Asymptotic Phasefront Extraction of High Frequency Wave Components from a Numerical Mesh

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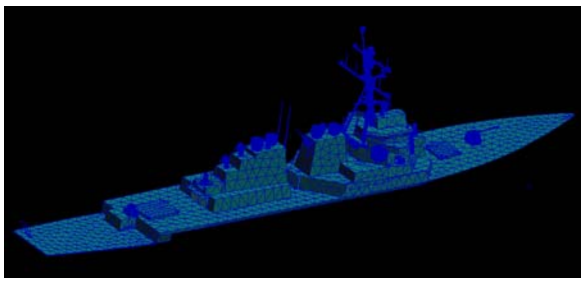
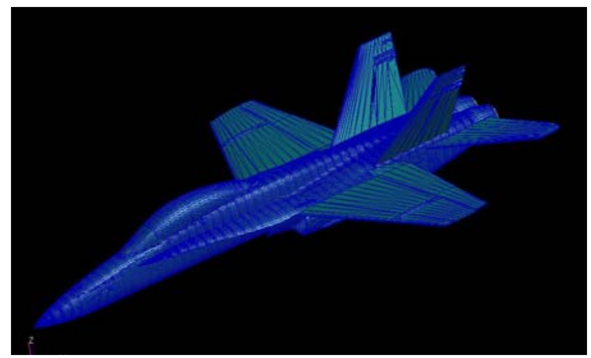
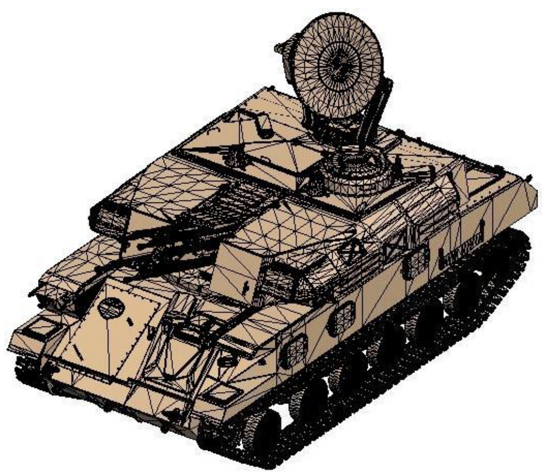
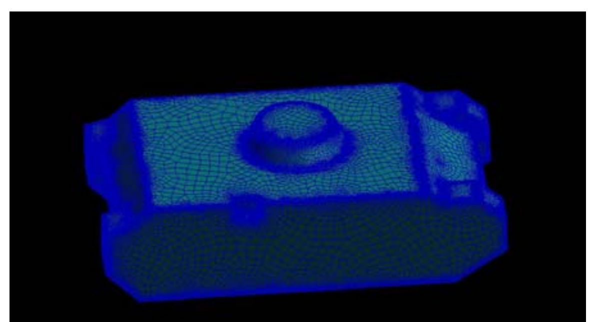
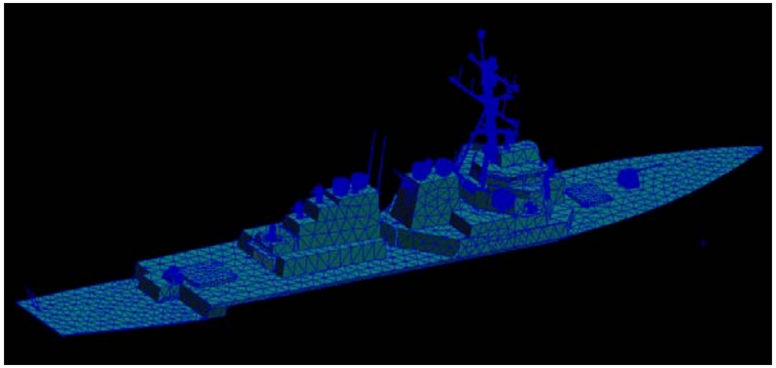


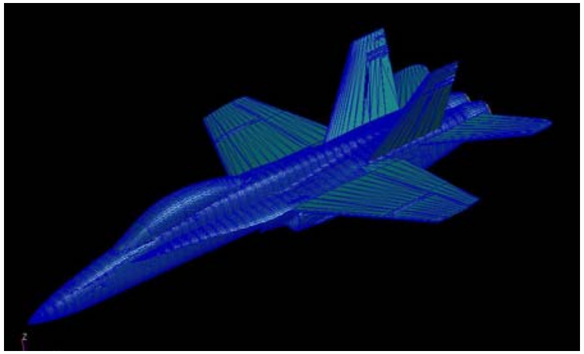






Motivation





- Realistic antenna platforms and radar targets are often hundreds or thousands of wavelengths in size at X-band frequencies and higher.
- Numerical methods require millions of unknowns, which, while possible, are far from routine.[1]
- Number of unknowns scales with the square of the frequency.
- Ray methods alone do not provide sufficient accuracy and generality.

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[1] S. Velamparmbil, W. C. Chew and J. M. Song, "10 million unknowns: Is that big?", *IEEE Antennas and Propagation Magazine*, 45(2): 43-58, April 2003.

Approaches for Extending Existing Numerical Methods

1. Faster computers
   - *Computer speed and memory resources continue to grow*
   - *Moore's Law can't keep up with frequency requirements*

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   - *Fast methods are already fairly mature*
   - *Unknowns still scale with square of frequency*
3. Reduce the number of unknowns
   - *Characteristic basis functions*[2]
   - *Asymptotic phasefront extraction*[3]

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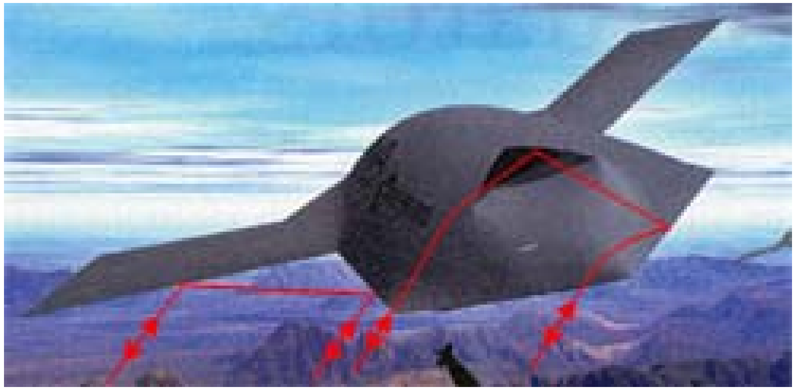
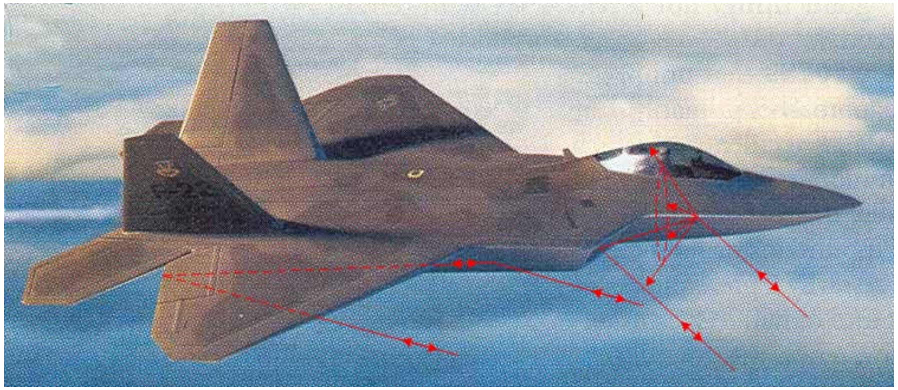
[2] V. V. V. Prakash and R. Mittra, "Characteristic basis function method: A new technique for efficient solution of method of moments matrix equations," *Microwave and Optical Tech. Letters*, 36(2): 95-100, Jan. 20, 2003.

[3] D.-H. Kwon, R. J. Burkholder and P. H. Pathak, "Efficient Method of Moments Formulation for Large PEC Scattering Problems Using Asymptotic Phasefront Extraction," *IEEE Trans. Antennas and Propagation*, 49(4) 583-591, April 2001.

Observations from Ray Physics

1. In smooth regions, the $O(f^2)$ dependence of the number of unknowns is due to the rapidly varying phase (~10 unknowns per wavelength).
2. Fields over smooth surfaces or in homogeneous material regions may be represented with a small number of ray wavefronts.
3. Ray paths are independent of frequency.
4. Amplitude of fields of each ray is slowly varying and relatively independent of frequency.





Principle of Asymptotic Phasefront Extraction (APEx)

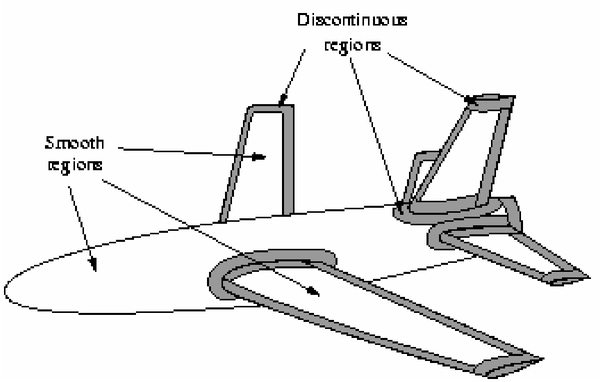
If the wavefronts over the smooth surface are known, the fast phase variation may be included in frequency-scalable traveling wave (linear phase) basis functions so that only the slowly varying amplitude is sampled:

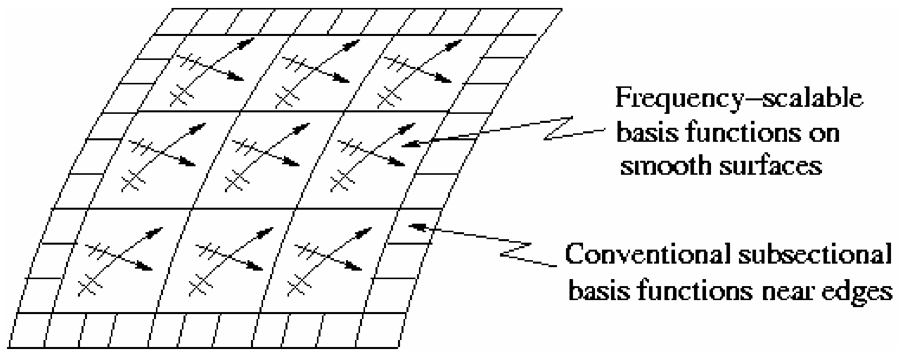
$$\mathbf{J}(\mathbf{r}) = \Sigma_i \, \mathbf{C}_i(\mathbf{r}) \exp(-j\mathbf{k}_i \bullet \mathbf{r})$$

- Since only the amplitude of the surface currents is sampled, the basis functions will be relatively frequency independent so they may be used over a very wide frequency band.

- Theoretically, subsectional basis functions may be larger than the electrical wavelength.

Basic Algorithm for TW Basis Functions





1. For a given CAD geometry, partition the surface into smooth regions and discontinuous regions near edges, tips, gaps, etc.
2. Expand the currents in discontinuous regions with conventional subsectional basis functions.
3. *Find the traveling waves over the smooth regions for a given excitation.*
4. Expand the currents in smooth regions using the frequency-scalable subsectional basis functions with linear phase variations (i.e., traveling waves).
5. Solve using method of moments. (Could also work with finite element method, but hasn't been tested.)

Approach 1 for Finding Phasefronts

Ray tracing and physical optics: Trace incident, reflected, and diffracted rays to all surface points in smooth regions.

- Not practical for complex CAD geometries.

K.R. Aberegg and A.F. Peterson, "Application of the integral equation-asymptotic phase method to two-dimensional scattering," *IEEE Trans. Antennas Propagat.*, vol. 42, pp. 534-537, May 1995.

M.E. Kowalski, B. Singh, L.C. Kempel, K.D. Trott, J.-M. Jin, "Application of the Integral Equation-Asymptotic Phase (IE-AP) Method to Three-Dimensional Scattering," *J. Electromagn. Waves and Appl.*, 15(7) 885-900, 2001.

E. Giladi and J.B. Keller, "A Hybrid Numerical Asymptotic Method for Scattering Problems," *J. Computational Physics*, 174(1) 226-247, Nov. 20, 2001.

Approach 2 for Finding Phasefronts

Low frequency phasefront extraction: Extract phasefront information from a numerical solution at a lower frequency.

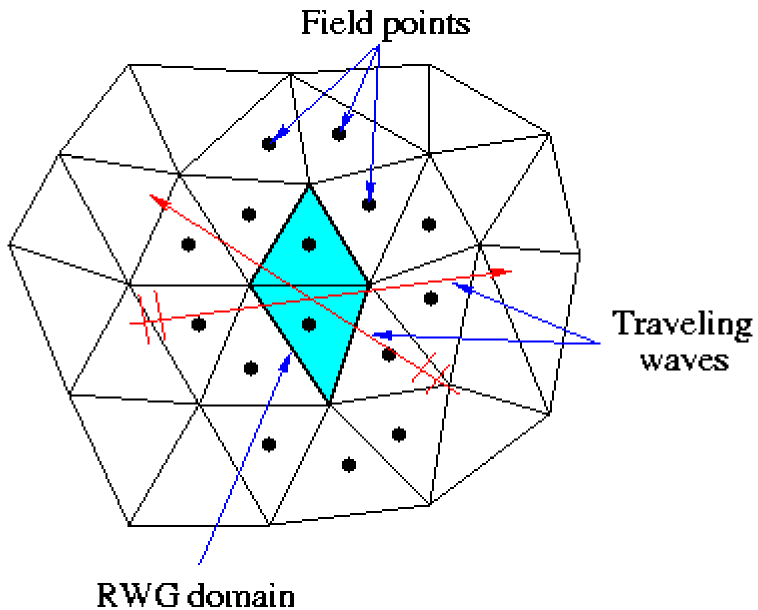
• Local surface must be electrically large enough to resolve overlapping wavefront components.

• Simple direction-of-arrival (DOA) analysis used to resolve wavefront vectors on smooth surfaces. Super-resolution techniques to improve accuracy.

D.-H. Kwon, R.J. Burkholder and P.H. Pathak, "Efficient Method of Moments Formulation for Large PEC Scattering Problems Using Asymptotic Phasefront Extraction (APE)," *IEEE Trans. Antennas Propagat.*, vol. 49, pp. 583-591, April 2001.

“Sensor Array” Technique for Phasefront Extraction

Find traveling waves (phasefronts) from a low frequency solution for the fields at a grid of points on the surface. Grid is a “sensor array” for direction-of-arrival (DOA) estimation.



- For each RWG basis function, use the field points of connected triangles as the sensor array.
- Triangles must be half-wavelength or smaller.
- The low frequency solution may use a coarser mesh because the phasefront vectors are relatively insensitive to numerical accuracy.
- Super-resolution techniques may be used for DOA estimation (Capon, Prony, GPoF, MUSIC, CLEAN).

Adaptation of CLEAN Algorithm (or "Extract and Subtract")

The local surface currents are assumed to have the form

$$\bar{J}(\bar{r}) = \sum_i \bar{C}_i \exp(-j\bar{k}_i \cdot \bar{r})$$

1. For P sensor grid points, find the phasefront vector k_1 and coefficient C_1 that minimize the function:

$$\sum_{p=1}^{P} \left| \bar{J}(\bar{r}_p) - \bar{C}_1 \exp(-j\bar{k}_1 \cdot \bar{r}_p) \right|^2$$

2. Subtract the first phasefront from the grid currents:

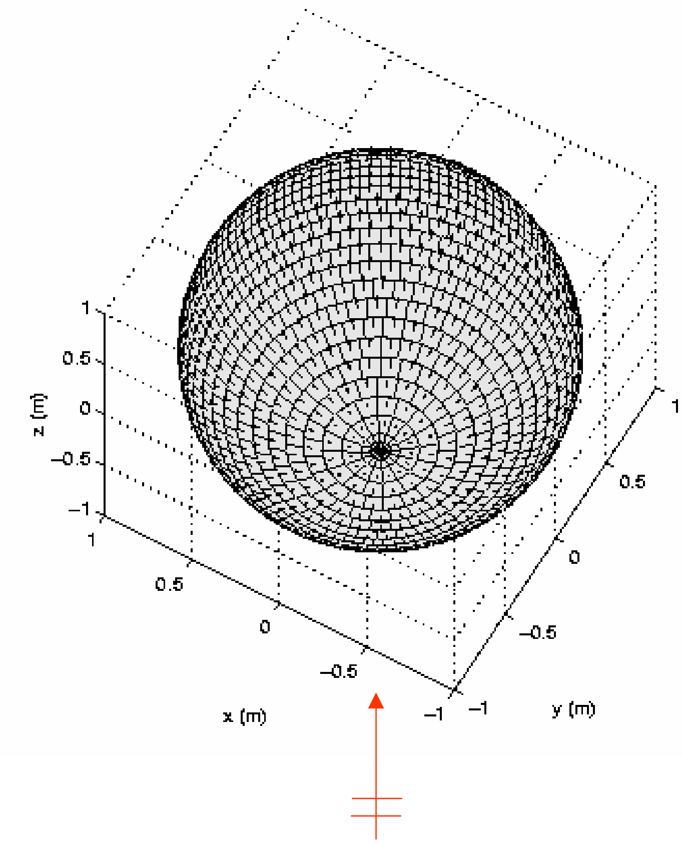
$$\bar{J}(\bar{r}_p) = \bar{J}(\bar{r}_p) - \bar{C}_1 \exp(-j\bar{k}_1 \cdot \bar{r}_p)$$

3. Repeat for each additional phasefront until C_i is sufficiently small.

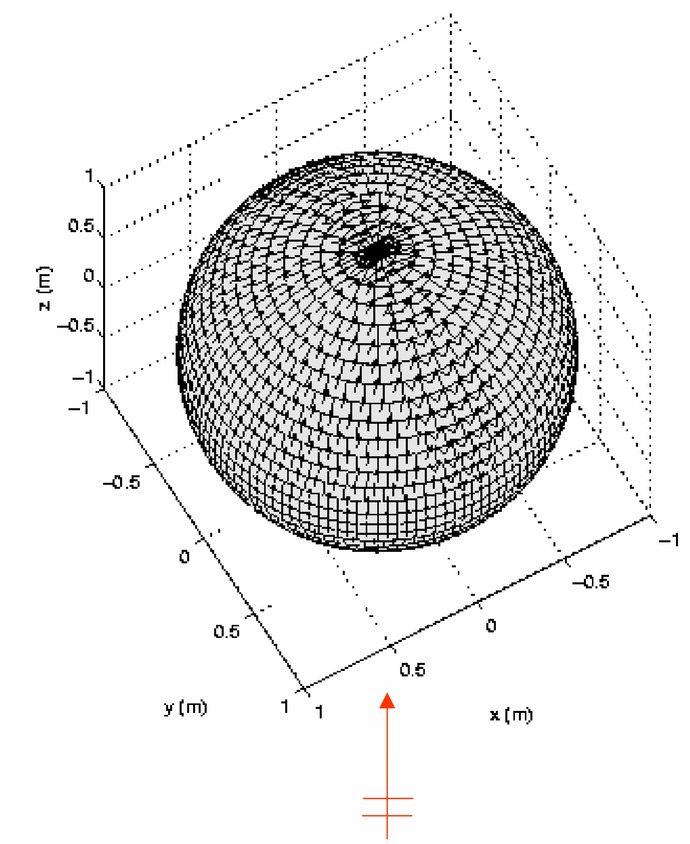
J. Tsao and B. D. Steinberg, "Reduction of Sidelobe and Speckle Artifacts in Microwave Imaging – The CLEAN Technique," *IEEE Trans. on Antennas and Propagation*, 36(4): 543-556, April 1988.

Phasefront Vectors on a Sphere from APEx

- Incident plane wave propagating in z-direction.



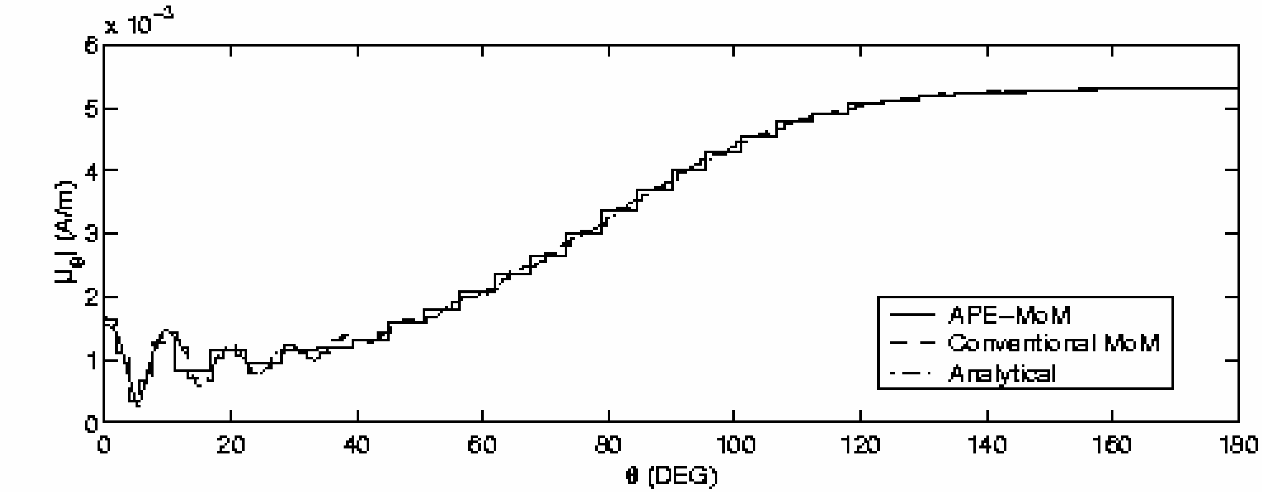
Lit region



Shadow region

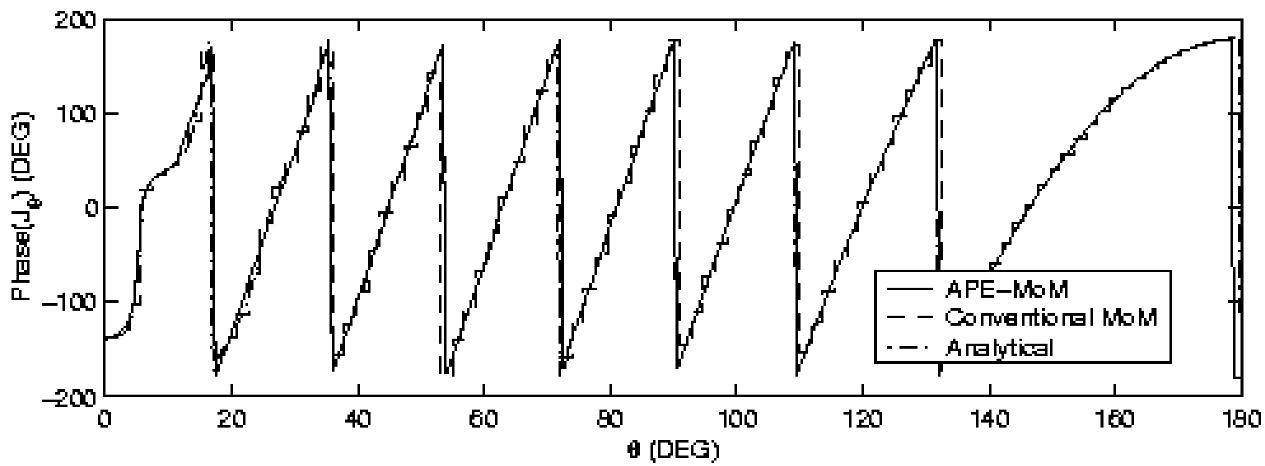
Currents on the 2m Sphere (900 MHz)

- Pulse basis for APEx-MoM and Conventional MoM (Method of Moments).



Amplitude

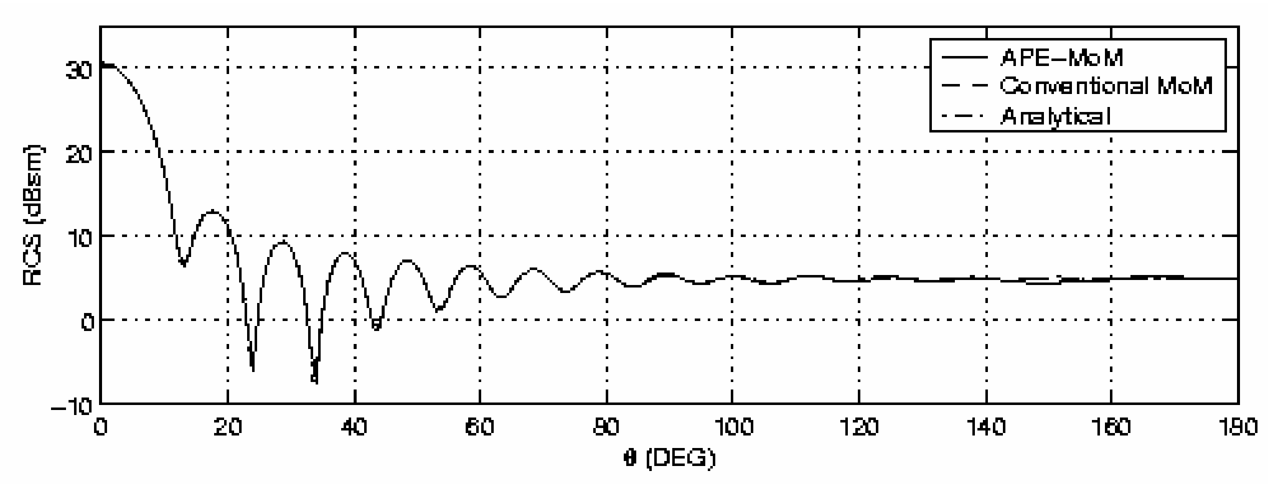
V-plane



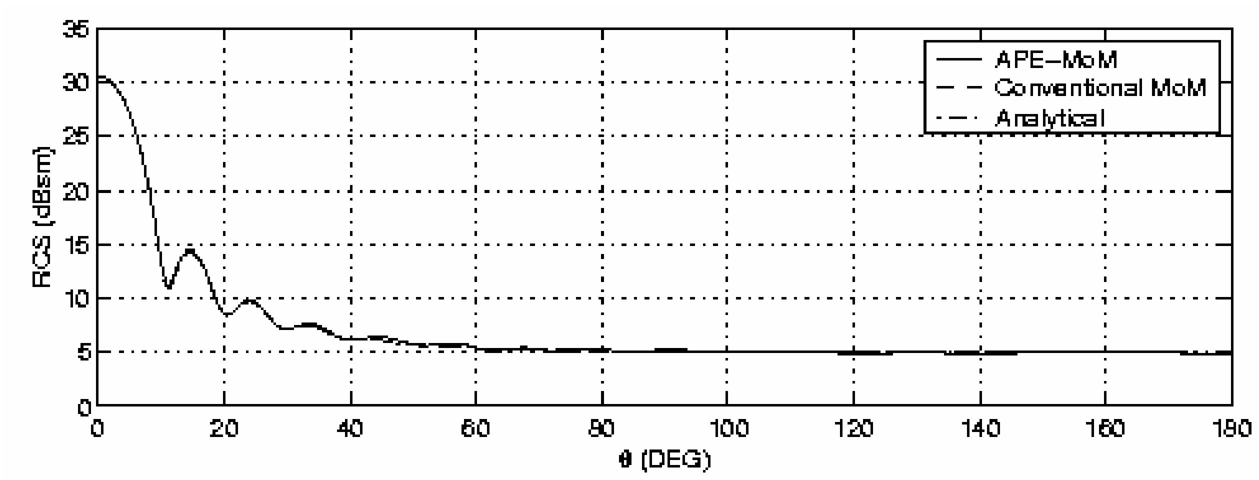
Phase

V-plane

Bistatic RCS of the 2m Sphere (900 MHz)



V-plane



H-plane

CPU Requirements for 2m Sphere

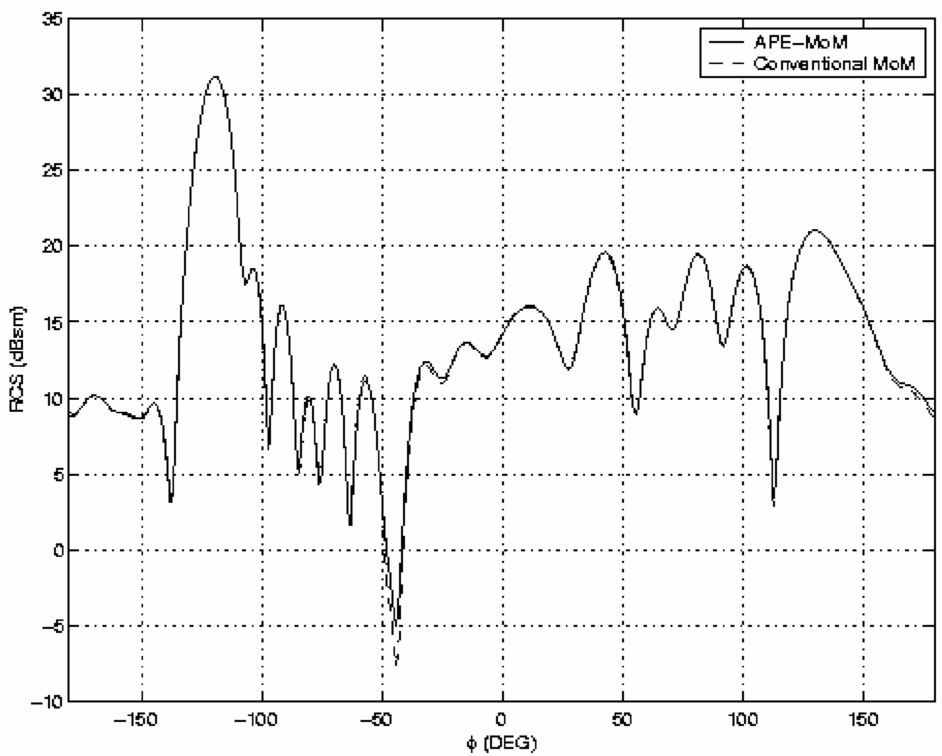
	Unknowns	CPU time	CPU memory
Low frequency solution at 300 MHz	2,668	2.6 mins	57 MB
APEx solution at 900 MHz	2,784	17.3 mins[1]	62 MB
Conventional solution at 900 MHz	22,950	361 mins[2]	4,213 MB[2]

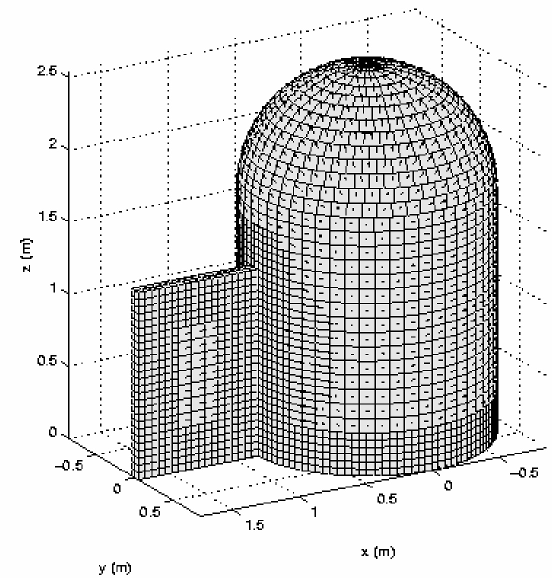
[1]Includes phasefront extraction time.

[2]Full matrix iterative solution.

Bistatic RCS Pattern of Finned Cylinder with APEx-MoM

- 600 MHz plane wave incident from theta=90, phi=60 deg
- Phasefronts extracted from 300 MHz MoM solution (*N*=*3,872*)
- Conventional MoM uses ***14,938*** basis functions
- APEx-MoM uses ***7,626*** basis functions





Surface grid and phasefront vectors at 600 MHz

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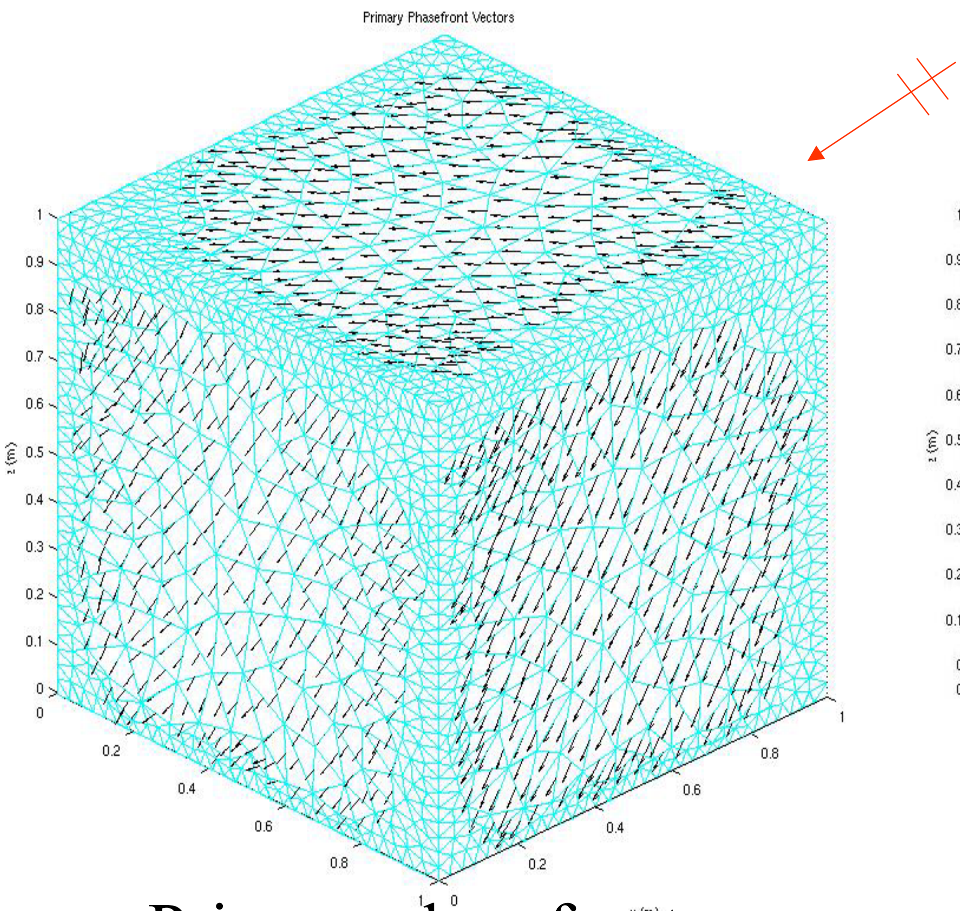
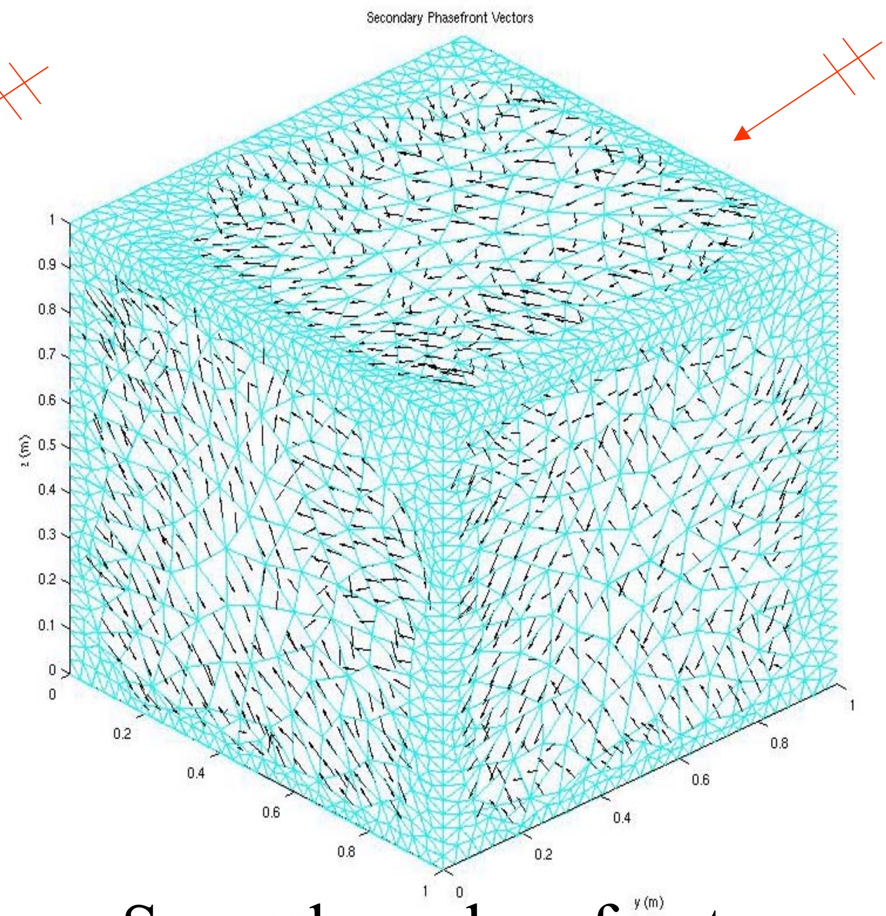
Phasefront Vectors on a Cube

45° azimuth and elevation 1.2 GHz

• Coarse grid is λ/3, fine grid is λ/8



Primary phasefronts



Secondary phasefronts

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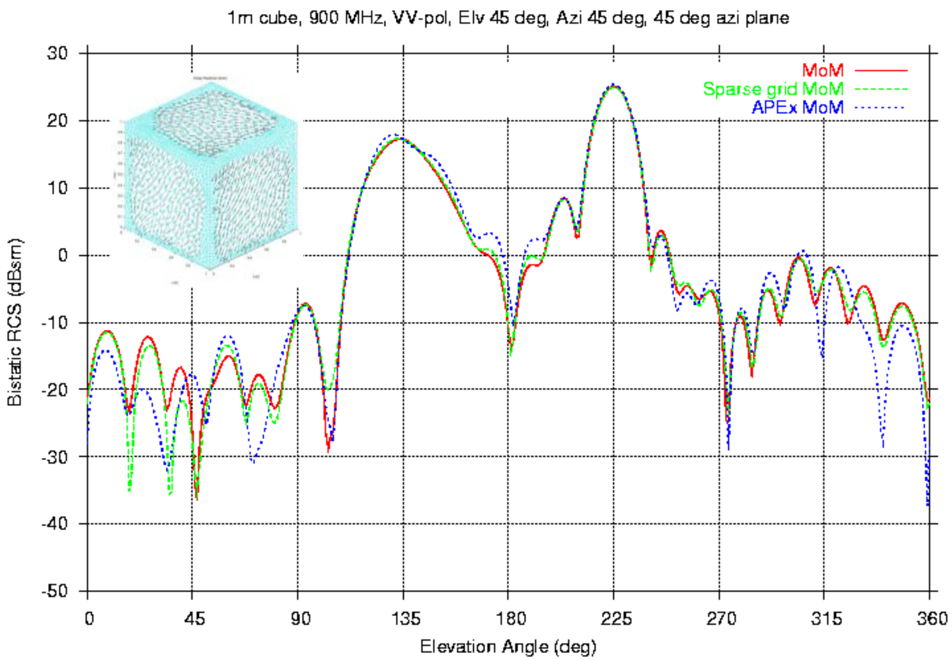
Bistatic RCS of 1 m Cube at 900 MHz

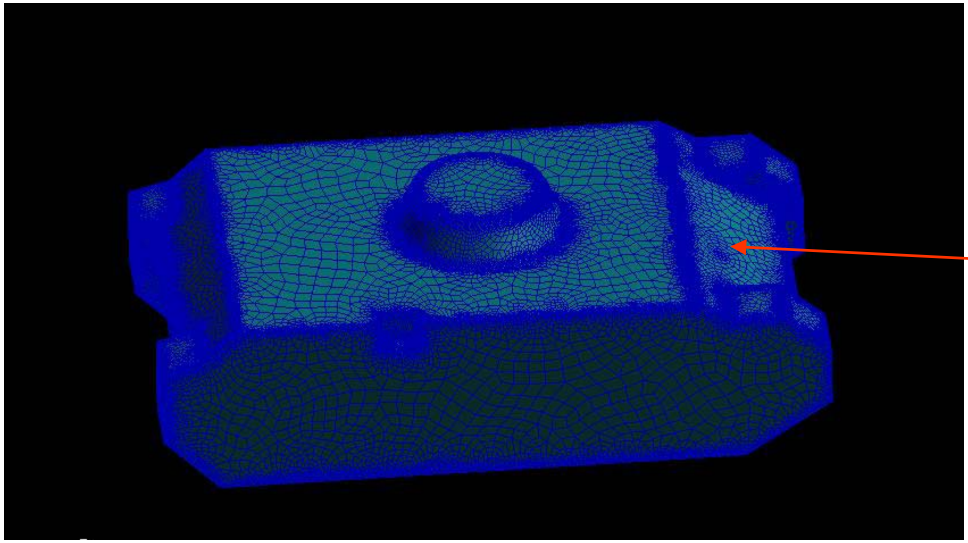


- 1 phasefront per RWG basis function domain.
- Phasefront vectors obtained from coarse grid method of moments (MoM) solution (for demonstration purposes).
- Direct LU factorization used for these results.
- Matrix fill would need to be repeated for each incidence angle for monostatic RCS with APEx-MoM.

	Sampling	Unknowns	Fill Time*	Solve Time*
MoM	λ/8	11,790	1,313 sec	13.1 hours
Coarse MoM	λ/3	5,454	393 sec	44 mins
APEx MoM	λ	1,944	100 sec	38 secs

*CPU times are for a 3 GHz Pentium IV workstation.

*Generic Tank Model**



Patran® used to generate APEx mesh using λ/8 elements along edges and 3λ elements away from edges.

*Scale model designed and built by Bill Spurgeon at Army Research Lab

• 1/16th scale model RCS measurements in OSU-ESL compact range at X, K, and W bands.

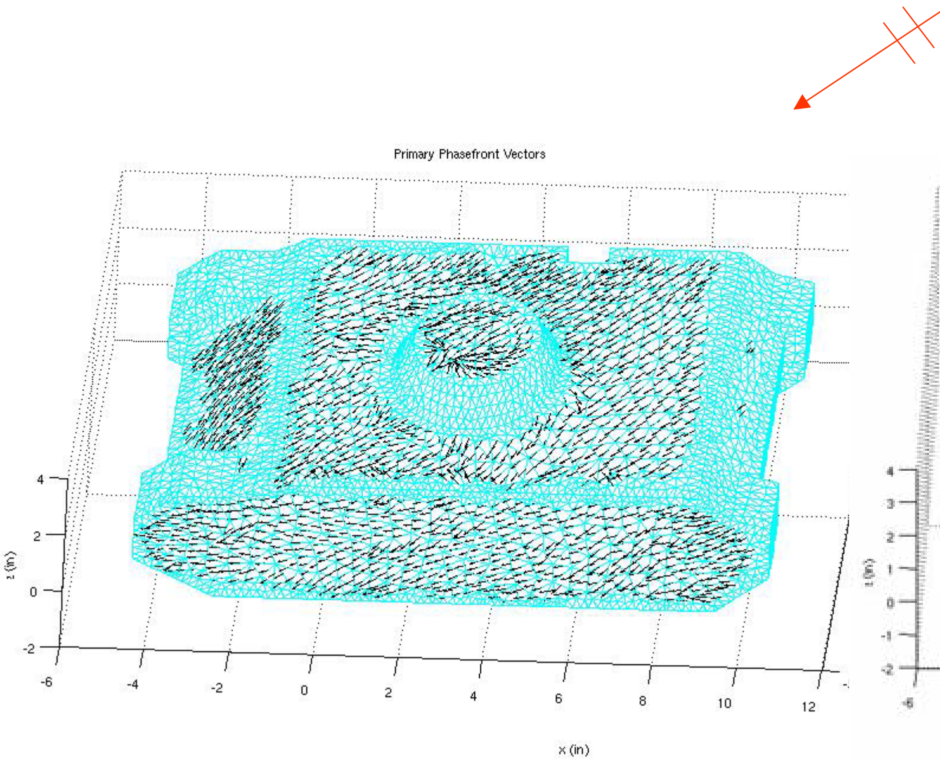
• Scale model is 15" long x 8" wide x 5" high.

• Full-scale model is 20' long x 11' wide x 7' high.

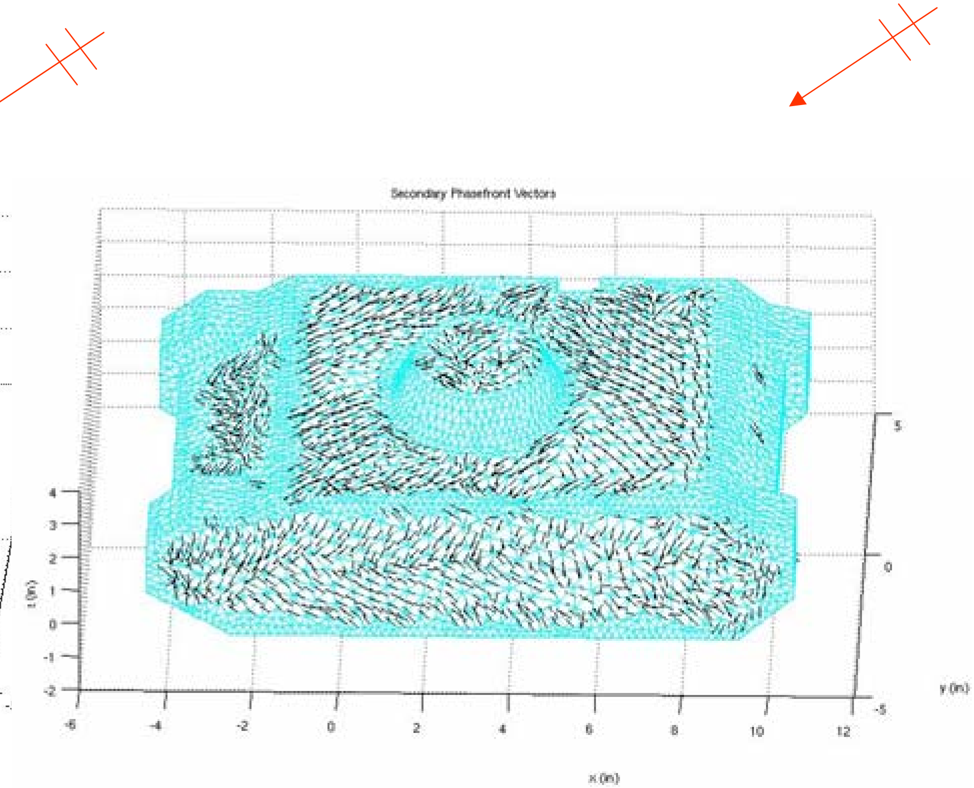
Phasefront Vectors on the Generic Tank

45° azimuth and 10.5° elevation   8 GHz

- Coarse grid is $\lambda/3$, fine grid is $\lambda/8$



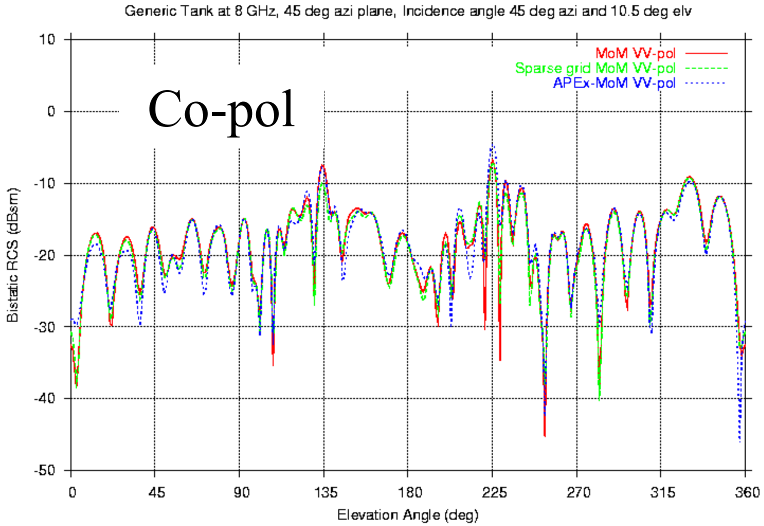
Primary phasefronts

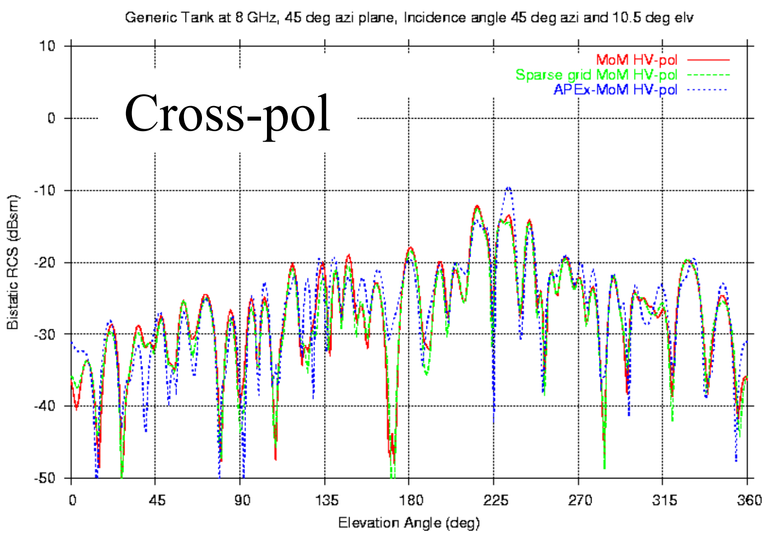


Secondary phasefronts

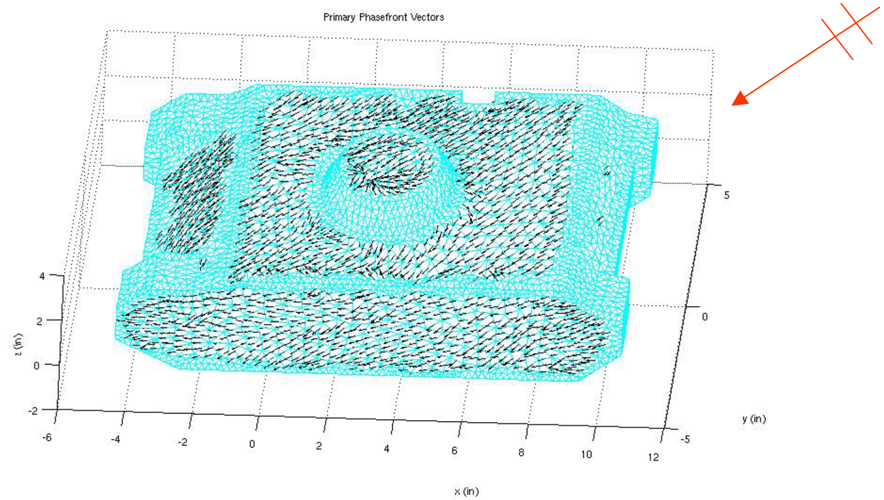
Bistatic RCS of a Generic Tank Model

8 GHz, Incidence angle 45 azi, 10.5 elv





1/16th scale model



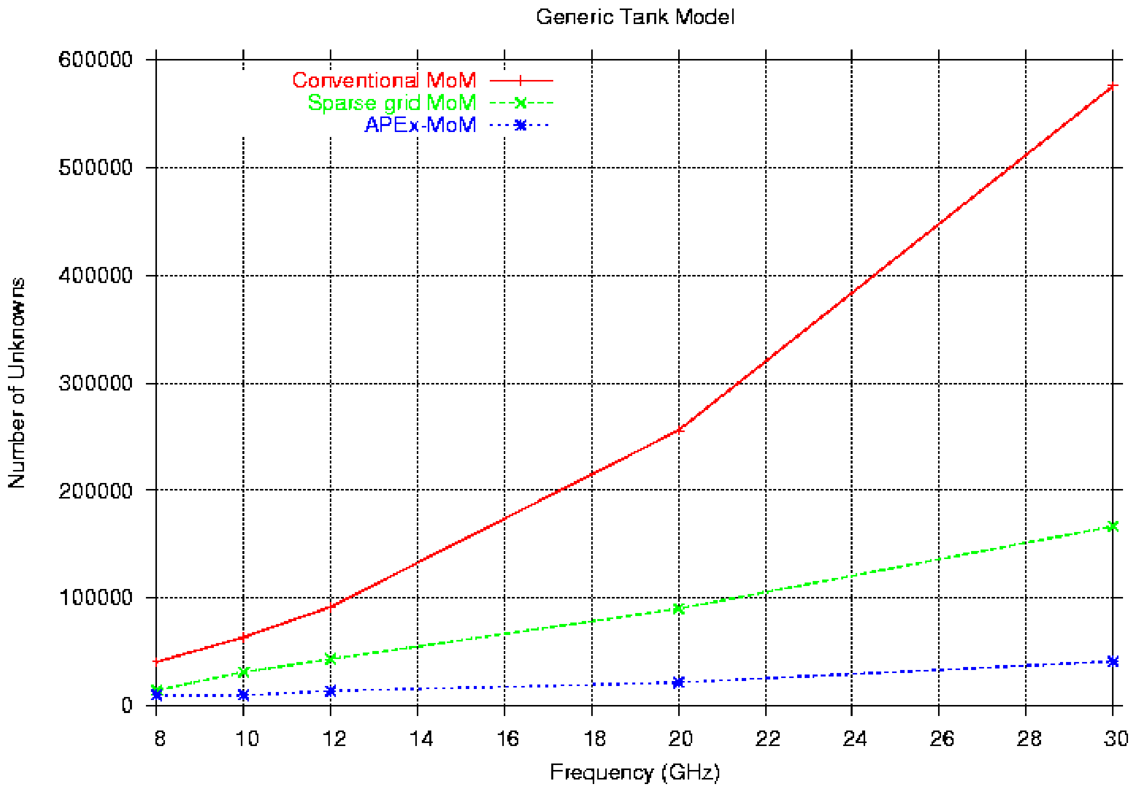
	N	Memory[1]	CPU time[2]
MoM	40,929	1348 MB	37 min
Coarse MoM	14,772	343 MB	9 min
APEx-MoM[3]	10,089	200 MB	6 min

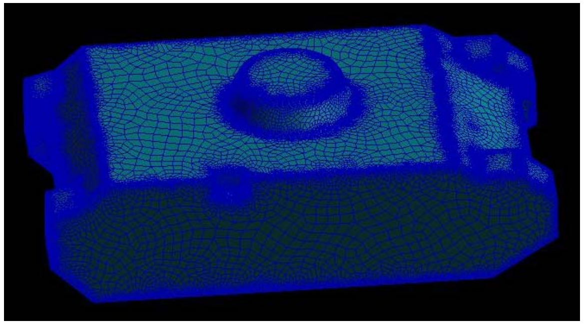
[1]Adaptive cross approximation for matrix compression.

[2]On 3 GHz Pentium IV workstation.

[3]One phasefront per patch.

Scaling of Unknowns for Generic Tank Model

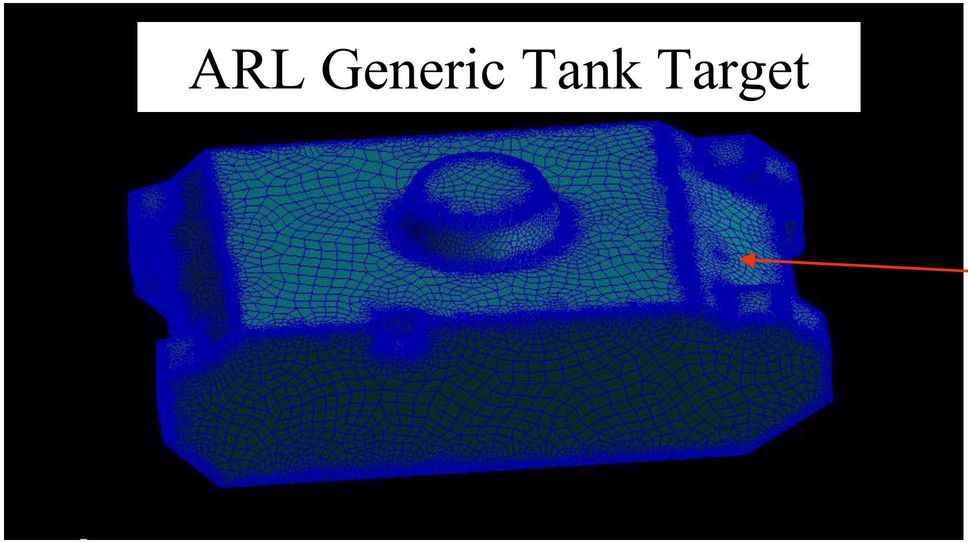




Freq	MoM	Coarse MoM	APEx MoM
8	40,929	14,772	10,089
12	92,092	43,653	14,043
20	255,806	90,352	21,730
30	575,564	166,816	41,395

- Meshes generated with Patran®
- Edge regions meshed with λ/8 sampling
- Coarse grid MoM meshed with λ/3 sampling away from edge regions
- APEx-MoM meshed with 1” sampling away from edge regions

Computational Estimates for a Full-Size Vehicle



Large basis APEx/MoM mesh for full-scale tank at X-band (10 GHz)

- Full-scale model is 20' long x 11' wide x 7' high.

Full scale at X-band	Basis Functions
MoM	7.5 million
Coarse-Grid APEx-MoM	1.2 million
Large basis APEx-MoM	275,000*

*Using 3λ surface patches.

Efficient Numerical Integration of Large Basis Functions

- The efficiency of the APEx-MoM approach relies heavily on the integration of large basis functions.
- Even with a greatly reduced set of basis functions, numerical integration requires a certain frequency-dependent sampling density.
- Basis function interactions (i.e., impedance matrix elements) need to be regenerated for each excitation.
- Much more efficient evaluation of the free space radiation integral is needed.
- Higher order surface patches needed for modeling curved surfaces.

Efficient Numerical Integration of Large Basis Functions

Methods under investigation:

- *Coarse grid sampling (doesn't require matrix regeneration)*
- *Surface to edge integral transform*
- *Closed-form far-field or asymptotic expansions*
- *Stationary phase methods*

O. P. Bruno, "Fast, high-order, high-frequency 'Accurate Fourier Methods' for scattering problems," 2002 IEEE Antennas and Propagation Symposium, June 16-21.

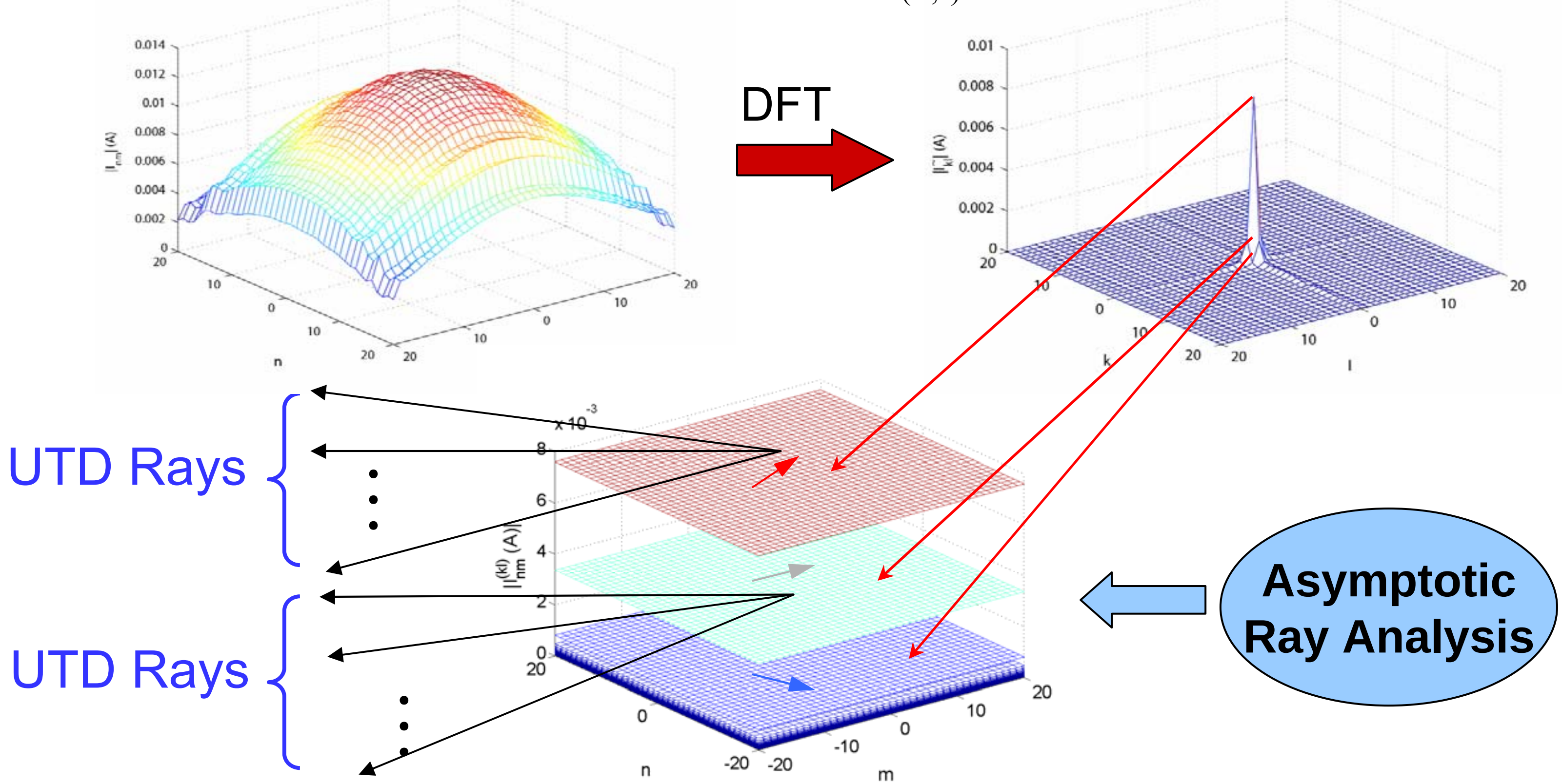
R. J. Burkholder and T.-H. Lee, "Adaptive sampling for fast physical optics numerical integration," *IEEE Trans. on Antennas and Propagation*, May 2005.

S. J. Leifer and R. J. Burkholder, "An Inverse Power Series Representation for the Free Space Green's Function," submitted to *Microwave and Optical Tech. Letters*.

Traveling Wave Basis Functions for Large Arrays

- Used to describe real aperture distributions in terms of smooth functions

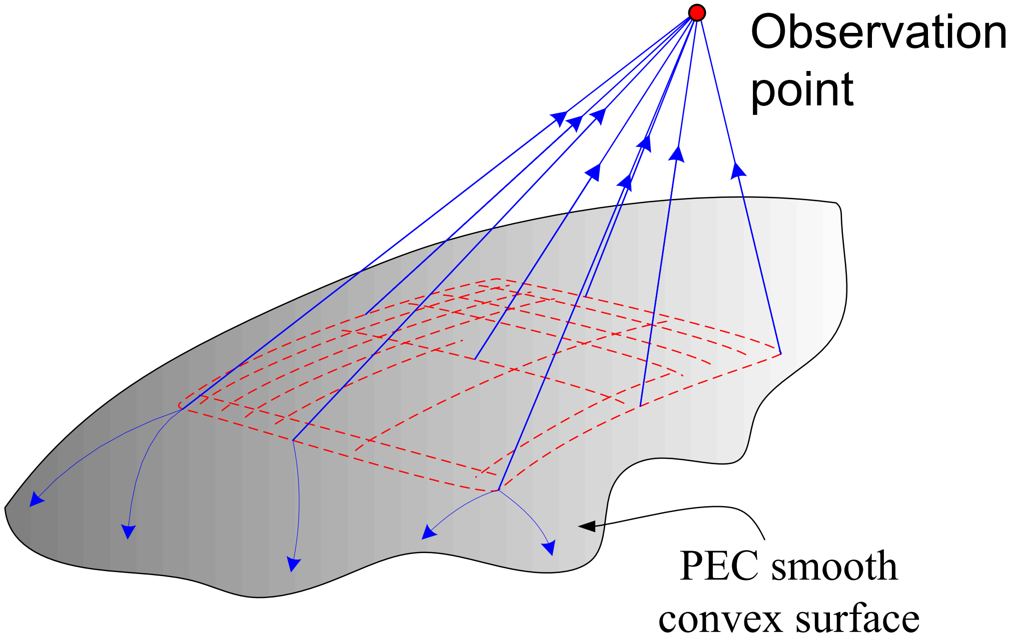
$$P_{nm} = \sum_{k=0}^{N-1}\sum_{l=0}^{M-1} \widetilde{P}_{kl} e^{-jn(2\pi k/N)} e^{-jm(2\pi l/M)} \approx \sum_{(k,l)\in D} \widetilde{P}_{kl} e^{-jn(2\pi k/N)} e^{-jm(2\pi l/M)}$$



P. Janpugdee, P. H. Pathak, et al., "Ray Analysis of the Radiation from a Large Finite Phased Array of Antennas on a Grounded Material Slab," *2001 AP-S/URSI Symp.*, Boston MA, Jul. 2001.

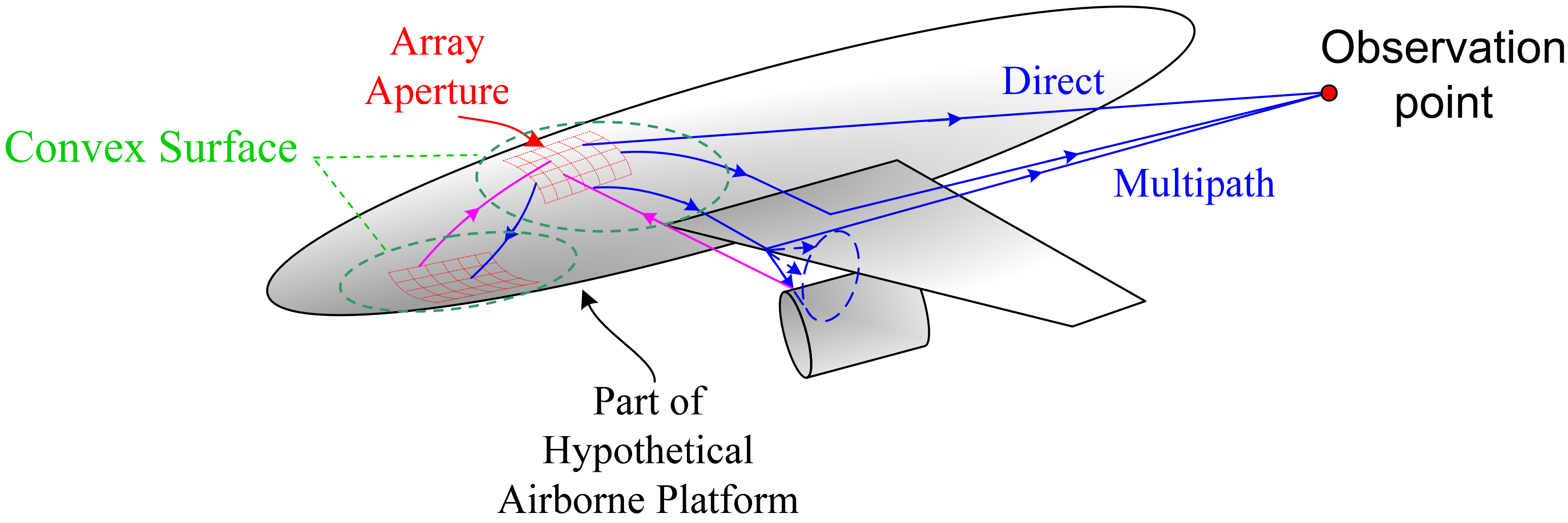
External Coupling via Ray Mechanisms

- Representation is called Collective UTD Array Field
- Complete aperture description given at once in terms of only a few UTD rays typically arising from one interior and several boundary points of aperture.



*Aperture must be described in terms of smooth analytical functions

Full External Platform Interaction



- UTD Green's function for predicting the external platform interactions and array-array coupling via rays.
  - It is assumed that the platform is much larger than the array aperture.

Conclusions

- Frequency-scalable basis functions may be constructed by superimposing multiple linear phases on conventional basis functions.
- Phasefronts may be extracted from low frequency coarse-grid data.
- Phasefront vectors illustrate high-frequency wave propagation.
- No upper frequency limit in theory, but fine mesh near edges and numerical integration limit practical cases.
- Very good accuracy in the method of moments may be obtained using a coarse mesh away from edges and discontinuities.
- APEx method may be applied to large arrays to extract traveling waves for UTD framework.