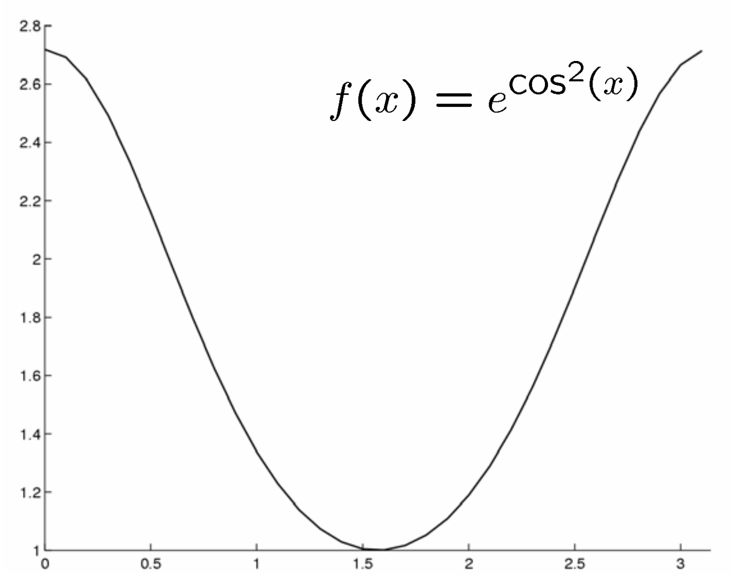
N	Rel. Error	Ratio
1	2.5(-1)	
2	9.5(-2)	2.6
4	3.5(-2)	2.7
8	1.3(-2)	2.7
8192	4.2(-7)	

$$\int_0^{\frac{\pi}{4}} e^{\cos^2(x)}dx$$

N	Rel. Error	Ratio
1	4.8(-2)	
2	1.2(-2)	4.0
4	2.9(-3)	4.0
8	7.4(-4)	4.0
8192	7.0(-10)	

$$\int_0^{\pi} e^{\cos^2(x)}dx$$

N	Rel. Error	Ratio
1	5.5(-1)	
2	6.0(-2)	9.1
4	3.1(-4)	1.9(+2)
8	7.2(-10)	4.3(+5)
16	2.1(-23)	3.4(+13)



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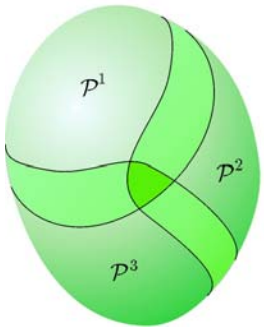
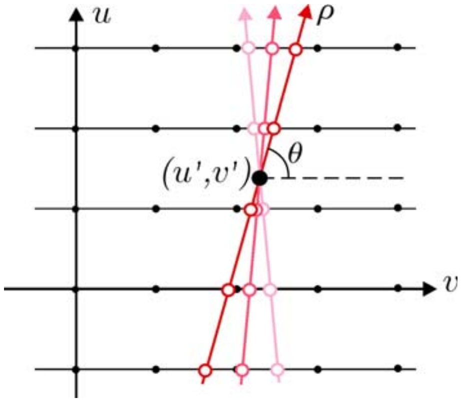
Resolution of singularities

(Basic, high-order solver; adjacent interactions)



$$\cos k\,|\mathbf{R}|\,\frac{\mathbf{R}\cdot\boldsymbol{\nu}(r)}{\mathbf{R}^3}$$

A polar-coordinate jacobian regularizes the integration problem

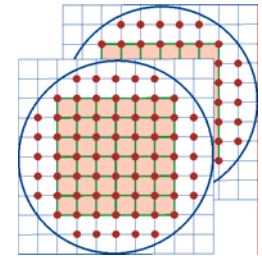


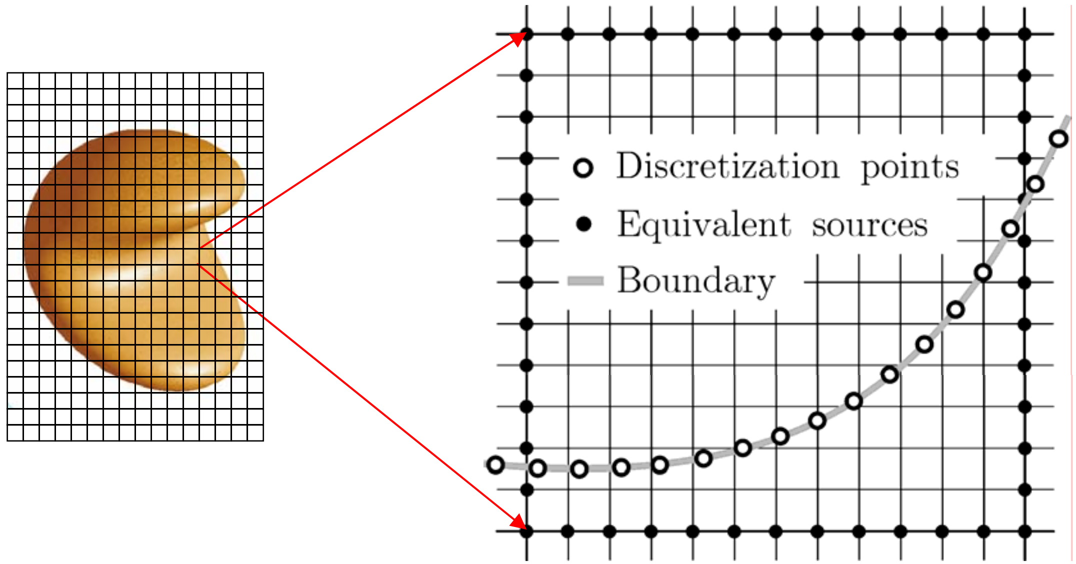
$$L(u',v',\theta)=\int_{-r_1}^{r_1} f_k^*(\rho,\theta)\frac{|\rho|}{|\mathbf{R}|}\cos k\,|\mathbf{R}|\,\frac{\mathbf{R}\cdot\boldsymbol{\nu}(r)}{\mathbf{R}^2}d\rho$$

…TOGETHER with an acceleration strategy…

Equivalent Sources

(Acceleration; Non-adjacent interactions)





$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} = ik \sum_{n=0}^{\infty} \sum_{m=-n}^{n} h_n^{(1)}(k\,|\mathbf{r}|) Y_n^m(\mathbf{r}/\,|\mathbf{r}|) j_n(k\,|\mathbf{r}'|) \overline{Y_n^m(\mathbf{r}'/\,|\mathbf{r}'|)}$$

Remark 5. The last theorem proves the convergence of the discretized approximated kernel which is used numerically. Unfortunately, because of roundoff errors, this convergence is not numerically attained...

$$G(x;x') \approx G_N^D(x;x0) := \frac{1}{2\pi N_T} \sum_{n_T=1}^{N_T} e^{(ik(x-z_i)\cdot U(\theta_{n_T}))}$$

$$\cdot e^{(-ik(x'-z_j)U(\theta_{n_T}))} \cdot$$

$$\left[\sum_{m=-N}^{N} e^{(im(\theta_{n_T}-arg(z_i-z_j)))} K_{|m|}(-ik|z_i - z_j|)\right] \qquad (6)$$

$$\left(\theta_{n_T} = \frac{2\pi}{N_T} n_T\right)$$

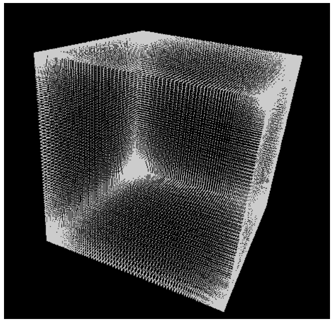
The main difficulty we face in studying Rokhlin's method lies in the fact that, even if from a theoretical point of view (see Theorems 2, 4, 6 and 7) the greater N the more accurate the approximation, N must (in numerical simulations) belong to a fixed range of integers. If N is too small, the overall accuracy is not good, which is quite logical. But if N is too large, then (6) is not numerically accurate... Hopefully, there is a range of integer values N such that the accuracy of Rokhlin's formula (6) is quite good...

C. Labreuche, "***A convergence theorem for the fast multipole method for 2-dimensional scattering problems***", *Math. Comp.* ***67****, 553-591 [1998]*

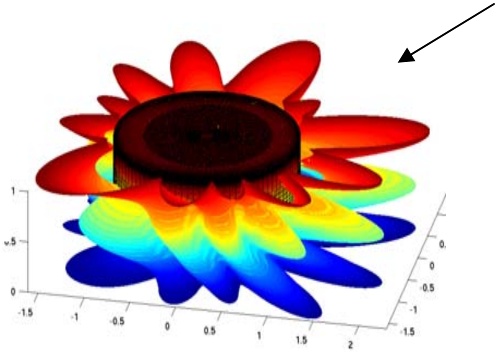
Large spheres

(comparison w/ O(N log(N)) FISC)

Algorithm	Diameter	Time	RAM	Unknowns	RMS Error	Computer
FISC	120λ	$32 \times 14.5h$	$26.7Gb$	9,633,792	4.6%	SGI Origin 2000 (32 proc.)
Present	80λ	$55h$	$2.5Gb$	1,500,000	0.005%	AMD 1.4GHz (1 proc.)
Present	100λ	$68h$	$2.5Gb$	1,500,000	0.03%	AMD 1.4GHz (1 proc.)



Singular Scatterers



Geometry	Diameters	Time	Unknowns	RMS Error	Computer
Cube (Present work)	$10\lambda \times 10\lambda \times 10\lambda$	$21h$	96,774	0.049%	AMD 1.4GHz (1 proc.)
Flying Saucer (Present work)	$42\lambda \times 42\lambda \times 17\lambda$	$53h$	290,874	0.0045%	AMD 1.4GHz (1 proc.)

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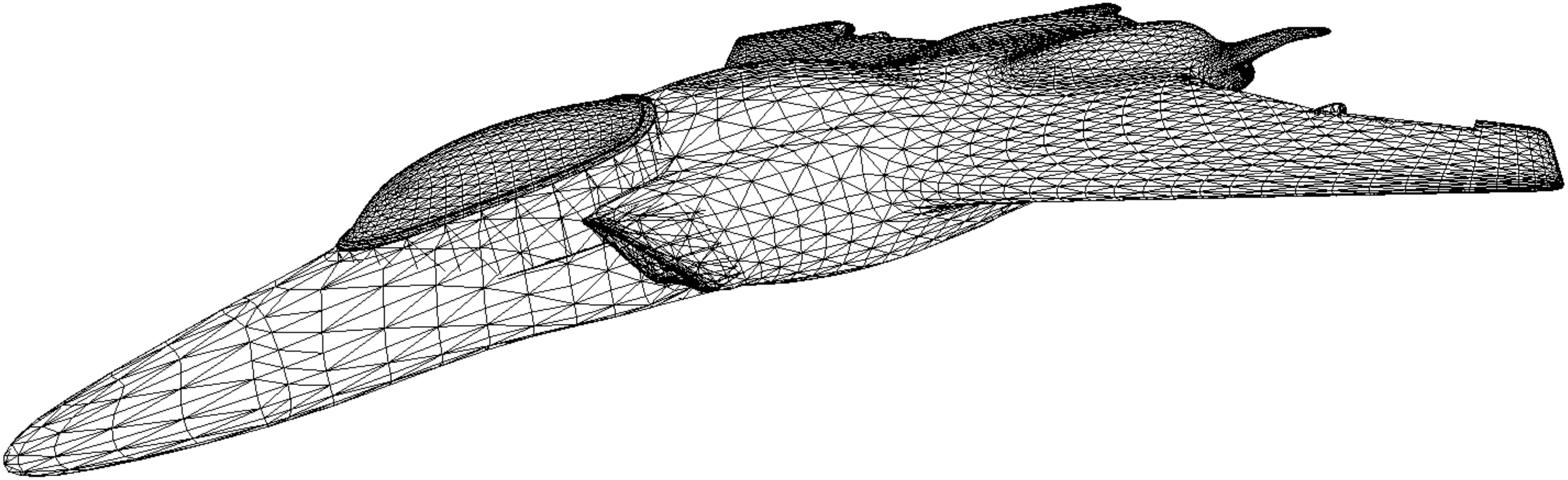
High-Order Surface Representation



Bruno, Han and Pohlman, in progress

Generation of Smooth Surfaces

A problem of present interest in the computer science literature

For general irregular triangulations, previous methods produce (at best) <u>C^1 surfaces only</u>

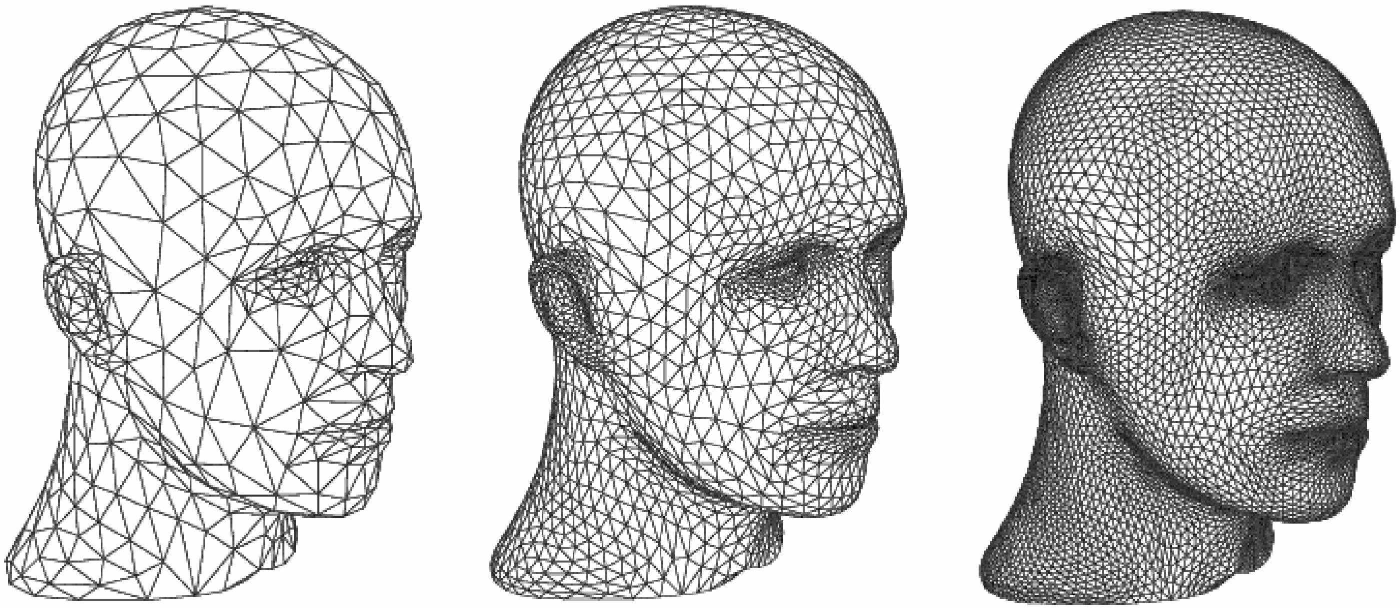


Figure 4. Two-dimensional loop subdivision is used to generate smooth surfaces from a coarse description.

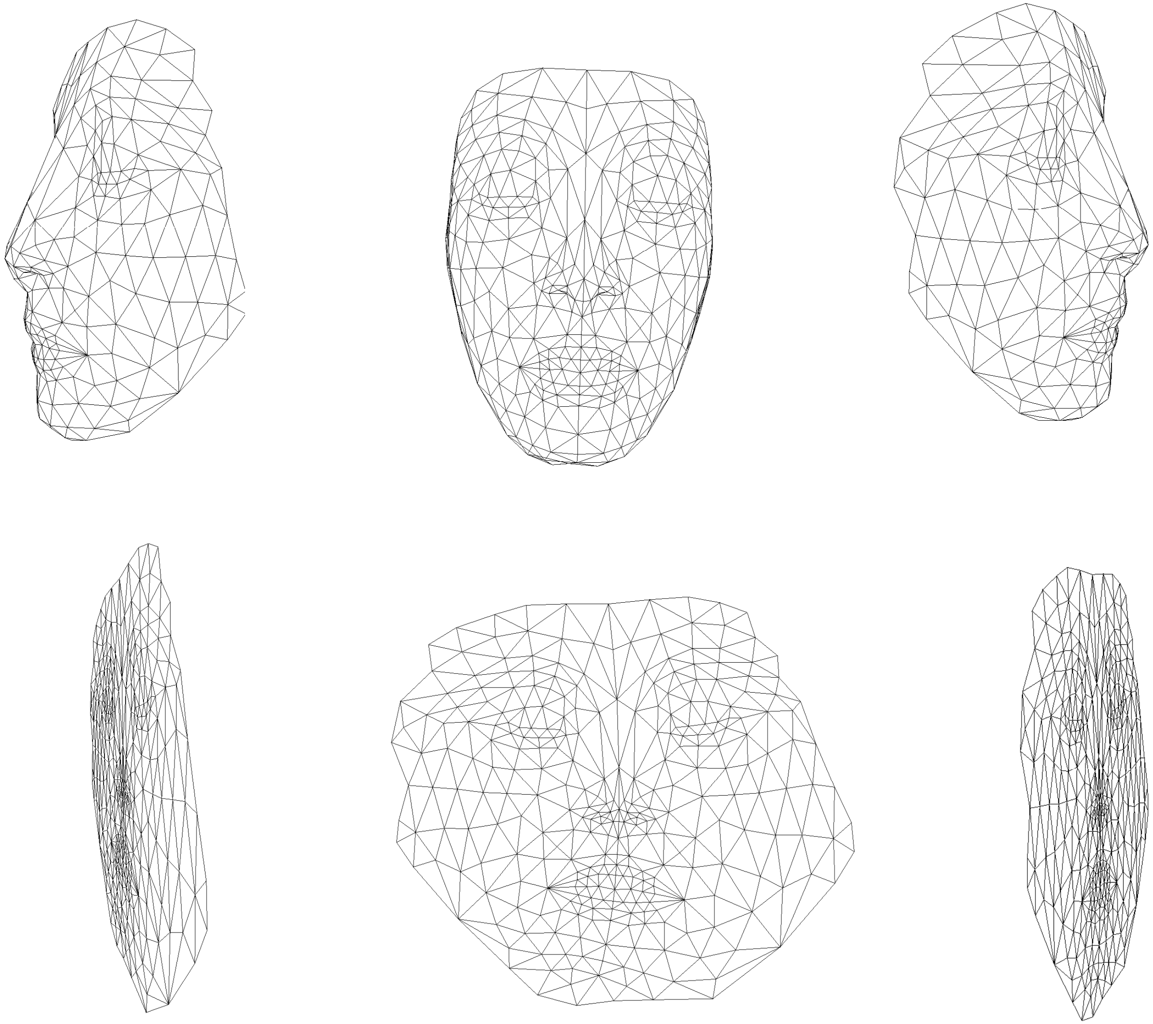
Daubechies, Guskov, Schröder and Sweldens, [1999]

Present Approach

Interpolation via Fourier series, using

- *Unequally spaced FFTs (USFFT), and*
- *A "Continuation Method" for trigonometric representation of non-periodic functions with spectral accuracy (thus, overcoming the Gibbs phenomenon)*

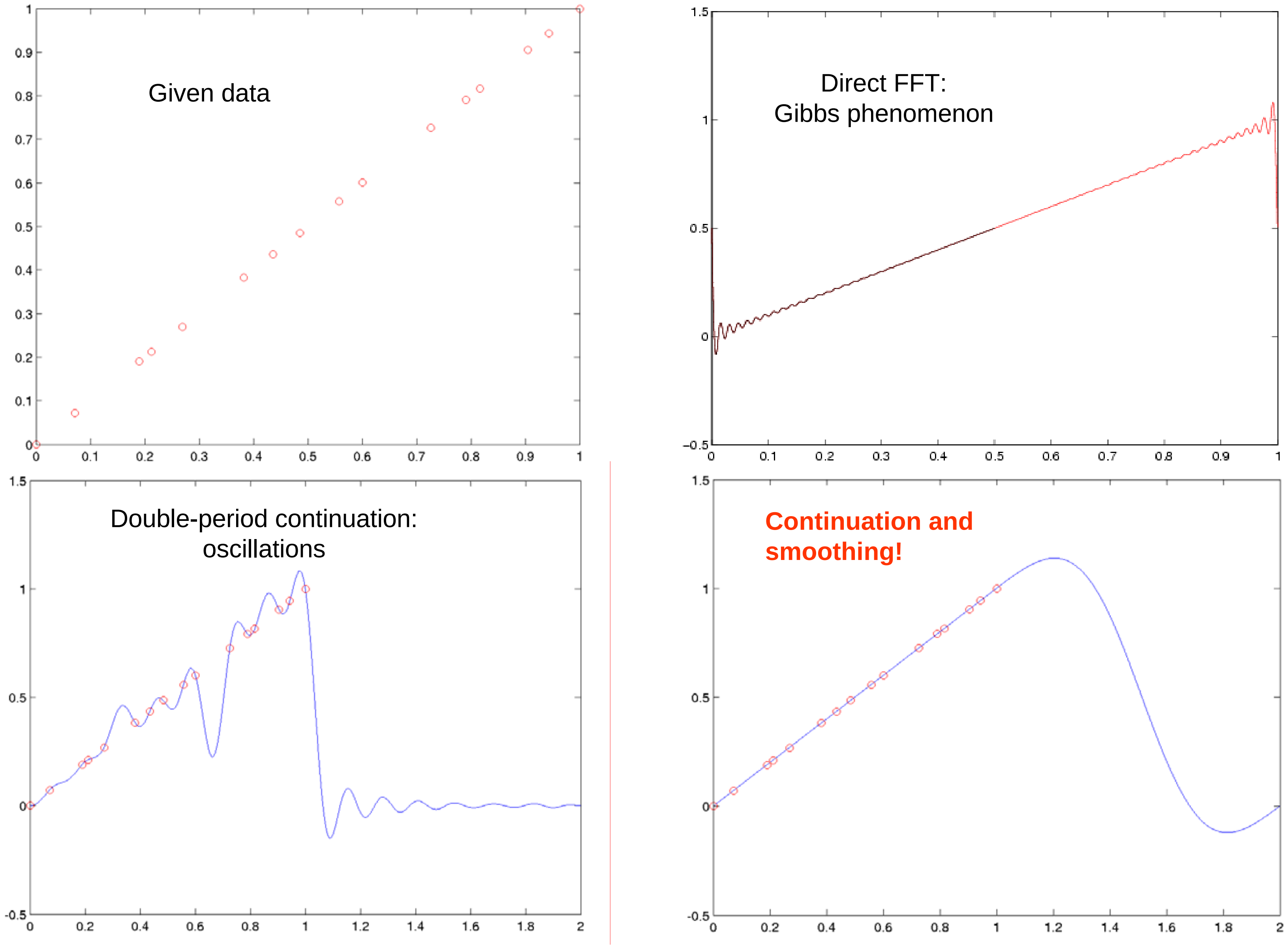
Intrinsic Parameterizations



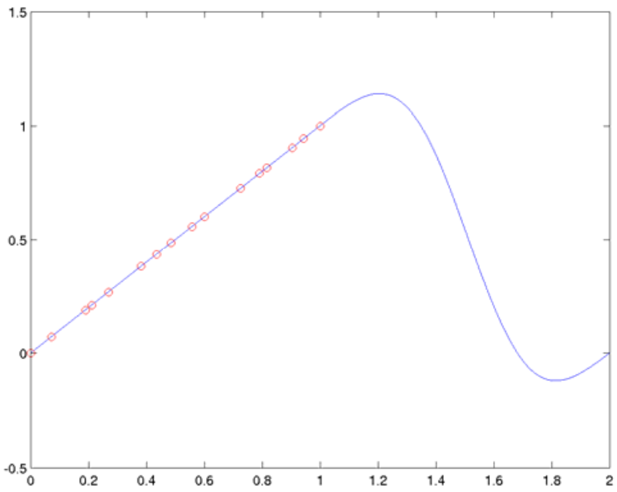
Desbrun, Meyer and Alliez, [2002]

Fourier Representation

POUs for boundary regions (Gibbs resolution)



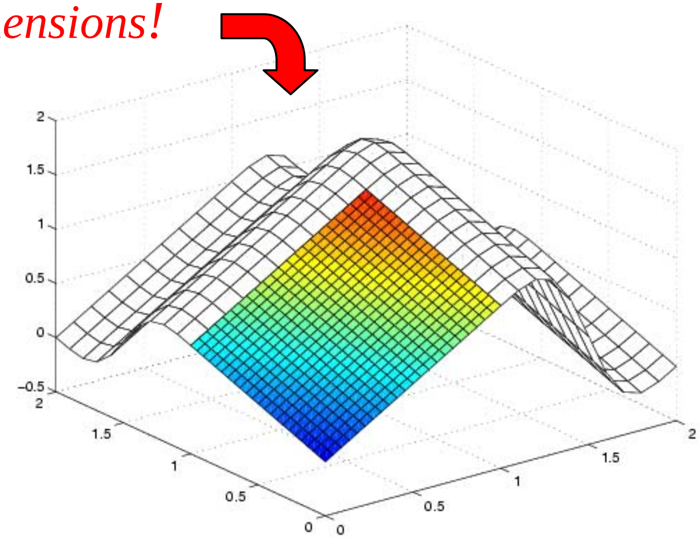
Fourier Representation



N	$f(x)$	ratio	df/dx	ratio	d^2f/dx^2	ratio
8	3.3e-03		1.3e-01		3.3e-00	
16	1.1e-05	3.0e+2	1.3e-03	9.9e+1	1.0e-01	3.2e+1
32	5.1e-10	2.2e+4	1.5e-07	8.6e+3	3.1e-05	3.3e+3
64	2.8e-13	1.8e+3	1.5e-10	9.7e+2	6.0e-08	5.3e+2
128	8.8e-15	3.2e+1	8.4e-12	1.9e+1	4.6e-09	1.3e+1

Generalizes to any number of dimensions!

N	$f(x,y)$	ratio	$\partial f/\partial x$	ratio	$\partial^2 f/\partial x^2$	ratio
8^2	2.9e-02		8.2e-01		1.4e+1	
16^2	3.5e-03	8.4e+1	2.7e-01	3.0e+0	1.4e+1	1.0e+0
32^2	1.2e-07	2.8e+4	3.0e-05	9.0e+3	4.8e-03	2.9e+3
64^2	2.8e-12	4.4e+4	1.4e-09	2.1e+4	4.2e-07	1.1e+4



Also useful for coarse inner discretizations.

Does not require domain to be a square!!!!

Fast convergence of Fourier Series of discontinuous functions

(Elimination or amelioration of the Gibbs phenomenon)

- *Majda, McDonough, Osher, 1978 (Filtering of high-order Fourier coefficients)*

- *Mock and Lax, 1978 (Integration rule based on "Chebyshev-like" quadrature points)*

- *Gottlieb and Tadmor, 1985 (Smoothing)*

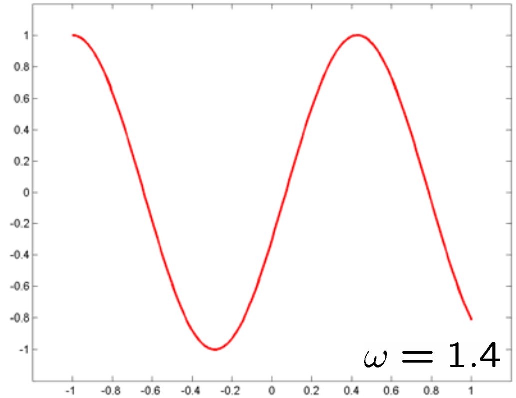
- *Gottlieb and Shu, 1992 (Gegenbauer Polynomials)*

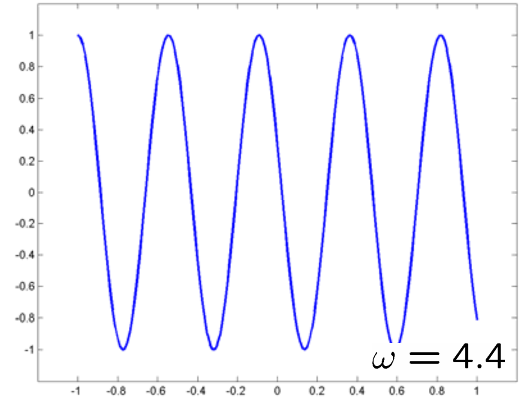
- *Geer and Banerjee, 1994 (Built in singularities, requires knowledge of jumps in function and derivatives)*

- *Geer, 1995; Fornberg 2000 (Pade approximants)*

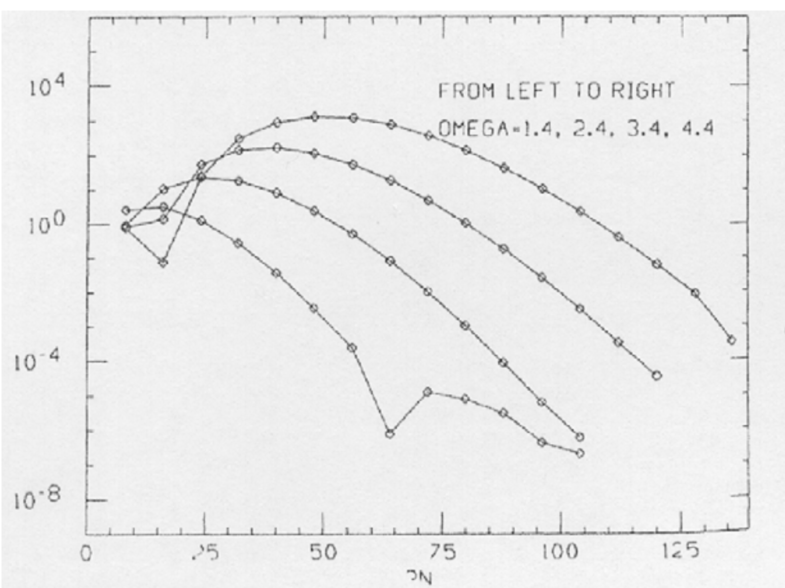
- *Gelb and Tanner, 2004 (New re-projection basis)*

$$f(x) = \cos(\pi\omega(x+1))$$

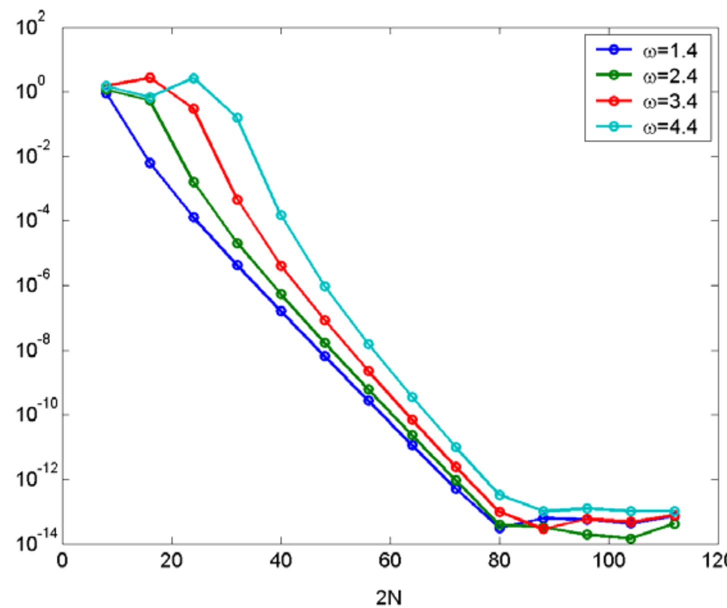




Error at $x = 1$

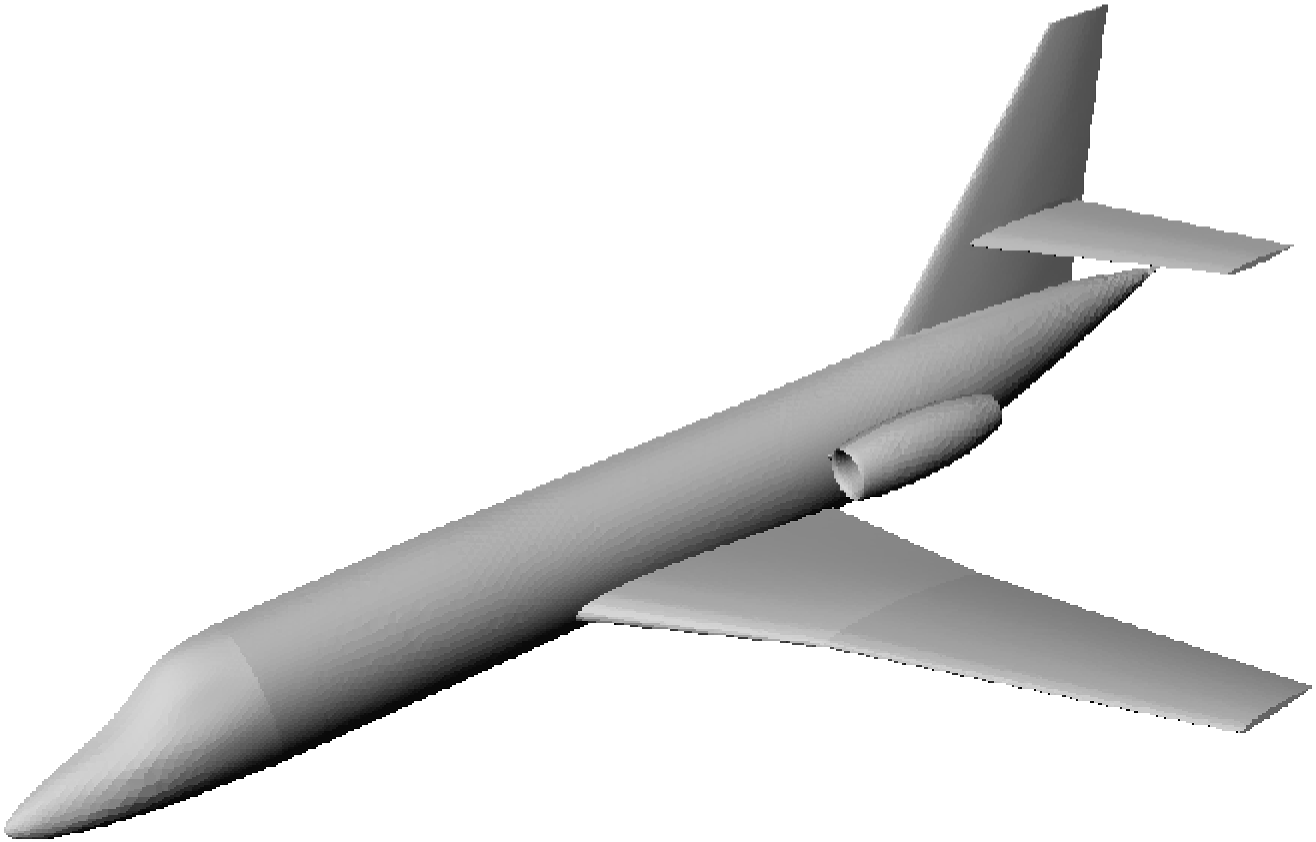


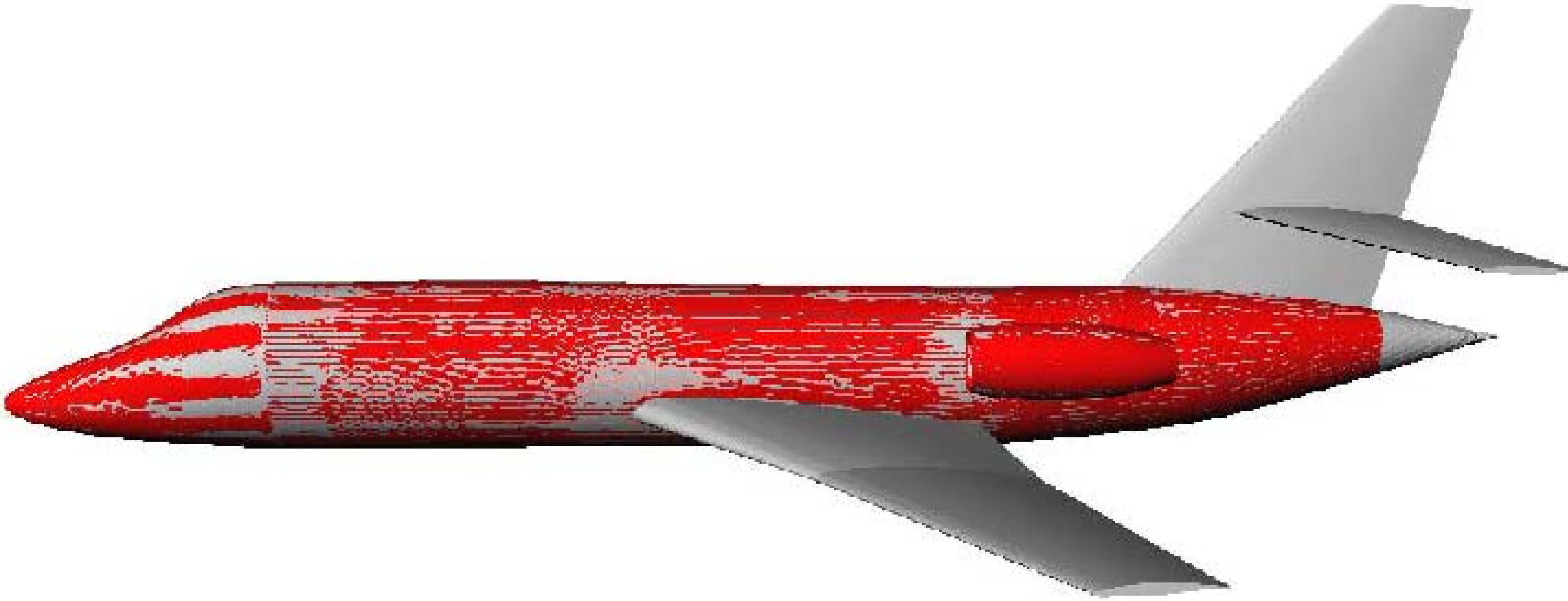
Gegenbauer Polynomials

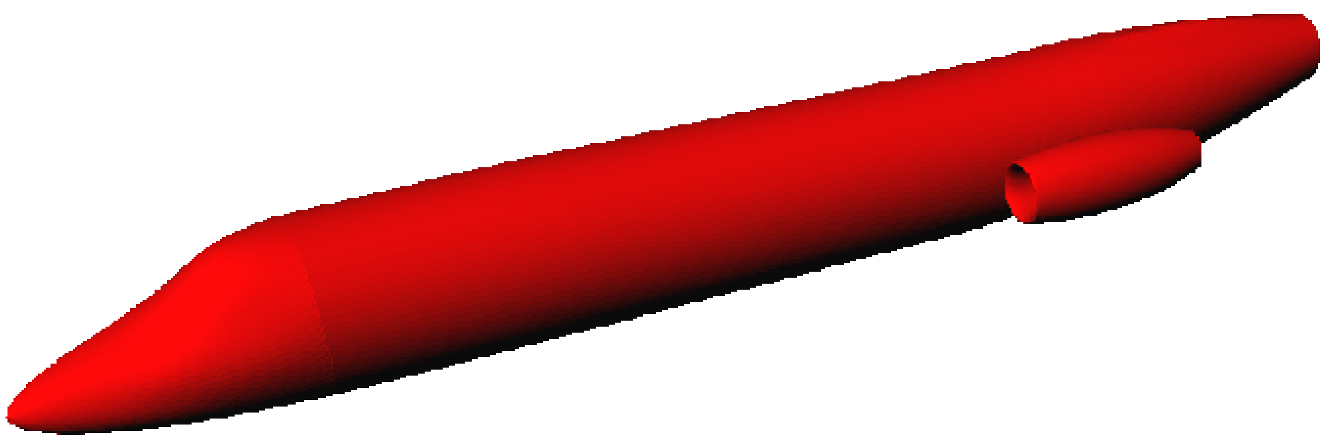


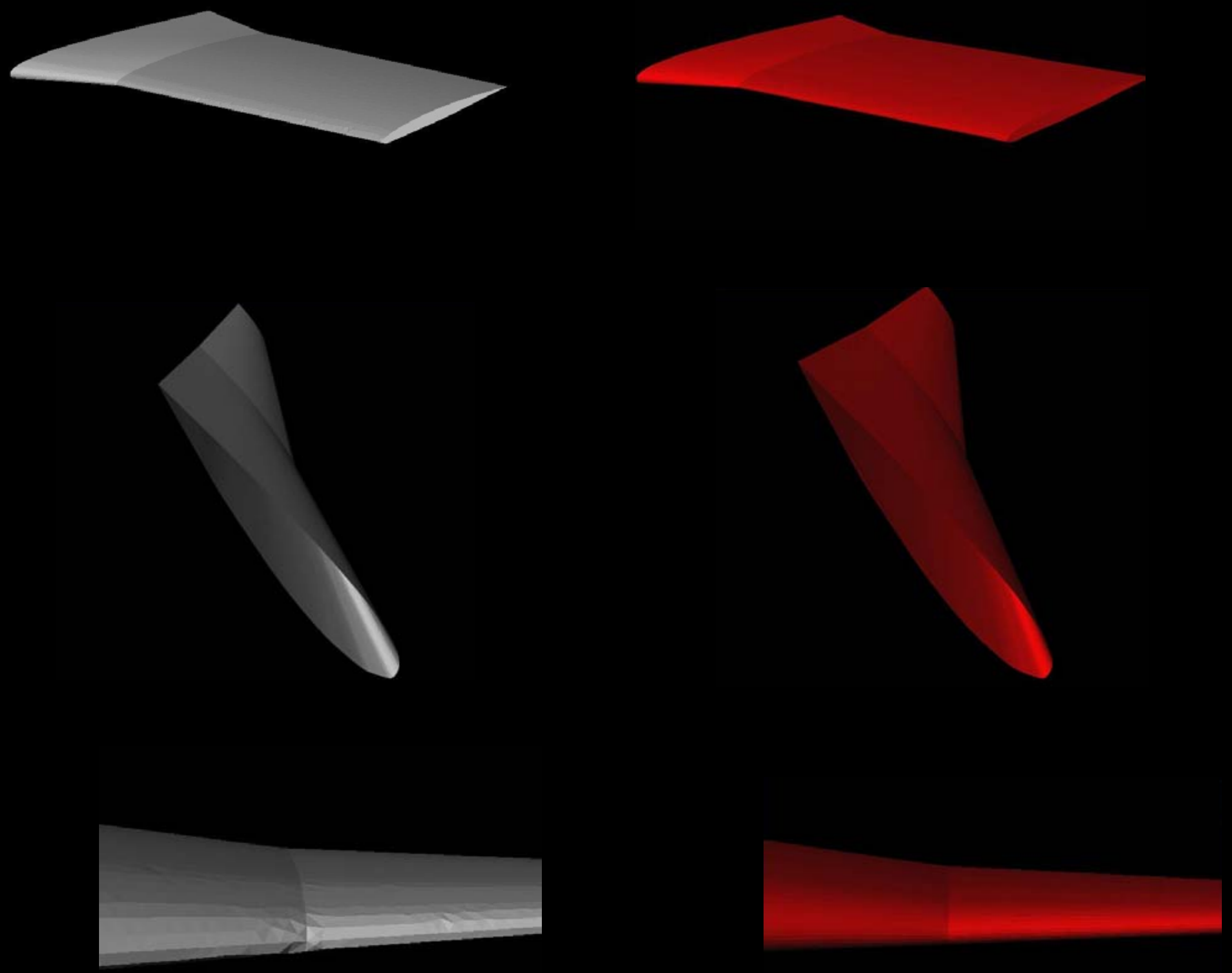
Present approach: continuation

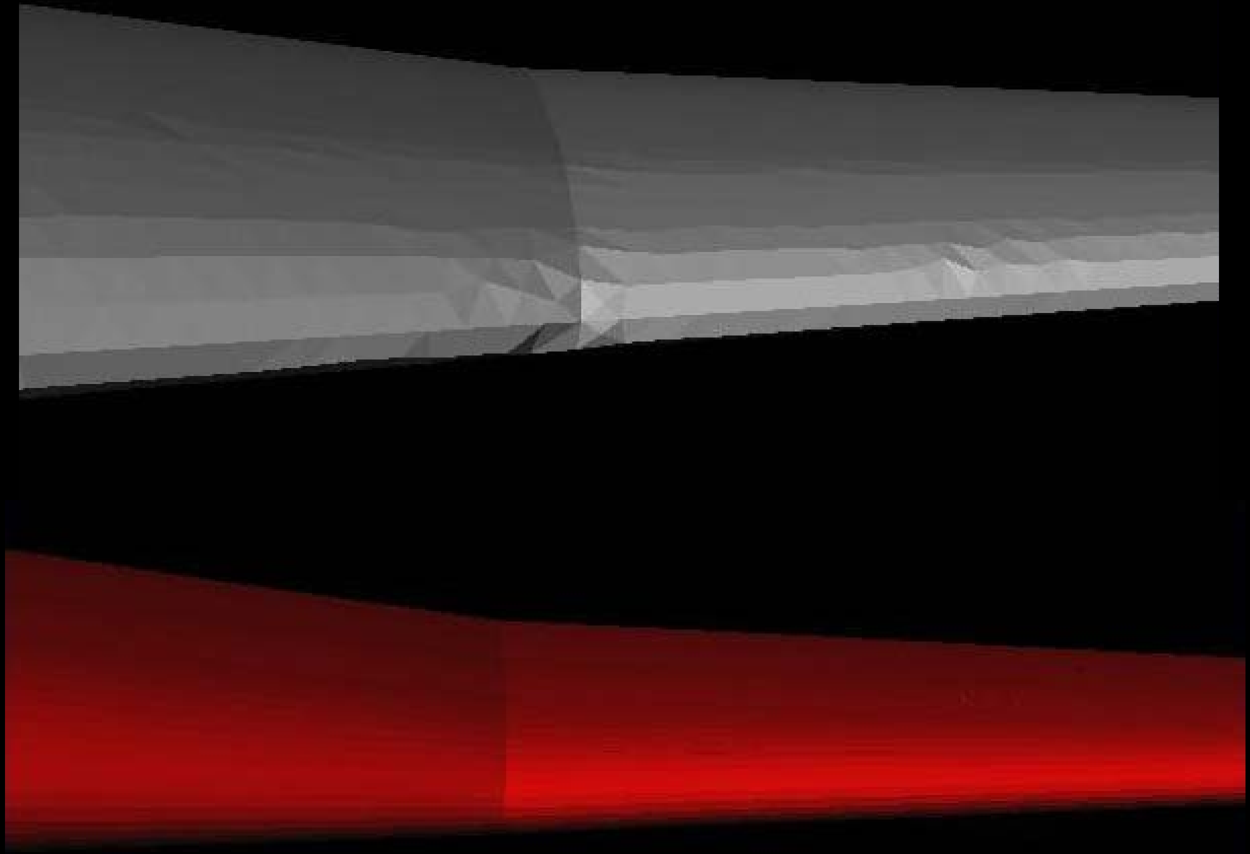
Falcon

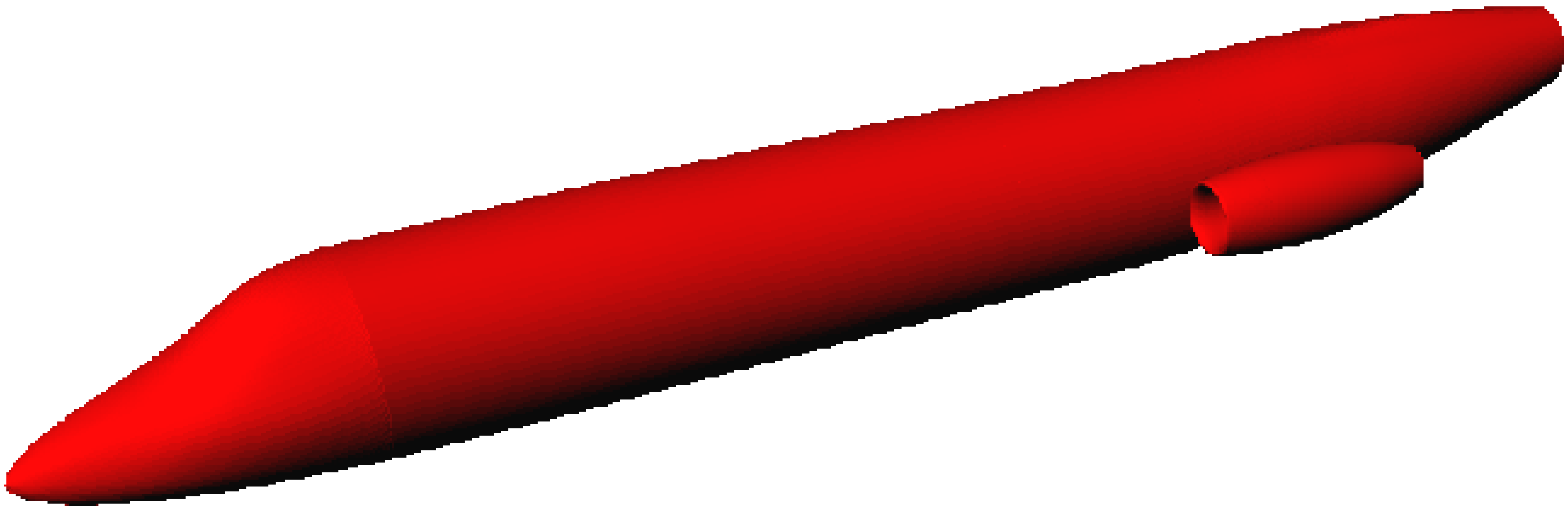


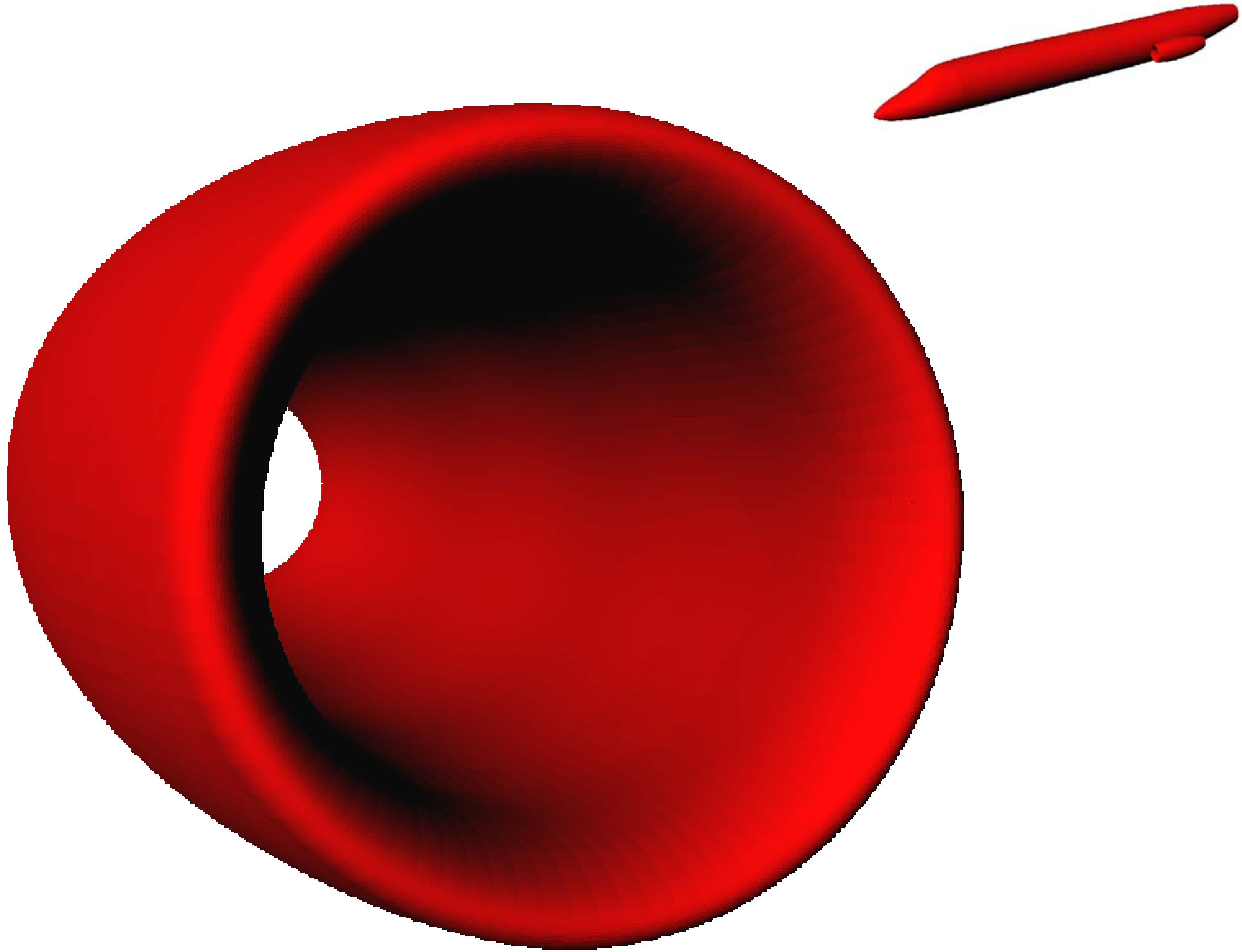


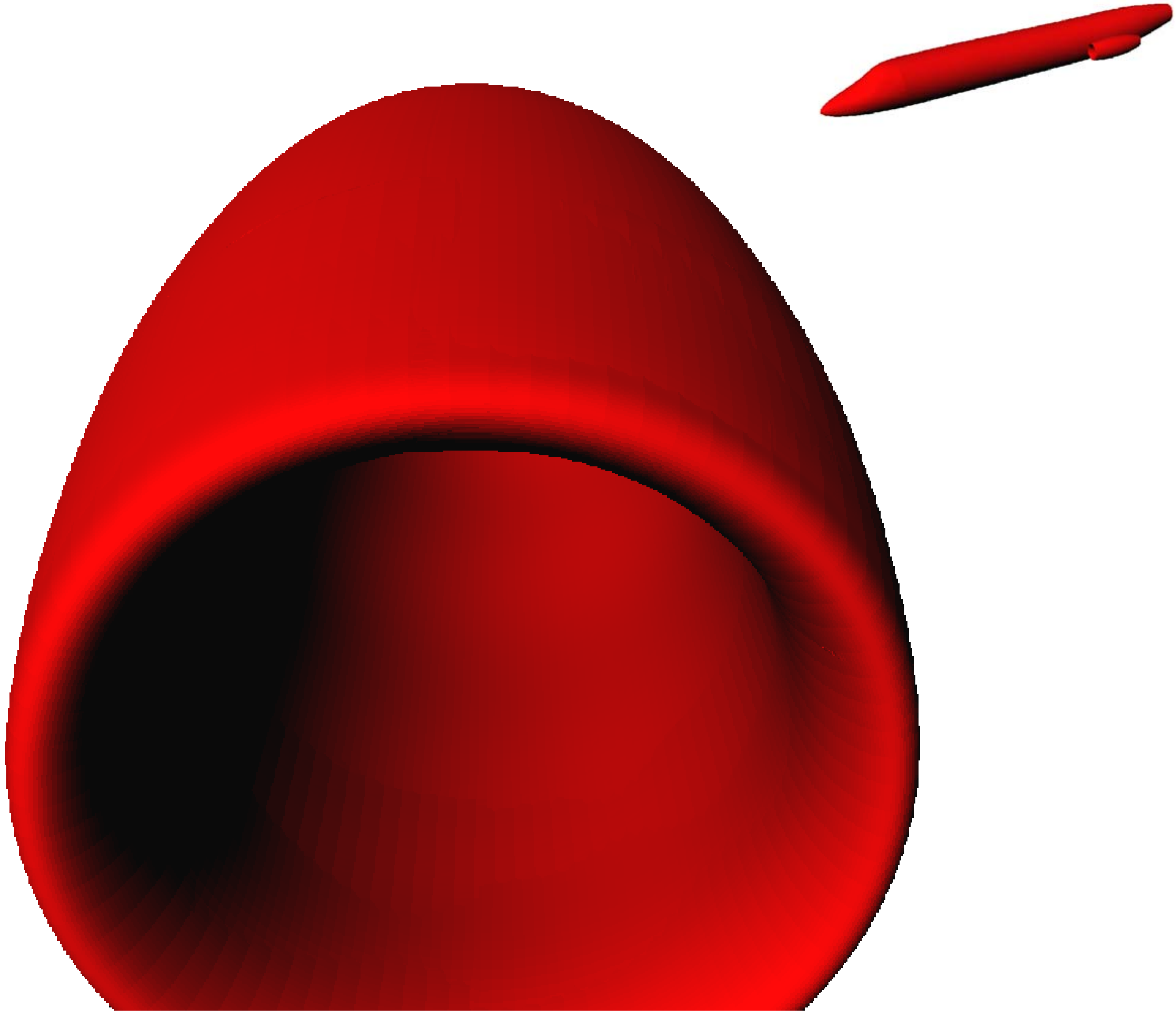












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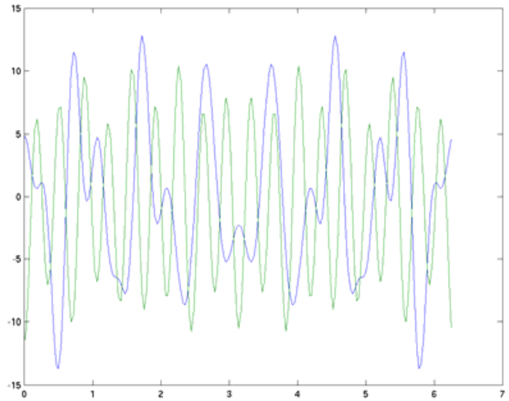
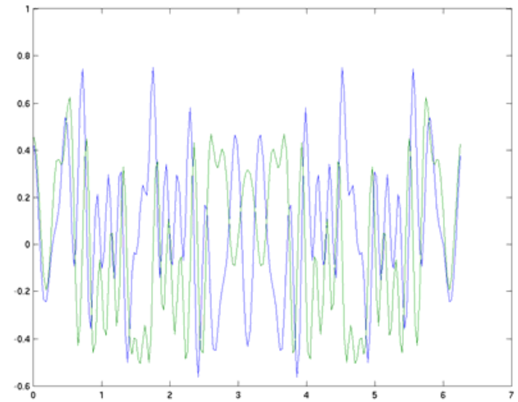
- *Surface Representation*
- *Shadow Boundaries* ←
- *Creeping-Waves, Diffraction* ←
- *Multiple Scattering* ←
- *Three-dimensionality*
- *Corners, Edges*

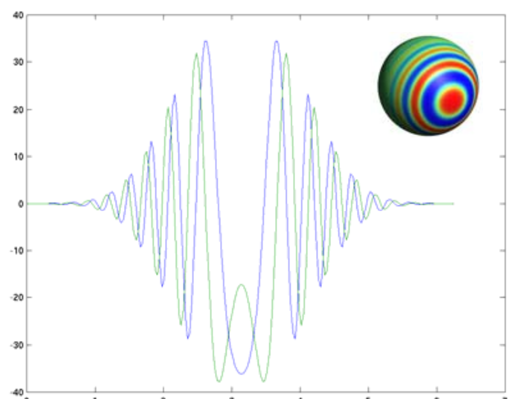
Shadow Boundary: Is μ_{slow} actually *slow*?

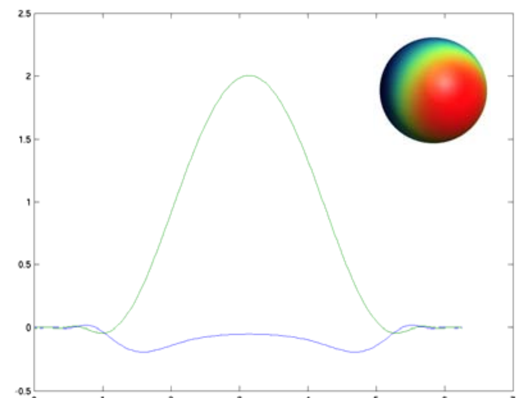
$$\frac{\varphi(\mathbf{r})}{2} = u^i(\mathbf{r}) - \int_{\partial D} \frac{\partial \Phi(\mathbf{r},\mathbf{r}')}{\partial \vec{\nu}_{\mathbf{r}'}} \varphi(\mathbf{r}')\, ds(\mathbf{r}') + i\gamma \int_{\partial D} \Phi(\mathbf{r},\mathbf{r}')\varphi(\mathbf{r}')\, ds(\mathbf{r}')$$





$$\frac{1}{2}\frac{\partial u}{\partial \vec{\nu}_{\mathbf{r}}}(\mathbf{r}) = \left(\frac{\partial u^i}{\partial \vec{\nu}_{\mathbf{r}}}(\mathbf{r}) + i\gamma u^i(\mathbf{r})\right) + \int_{\partial D} \frac{\partial \Phi(\mathbf{r},\mathbf{r}')}{\partial \vec{\nu}_{\mathbf{r}}} \frac{\partial u}{\partial \vec{\nu}_{\mathbf{r}'}}(\mathbf{r}')\, ds(\mathbf{r}') + i\gamma \int_{\partial D} \Phi(\mathbf{r},\mathbf{r}') \frac{\partial u}{\partial \vec{\nu}_{\mathbf{r}'}}(\mathbf{r}')\, ds(\mathbf{r}')$$

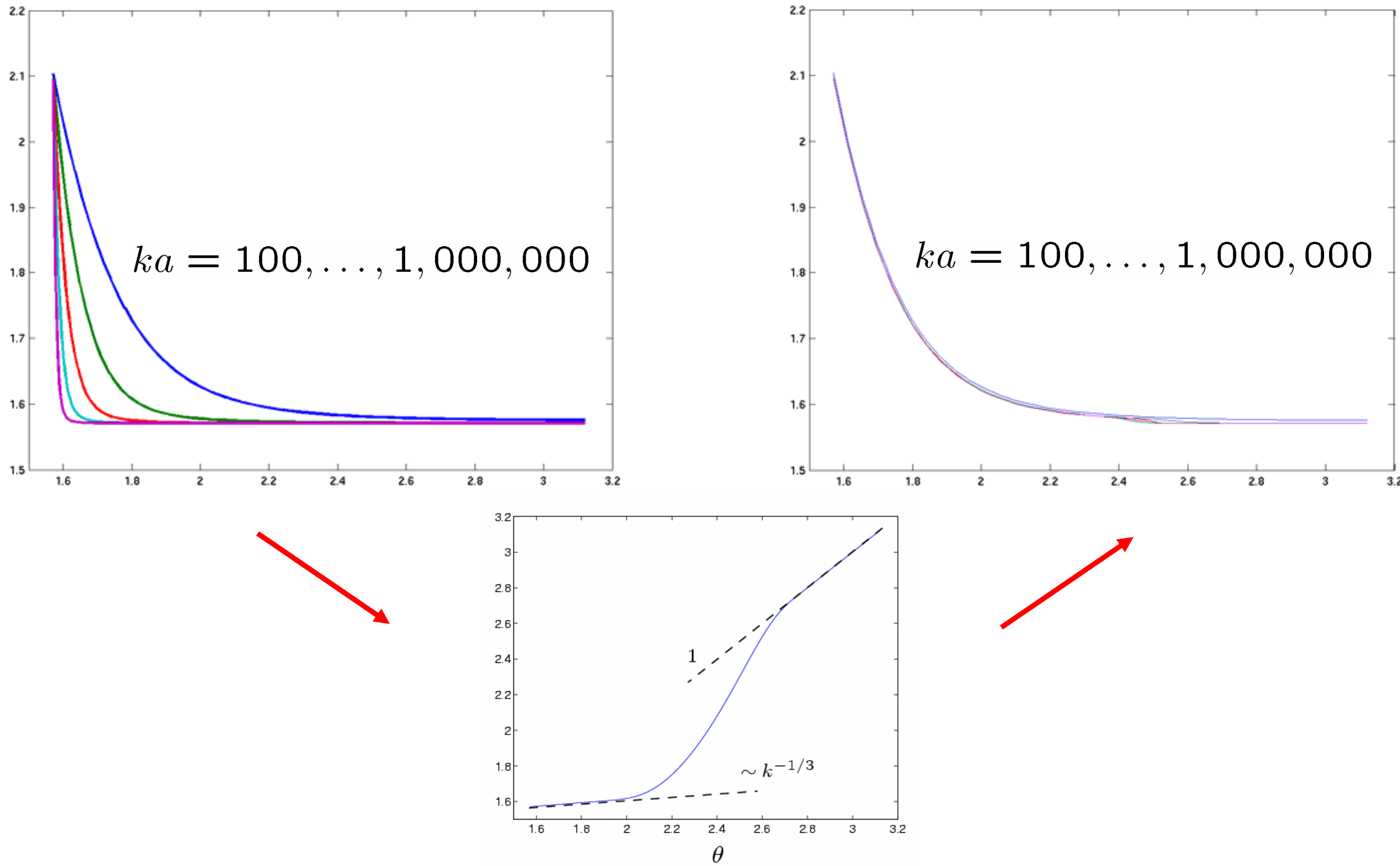




Creeping waves! Diffraction!

Cubic root ratios in the slow-density slopes

around shadow boundaries



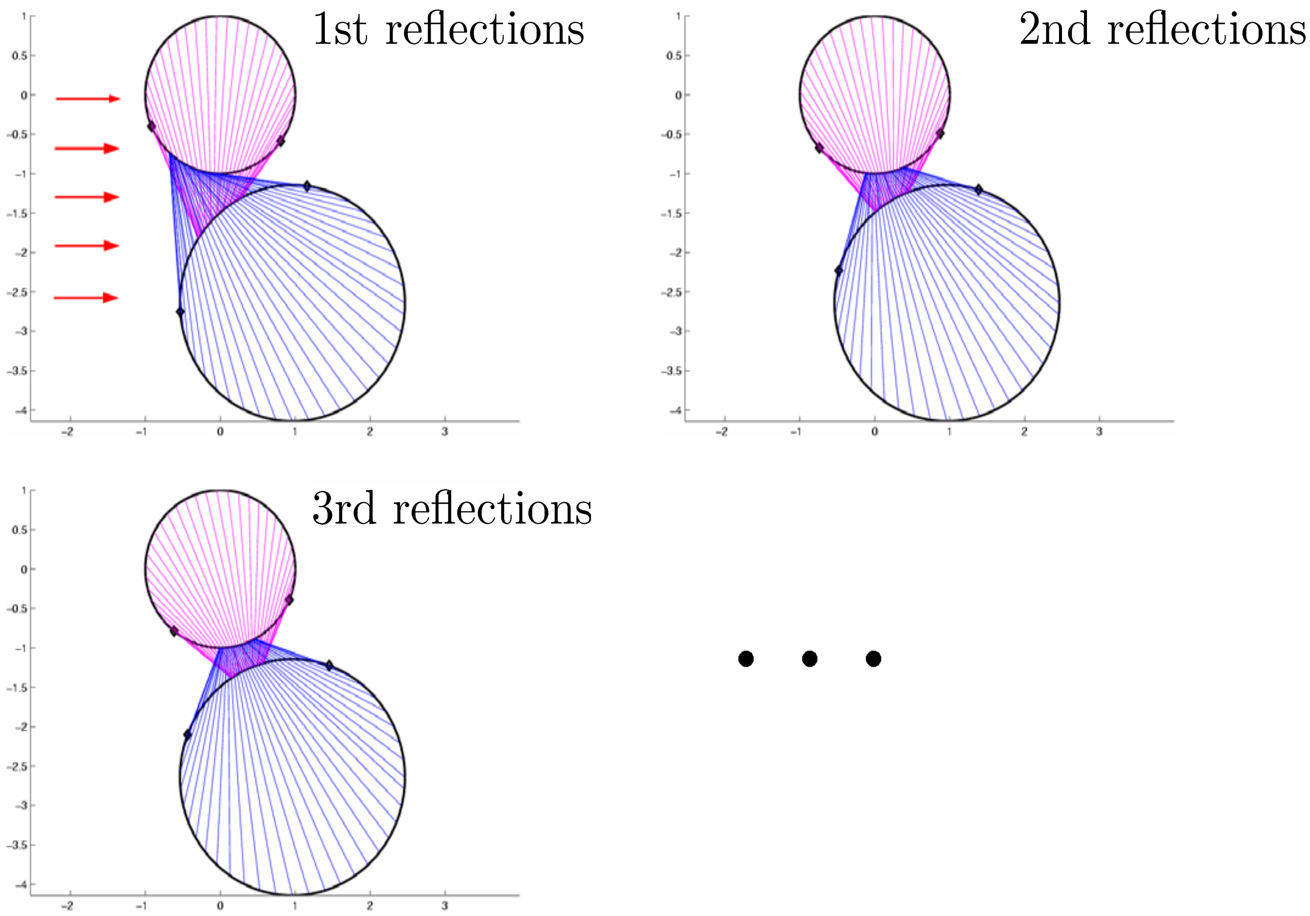
# of Fourier modes needed to represent μ_{slow} with a fixed accuracy

k	w/out chg. of vars.	w/ chg. of vars.
100	110	110
1000	230	220
10000	310	280
100000	350	280
1000000	> 500	280

Multiple Scattering

Automatic Multiple reflections

Ansatz Generation!!!



Bruno & Reitich [2003]

High-Frequency Integral Equation Method

Implementation: Multiple reflections

$$\begin{bmatrix} I - T_{11} & -R_{12} \\ -R_{21} & I - T_{22} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} e^{ik\phi^0(t)} \\ e^{ik\nu^0(\tau)} \end{bmatrix} \quad \Rightarrow$$

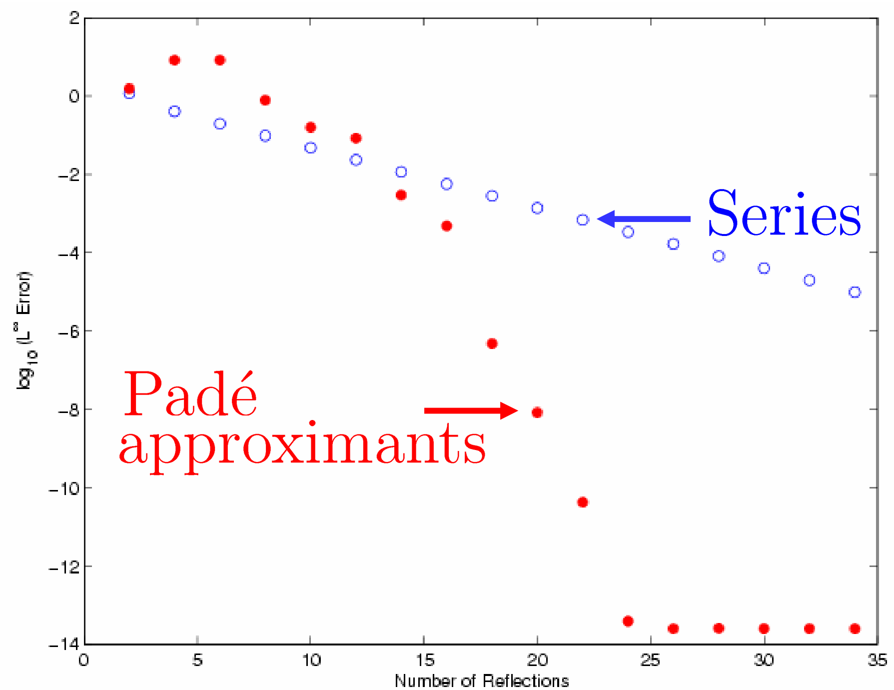
$$\underbrace{\begin{bmatrix} I & (I - T_{11})^{-1}(-R_{12}) \\ (I - T_{22})^{-1}(-R_{21}) & I \end{bmatrix}}_{I - A} \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \underbrace{\begin{bmatrix} (I - T_{11})^{-1} e^{ik\phi^0(t)} \\ (I - T_{22})^{-1} e^{ik\nu^0(\tau)} \end{bmatrix}}_{\begin{bmatrix} \mu^0_{1,slow} e^{ik\phi^0} \\ \mu^0_{2,slow} e^{ik\nu^0} \end{bmatrix}}$$

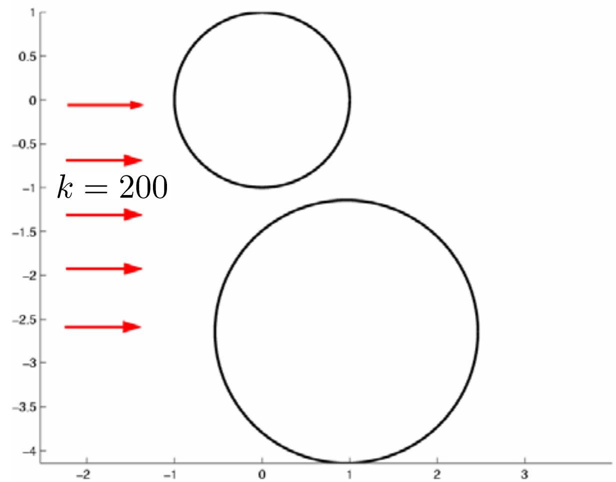
$$\Rightarrow \quad \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \mu^0_{1,slow} e^{ik\phi^0} \\ \mu^0_{2,slow} e^{ik\nu^0} \end{bmatrix}}_{\text{Isolated Obstacles}} + A \underbrace{\begin{bmatrix} \mu^0_{1,slow} e^{ik\phi^0} \\ \mu^0_{2,slow} e^{ik\nu^0} \end{bmatrix}}_{\text{First Reflections}} + A^2 \underbrace{\begin{bmatrix} \mu^0_{1,slow} e^{ik\phi^0} \\ \mu^0_{2,slow} e^{ik\nu^0} \end{bmatrix}}_{\text{Second Reflections}} + \cdots$$

$\cdots$

Enhanced Convergence
Acceleration by analytic continuation

Maximum Error:



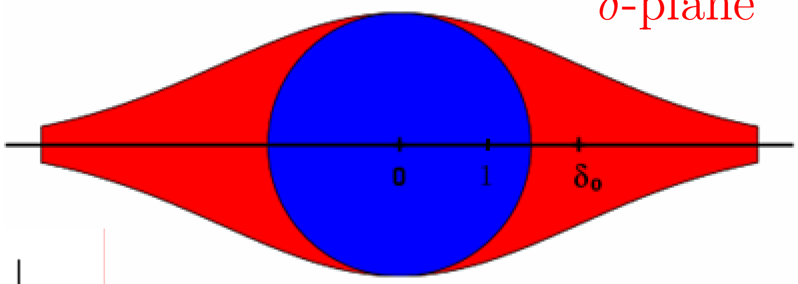


Bruno & Reitich [2004]

A Convergent High-Frequency Approach
Acceleration by analytic continuation

$$\mu(x) \approx \sum_{n=0}^{N_{refl}} \mu_{slow}^{n}(x) e^{ik\phi^n(x)}$$

$$= \sum_{n=0}^{N_{refl}} \mu_{slow}^{n}(x) e^{ik\phi^n(x)} \delta^n \Big|_{\delta=1}$$



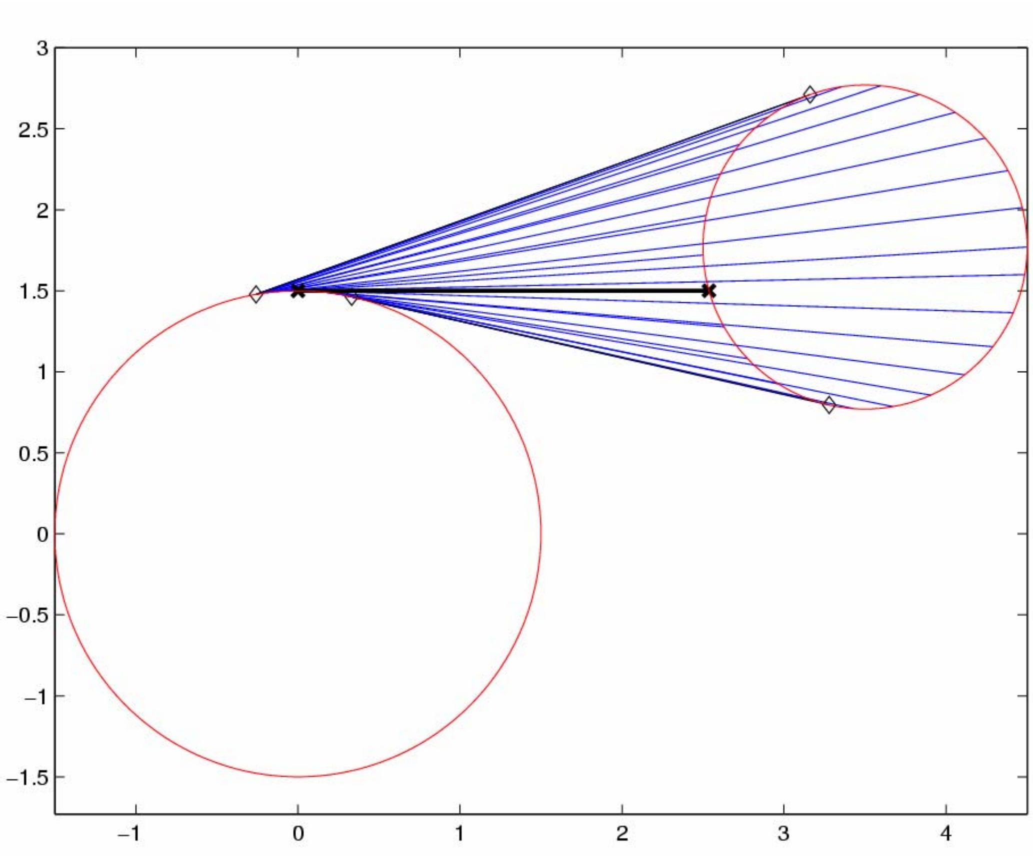
Analytic continuation $\Rightarrow$

- Sum series outside radius of convergence
- Accelerate convergence inside

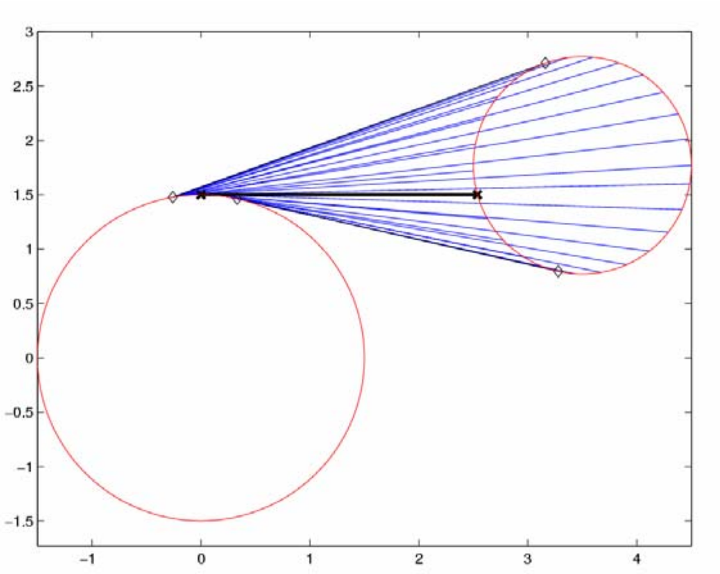
e.g. *Pade approximation*

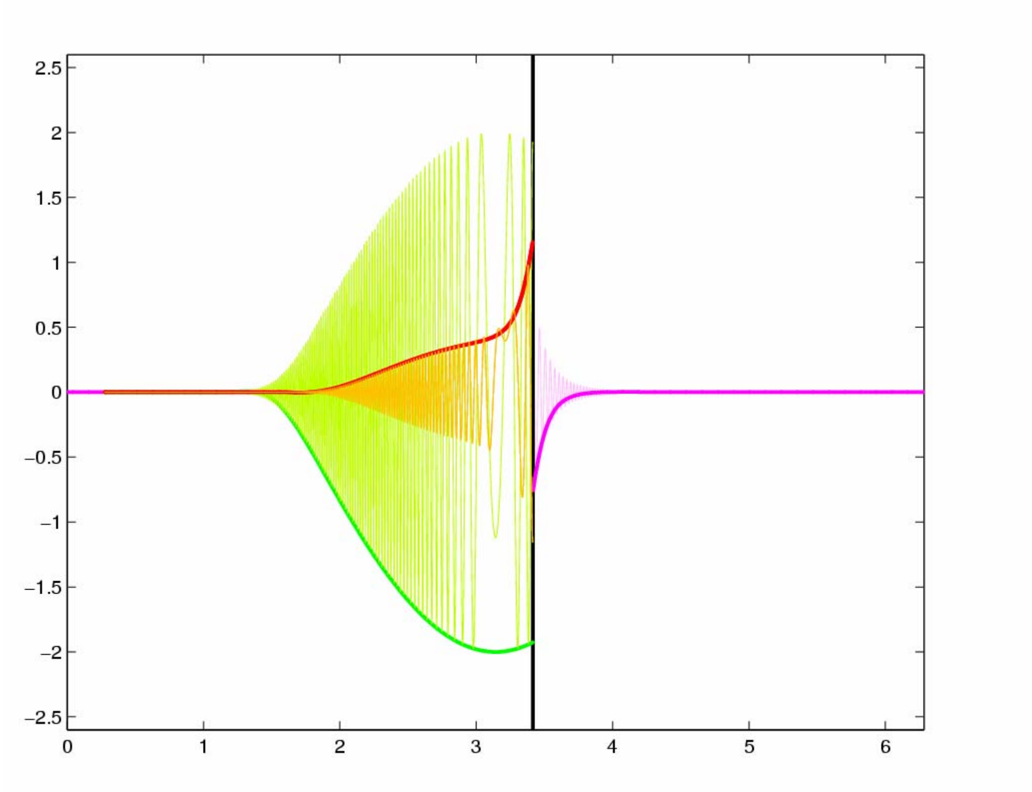
$$\mu(x) \approx \frac{a_0(x) + a_1(x)\delta + \cdots + a_m(x)\delta^m}{1 + b_1(x)\delta + \cdots + b_m(x)\delta^m}\Big|_{\delta=1}, \quad m = 1, 2, \cdots, N_{refl}/2$$

Multiple scattering + diffraction!



Diffraction ansatz



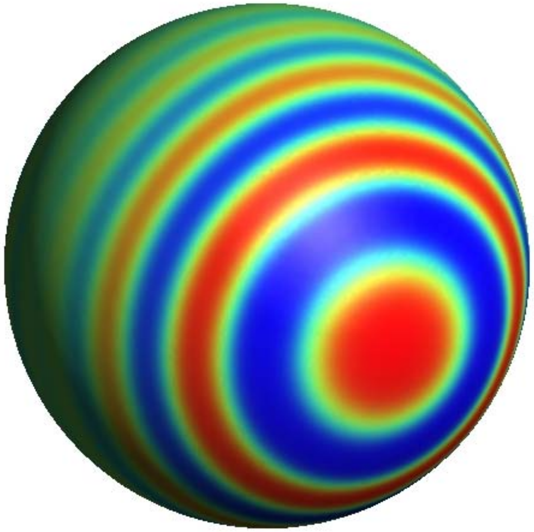


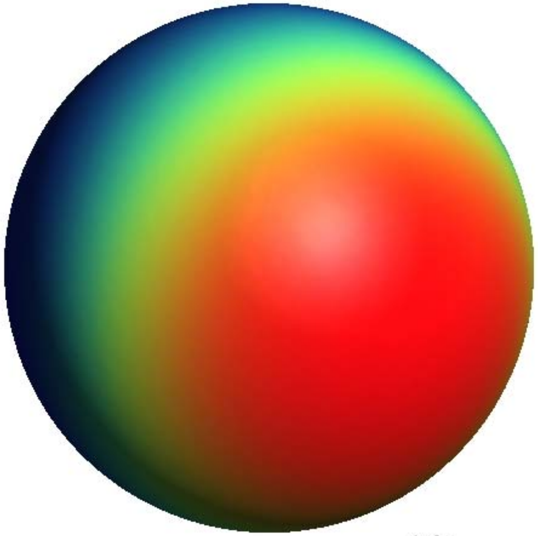
Bruno and Reitich, [2004]

Issues

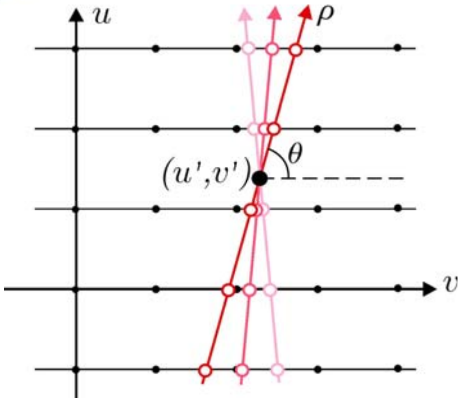
- *Kernel Singularities*
- *Surface Representation*
- *Shadow Boundaries*
- *Creeping-Waves, Diffraction*
- *Multiple Scattering*
- *Three-dimensionality*
- *Corners, Edges*

Three Dimensional Problem (Preliminary!)





A polar-coordinate Jacobian – same as previously

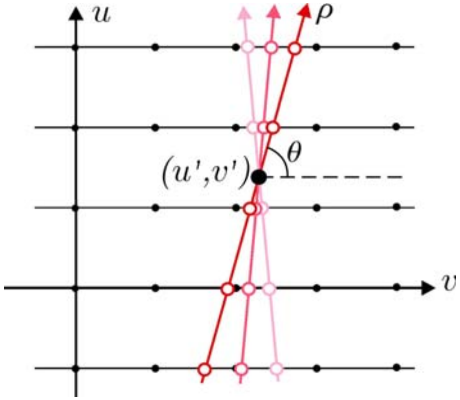


$$L(u', v', \theta) = \int_{-r_1}^{r_1} f_k^*(\rho, \theta) \frac{|\rho|}{|\mathbf{R}|} \cos k\,|\mathbf{R}|\, \frac{\mathbf{R} \cdot \boldsymbol{\nu}(r)}{\mathbf{R}^2} d\rho$$

Use "Canonical Integrals":

Re-express in the form

$$\int_0^{2\pi} d\theta \int_0^{\varepsilon} e^{ik\phi(\rho)} f(\rho,\theta)\,d\rho$$



$Change\ of\ variables$ $\phi(\rho) = t$

Slow

Fixed

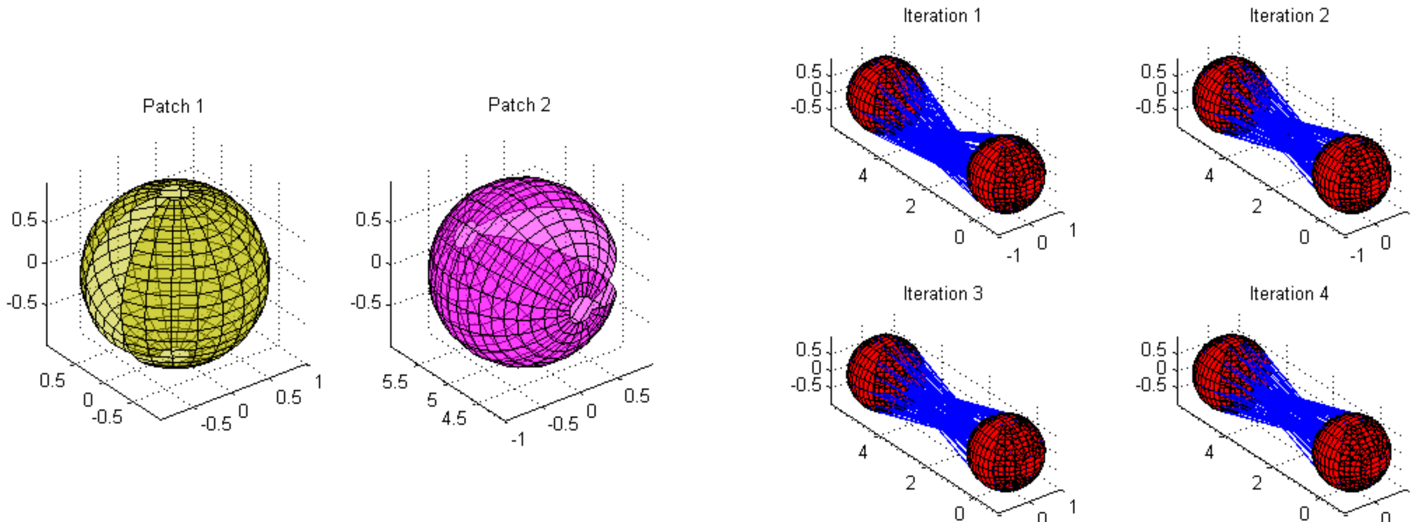
$$\int_0^{2\pi} d\theta \int_0^{b} e^{ikt} g(t,\theta)\,dt$$

Smooth

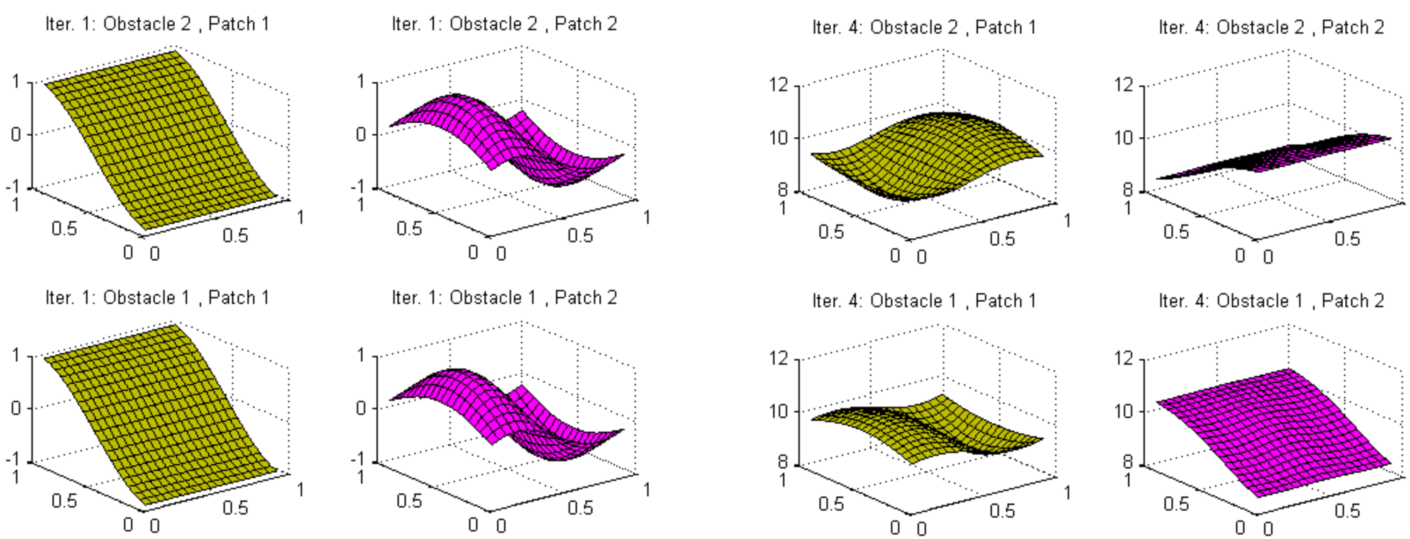
(indep. of k)

Critical points near shadow boundaries!

Bruno and Geuzaine, [2005]



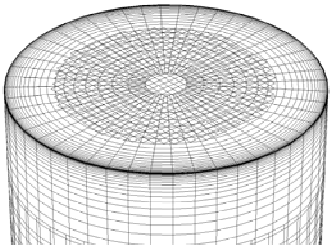
Spectral GO solver. Left: Parametric patches on a sphere. Right: Geometrical rays for the first through fourth reflections on a two-sphere configuration.



Spectral GO solver: Calculated phases on each patch for first and fourth reflections.

Issues

- *Kernel Singularities*
- *Surface Representation*
- *Shadow Boundaries*
- *Creeping-Waves, Diffraction*
- *Multiple Scattering*
- *Three-dimensionality*
- *Corners, Edges*



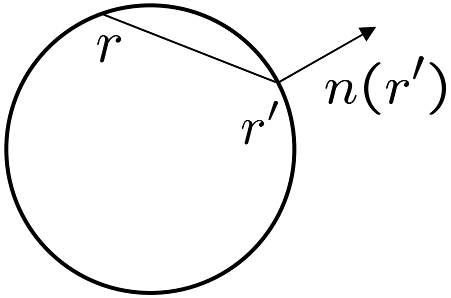
Singular surfaces

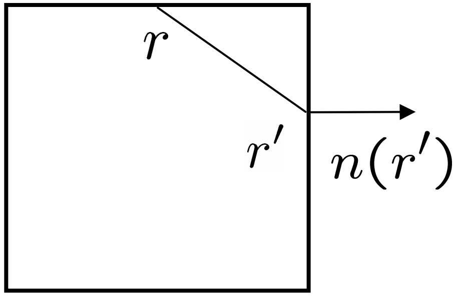
Example: Double layer potential
(soft acoustic scattering)

$$\int_{\partial\Omega} f(r') \frac{\partial}{\partial n(r')} \Phi_k(r, r') ds(r')$$

$$\frac{\partial}{\partial n(r')} \Phi_k(r, r') = \mathcal{O}\left(\frac{(r - r') \cdot n(r')}{|r - r'|^3}\right) = \mathcal{O}\left(\frac{1}{|r - r'|^2}\right)$$

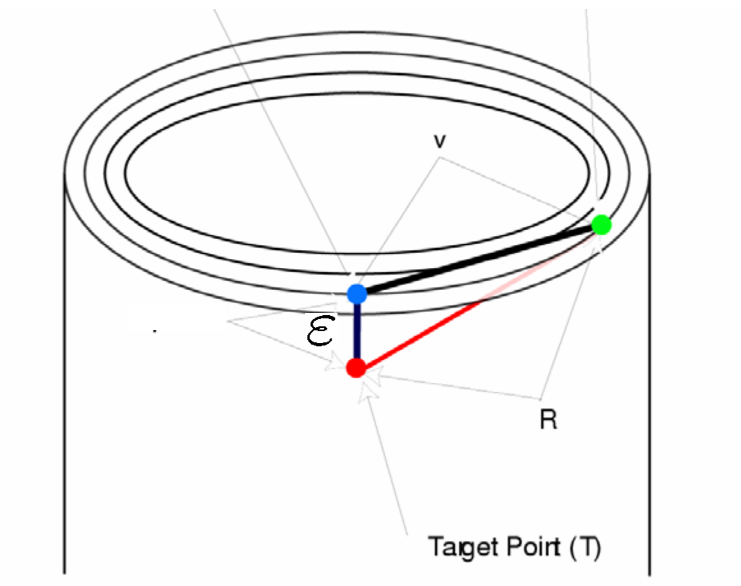
Non-integrably singular

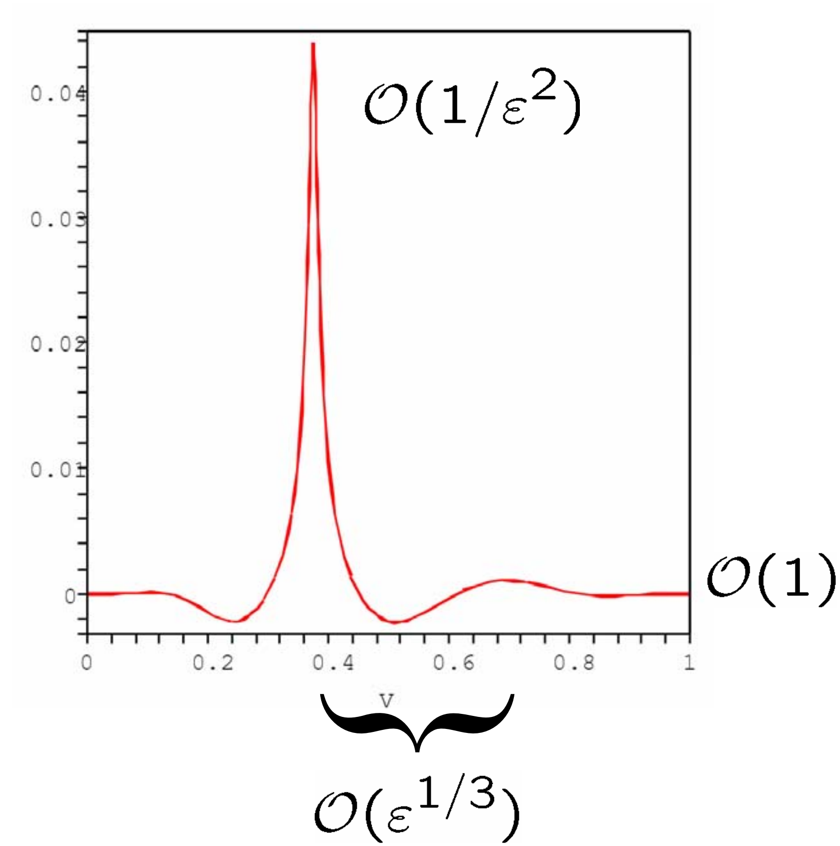




Integrable kernels are still nearly singular:

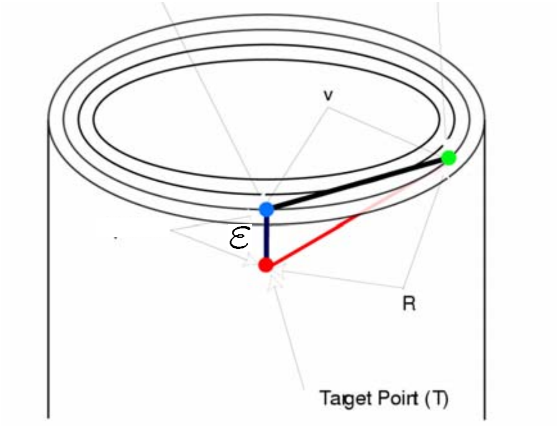
1-d-canonical integration





$$\varepsilon = \mathcal{O}(h^p), \ p = 4, 5, 6 \ldots !!!!$$

Solution: "1-d-canonical integration" of nearly-singular kernel



Integrand of interest:

$$g(v)/R^3$$

Note that

1) g is a smooth function of v, and
2) $R = \sqrt{\varepsilon^2 + v^2}$.

Use v as variable of integration!

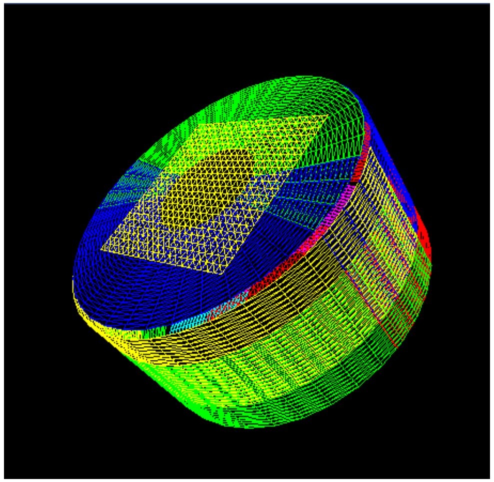
It suffices to integrate

$$v^p/(v^2 + \varepsilon^2)^{3/2}$$

This integral is

1) "Canonical" (closed form), and
2) Independent of the integration surface!

OB and R. Paffenroth [2004]



Cylinder of radius 1 and height 1

$$k = 2$$

(Normalized Max. Error = max. of field-error on a sphere of radius 2 divided by max of field)

Point source slightly off-center

Discretization	Iterations	Normalized Max. Error
$10 \times 9 \times 9$	13	$2.2 \cdot 10^{-2}$
$10 \times 17 \times 17$	11	$3.3 \cdot 10^{-4}$
$10 \times 33 \times 33$	11	$1.0 \cdot 10^{-5}$
$10 \times 65 \times 65$	11	$1.1 \cdot 10^{-6}$

Plane wave incidence

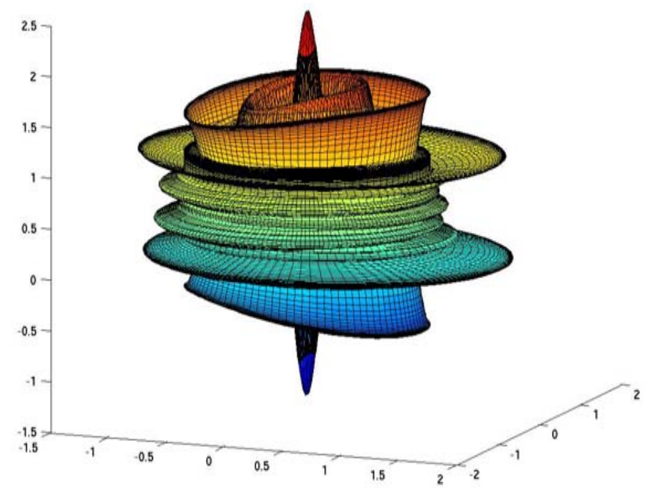
Discretization	Iterations	Normalized Max. Error
$10 \times 9 \times 9$	15	$1.5 \cdot 10^{-2}$
$10 \times 17 \times 17$	13	$3.1 \cdot 10^{-4}$
$10 \times 33 \times 33$	13	$9.5 \cdot 10^{-6}$
$10 \times 65 \times 65$		

Cylinder of radius 1 and height 1

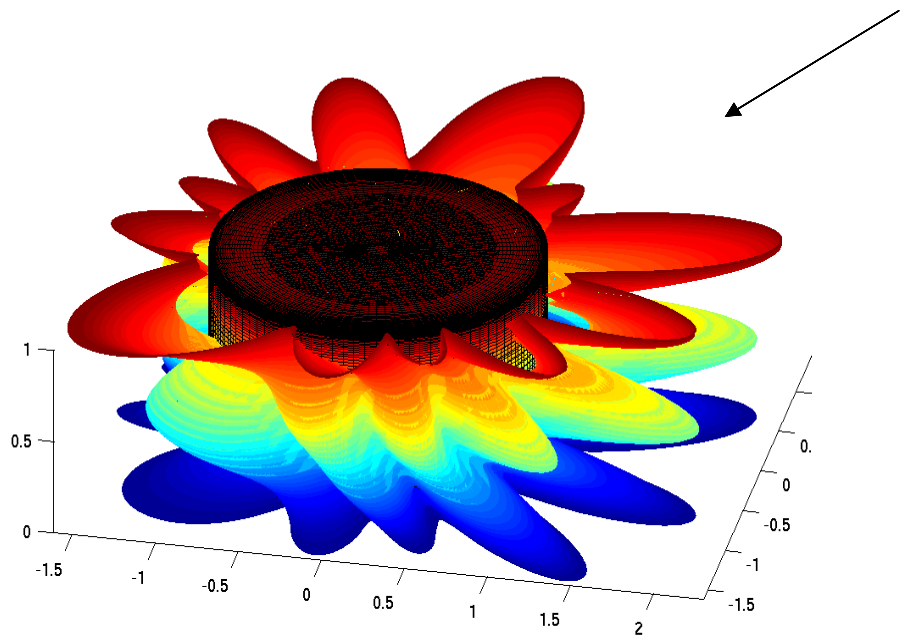
$$k = 10,\ 20$$

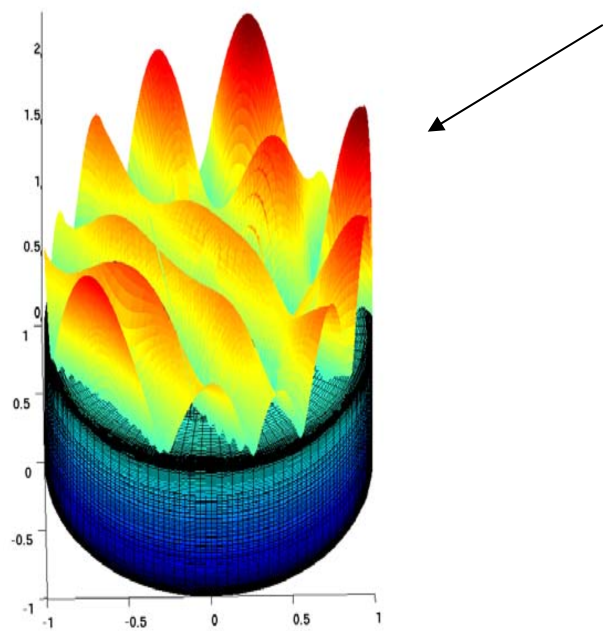
Discretization	k	Normalized Max. Error
$10 \times 65 \times 65$	10	$1.7 \cdot 10^{-4}$
$10 \times 65 \times 65$	20	$2.0 \cdot 10^{-3}$

a) Point source, k=10

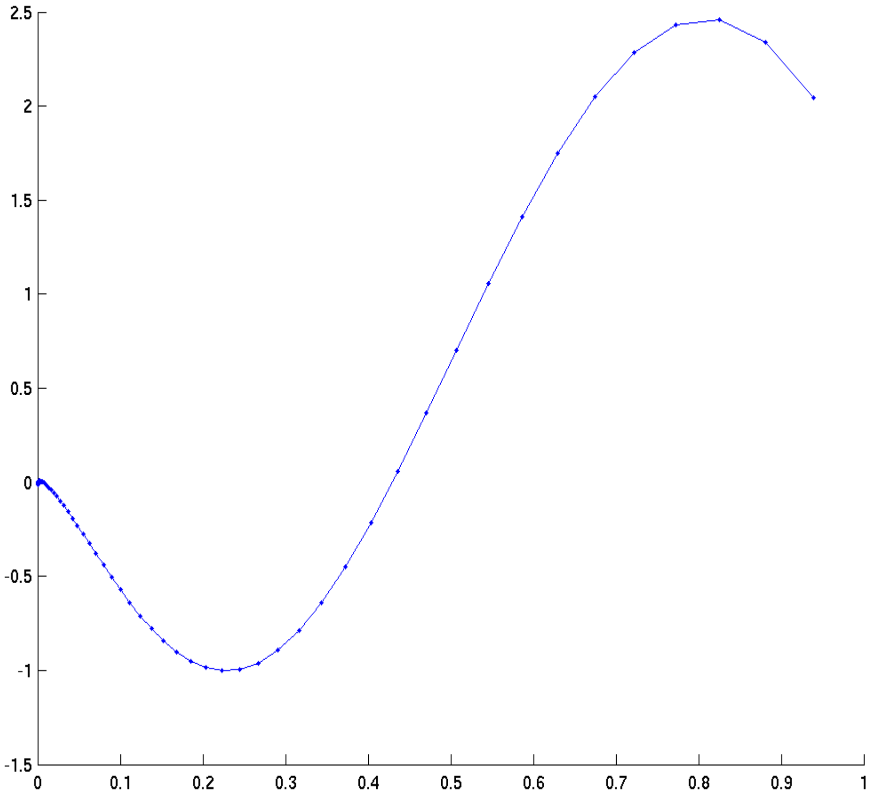


b) Plane wave incidence, k=10



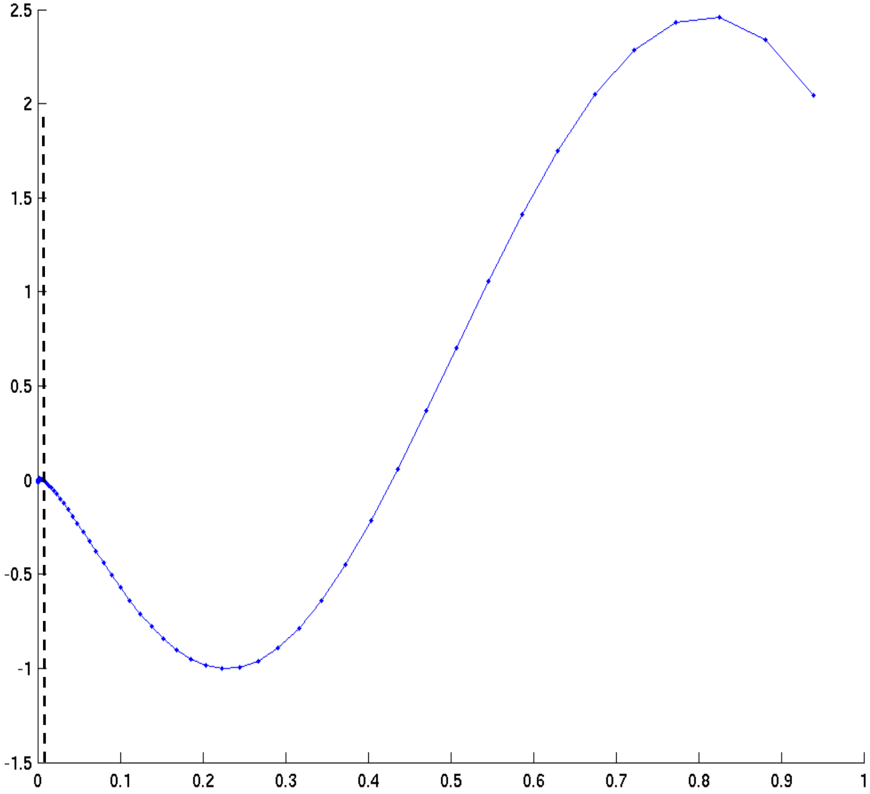


Current along a radius

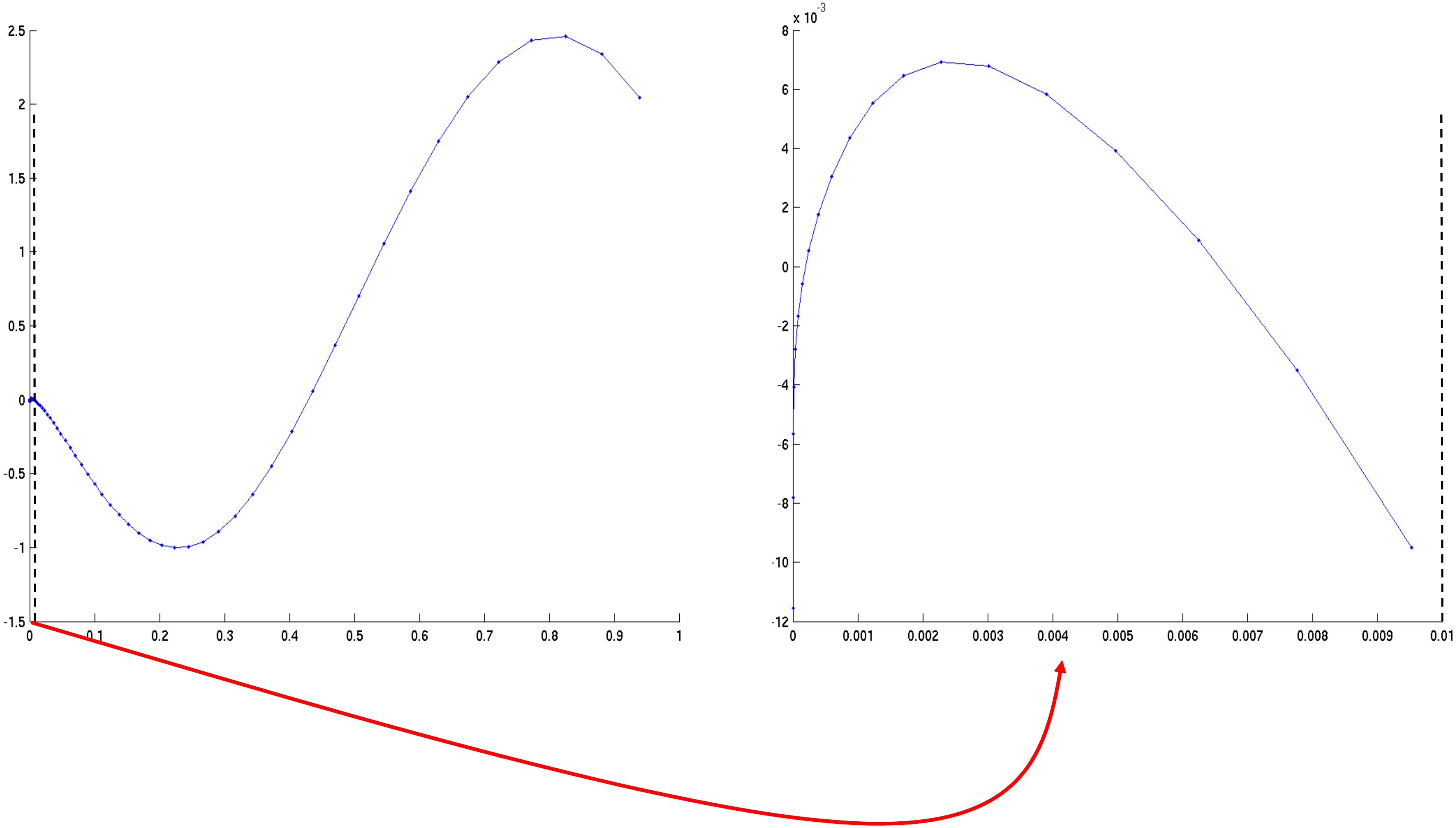


distance from the edge

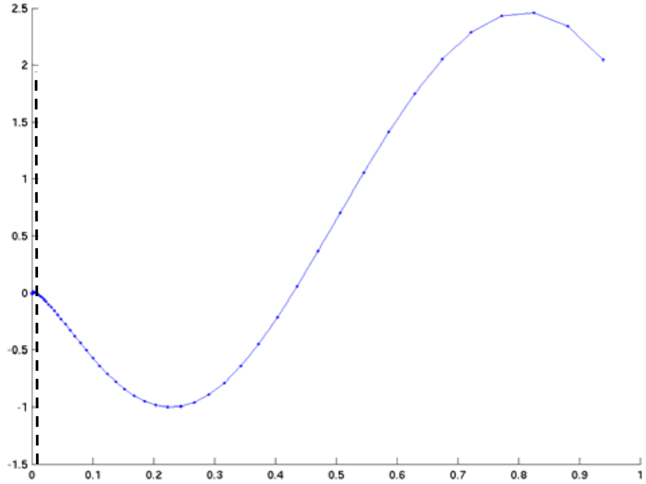
Current along a radius – focus near the edge

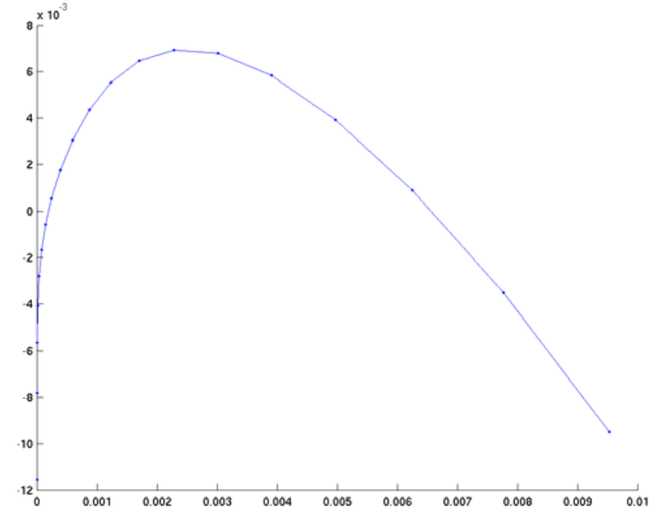


Current along a radius – focus near the edge

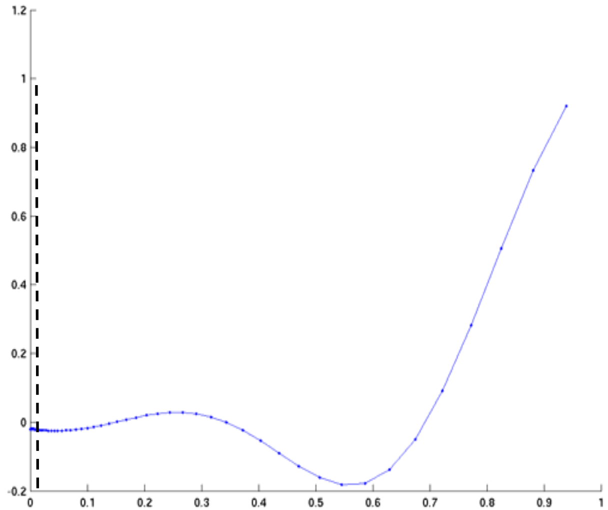


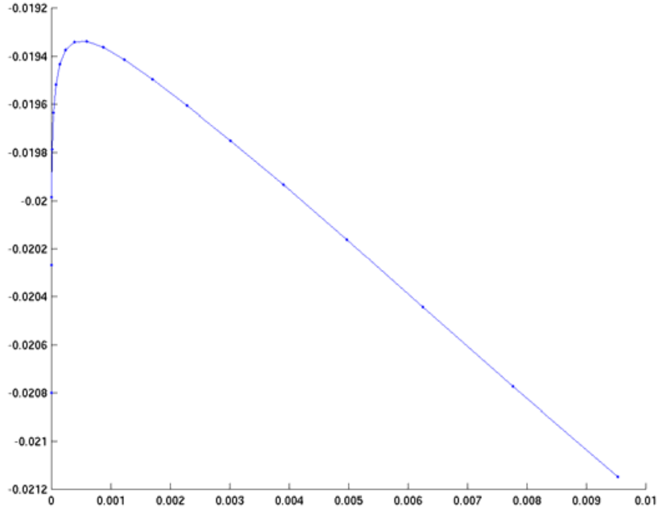
Plane-wave incidence



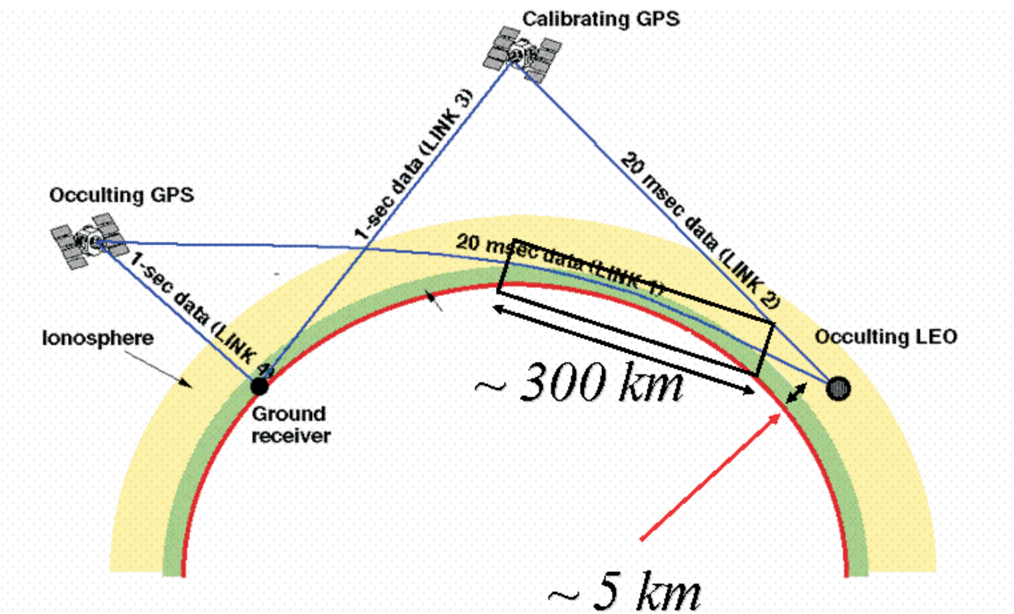


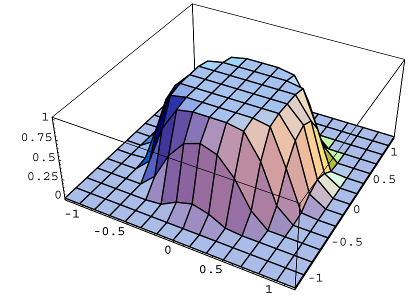
Point-source incidence (exact solution known)

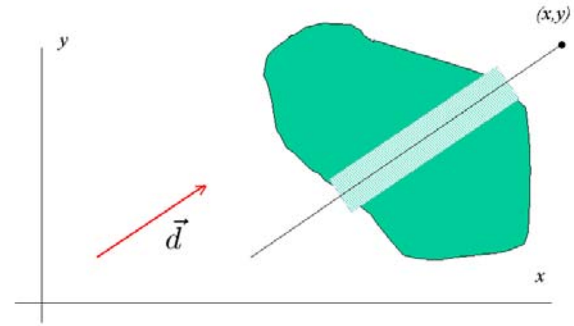




Occultation Retrievals: a high-frequency <u>penetrable</u> scattering problem





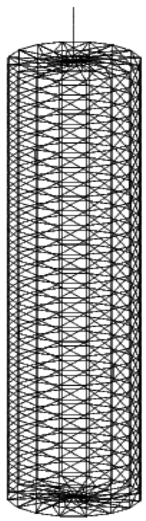


N_λ	ϵ	Relative Error	$N_\lambda \epsilon^2$	T(secs)	HF integrator (secs)
100	0.5	$\mathcal{O}(10^{-4})$	25	.11	.02
400	0.25	$\mathcal{O}(10^{-4})$	25	2.08	.08
1600	0.125	$\mathcal{O}(10^{-4})$	25	29.01	.31
6400	0.0625	$\mathcal{O}(10^{-4})$	25	450.39	14.45
200000	0.011	$\mathcal{O}(10^{-4})$	25	$\approx$ 5 days	37.73

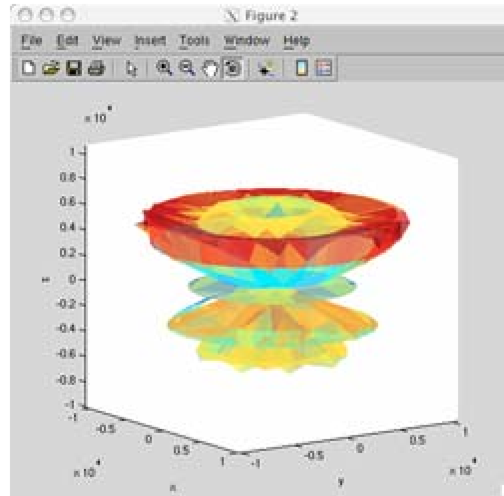
Computational cost of evaluation of the High-Frequency integral

OB and J. Chaubell, in progress

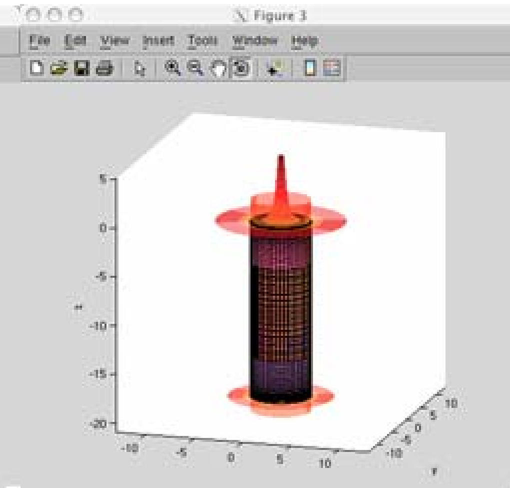
Antenna (wire) problem



Far Field

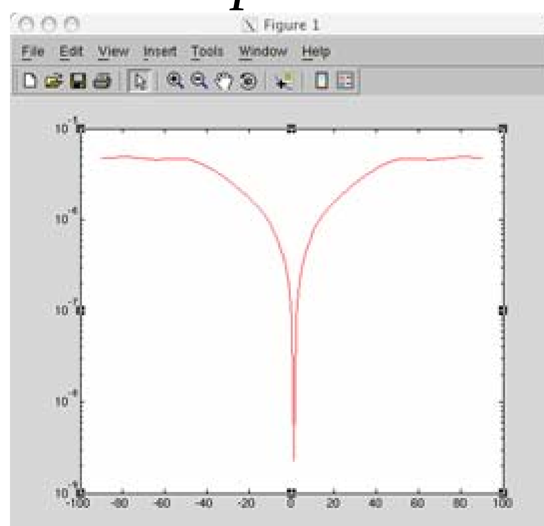


Surface and wire currents

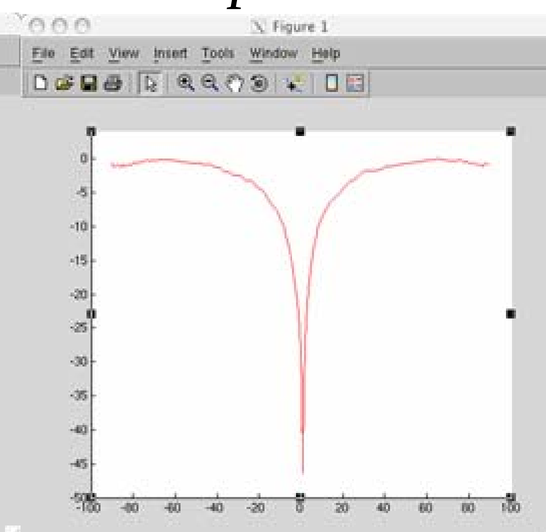


Sphere and wire, k = 0.5, dist = 5.0e-3

Computation



Experiment

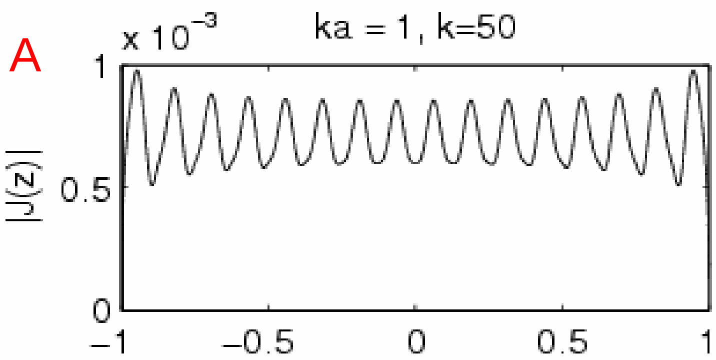


Experiments by Cable and Blezyunk, JPL, 2005

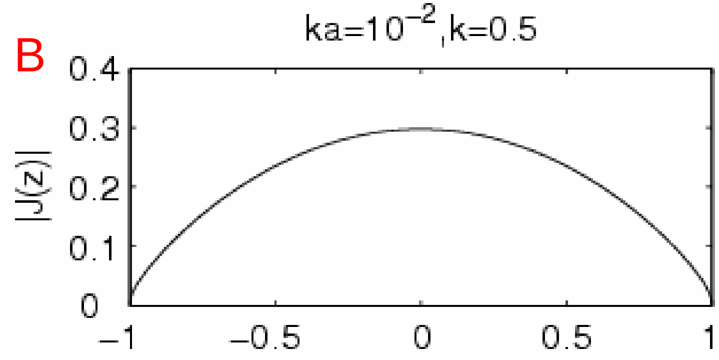
Wire problem

Method: Canonical integration

A



B



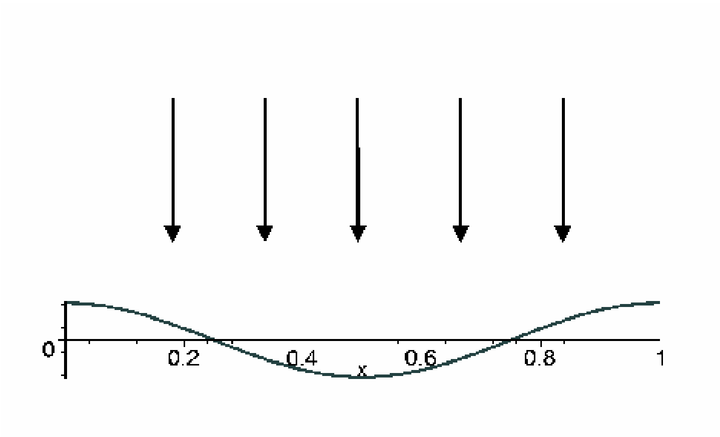
N	e_{max}	t_{exe} (s)
50	1.93×10^{-1}	0.101
60	1.20×10^{-3}	0.316
70	7.65×10^{-7}	0.469
80	1.56×10^{-8}	0.703
90	5.84×10^{-9}	0.936
100	7.18×10^{-11}	2.251
110	5.08×10^{-12}	2.786
120	4.59×10^{-13}	4.219

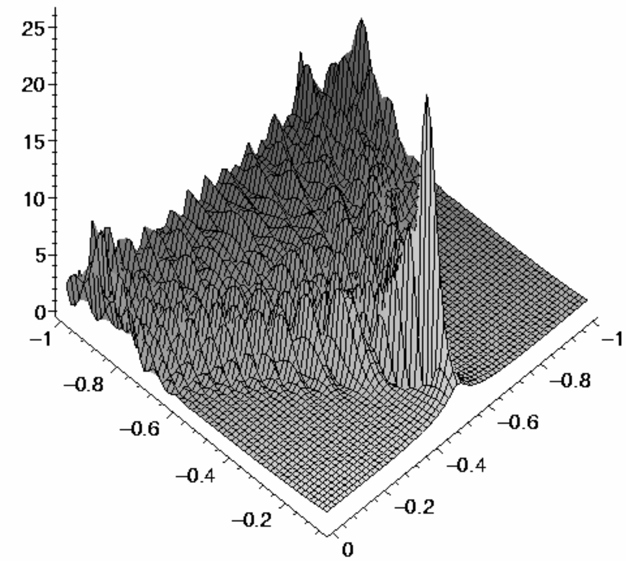
N	e_{max}	t_{exe} (s)
10	1.50×10^{-3}	0.015
20	3.73×10^{-4}	0.032
30	3.31×10^{-5}	0.044
40	2.12×10^{-6}	0.085
50	3.39×10^{-7}	0.127
60	4.70×10^{-8}	0.235
70	1.47×10^{-9}	0.349
80	2.82×10^{-10}	0.509
90	1.96×10^{-11}	1.152
100	2.83×10^{-12}	1.307

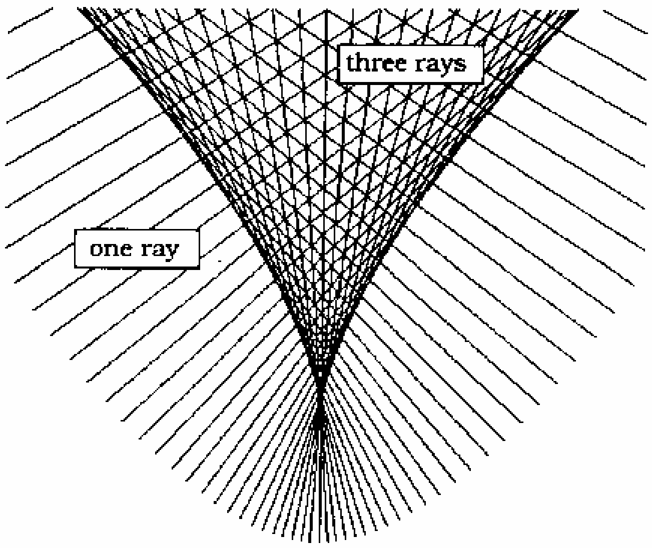
*Previous state of the art: Davies et. al. (2001) J Comput Phys **168**: 155-183.*
Require N = 700 for $O(10^{-5})$ maximum relative errors.

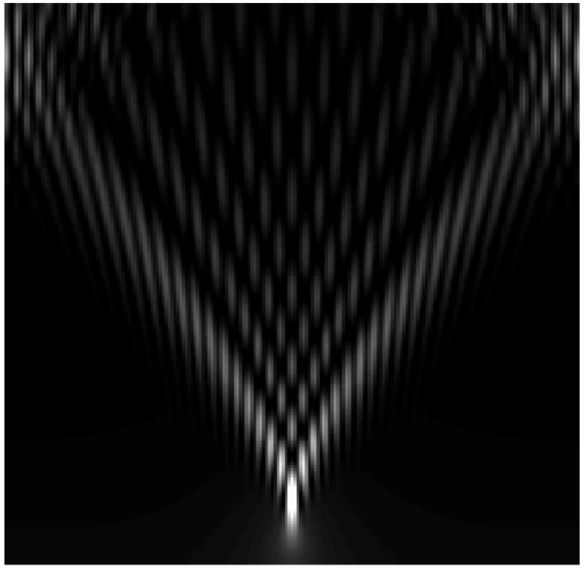
OB and M. Haslam [2005]

High Frequency and Caustics



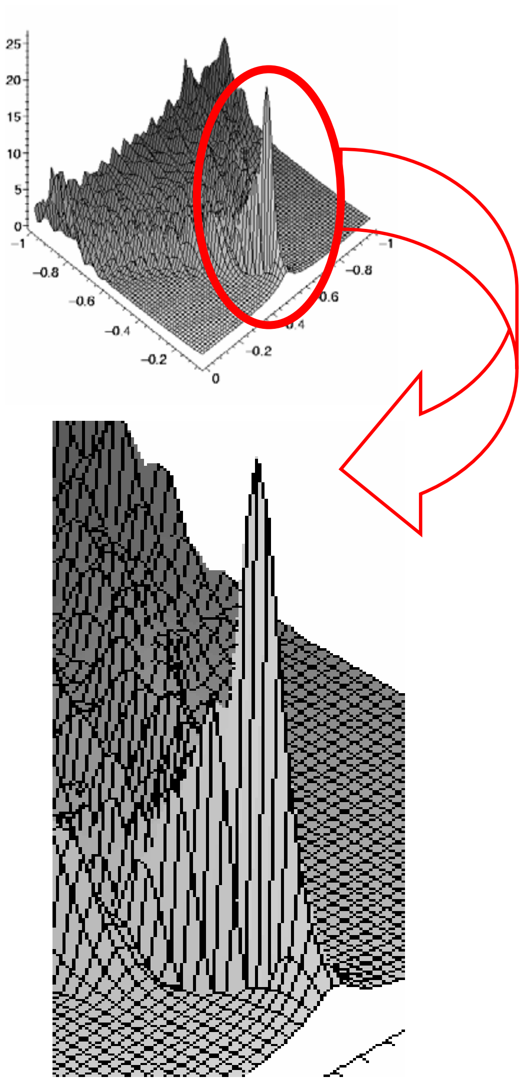


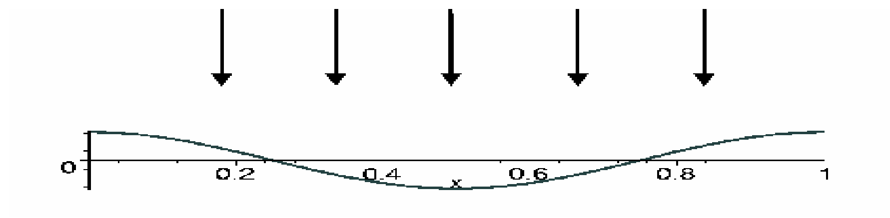


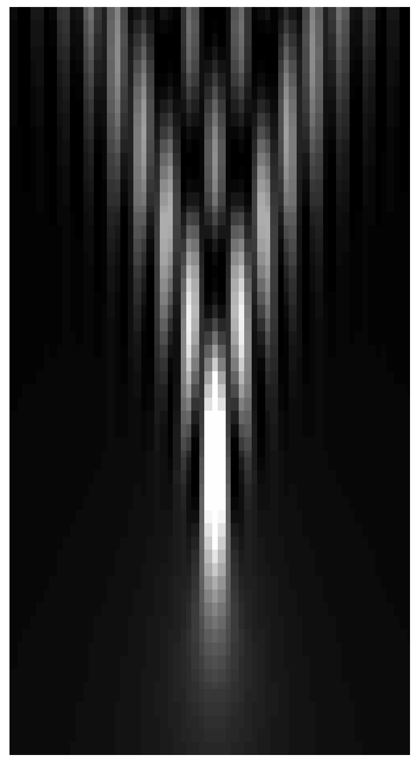


Bruno, Sei and Caponi

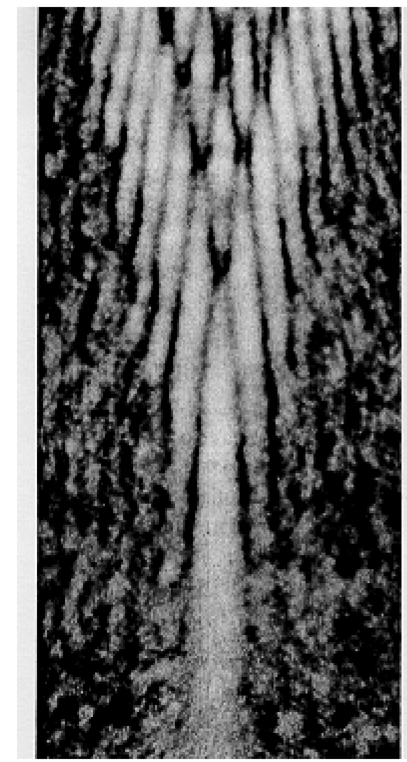
Application: High Frequency and Caustics







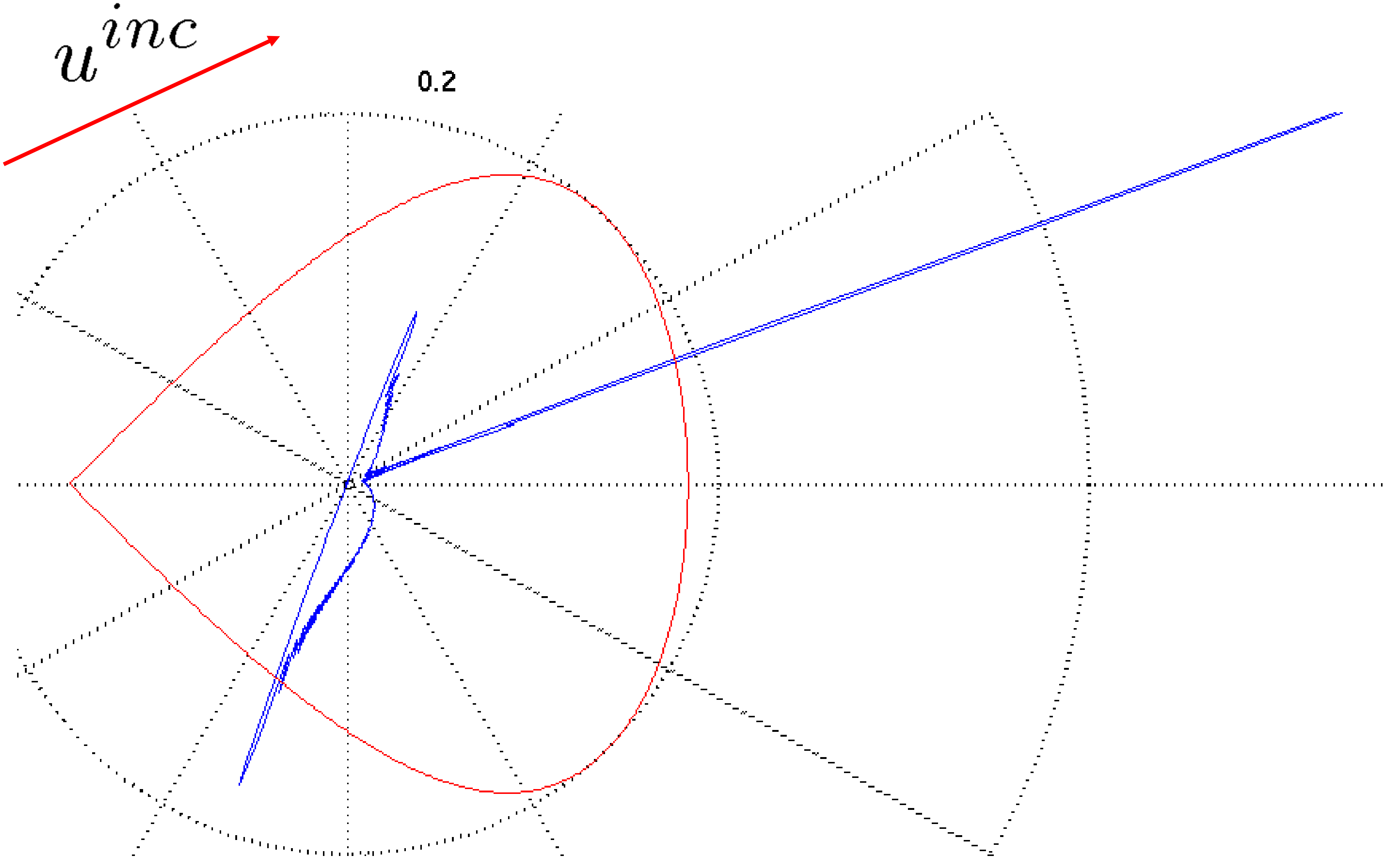
Numerical



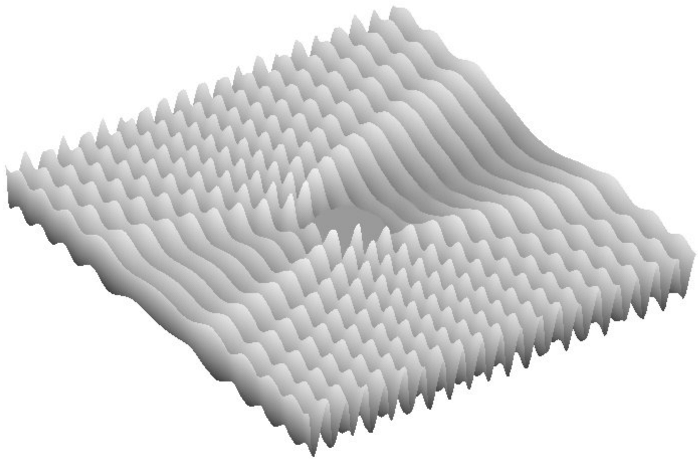
Experimental

Bruno, Sei and Caponi

DROP: Far Field; ka = 1000



Example: Combined Field IE



25 unknowns			
ka	GMRES iterations	Error	CPU time
1	9	$1.0e{-12}$	$< 1s$
10	11	$1.6e{-4}$	$< 1s$
100	13	$9.3e{-4}$	$3s$
1000	13	$8.3e{-3}$	$5s$
10000	15	$1.0e{-2}$	$6s$
100000	14	$1.1e{-2}$	$6s$

100 unknowns			
ka	GMRES iterations	Error	CPU time
1	9	$1.0e{-12}$	$< 1s$
10	17	$3.0e{-11}$	$5s$
100	22	$1.5e{-5}$	$11s$
1000	25	$3.1e{-5}$	$2m30s$
10000	27	$8.4e{-5}$	$3m12s$
100000	30	$8.8e{-5}$	$3m43s$

Convergence (Combined Field IE)

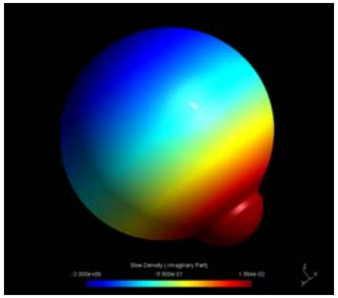
$$ka = 150$$

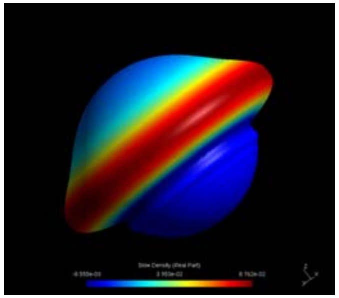
Unknowns	GMRES Iterations	Max. Error
25	13	4.4e−3
50	23	1.2e−3
100	31	1.2e−4
200	34	4.4e−6
400	39	1.0e−9
800	45/56	1.0e−12/1.3e−13

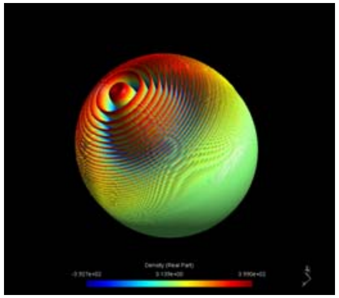
Preliminary numerical results

Single processor runs (1.7GHz pentium IV)

No code optimizations







$32 \times 32 \times 2$ unknowns

k	Diameter	GMRES iterations	Max. Error	CPU time
800	127λ	32	2.18e-2	164 min
1600	255λ	33	3.10e-2	169 min
3200	510λ	41	5.01e-2	212 min
Preprocessing time (critical points computation): 22 min				

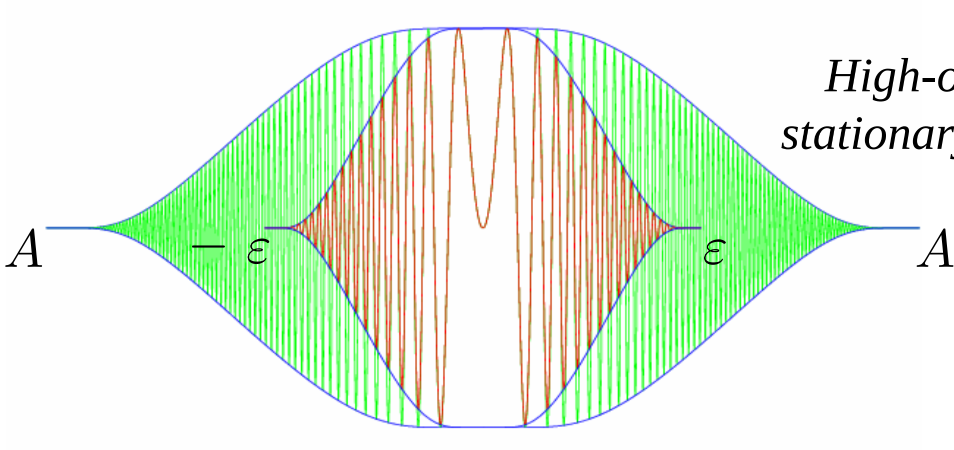
$$\text{Error} = \frac{\max_r |\mu_{slow}(r) - \mu_{slow}^{exact}(r)|}{\max_r |\mu_{slow}^{exact}(r)|}$$

Surface current error. Far field error should be one-to-two digits smaller.

OB and C. Geuzaine [2005] (preliminary)

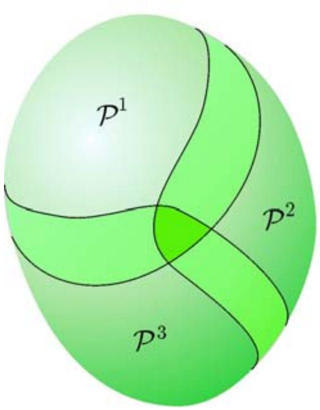
Recap

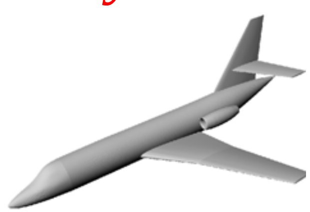
1) *Convergent **O(1)** High-Frequency Integral Method*

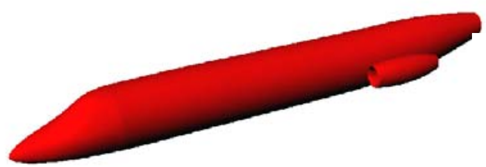


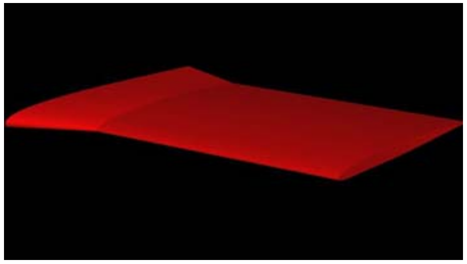
High-order stationary phase

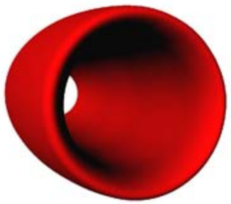
2) *Basic integration methods, surface representation and other issues addressed by means of novel Fourier-based approaches*

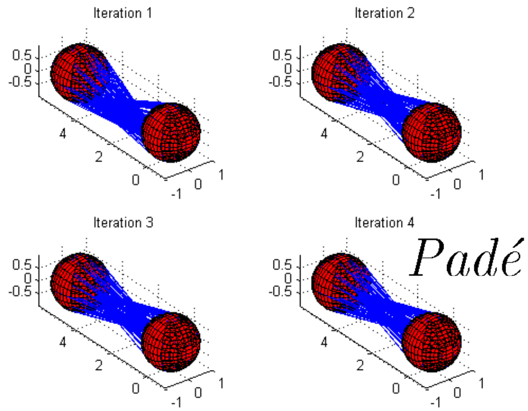












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