Learning Link Characteristics in a Boolean Network

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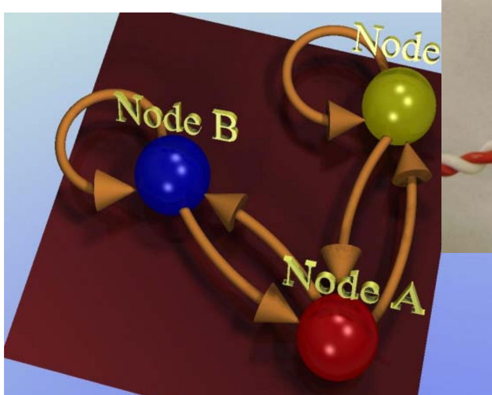
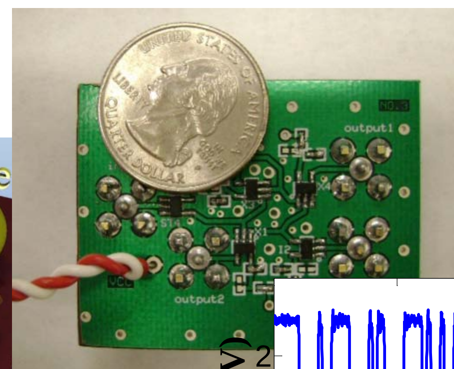
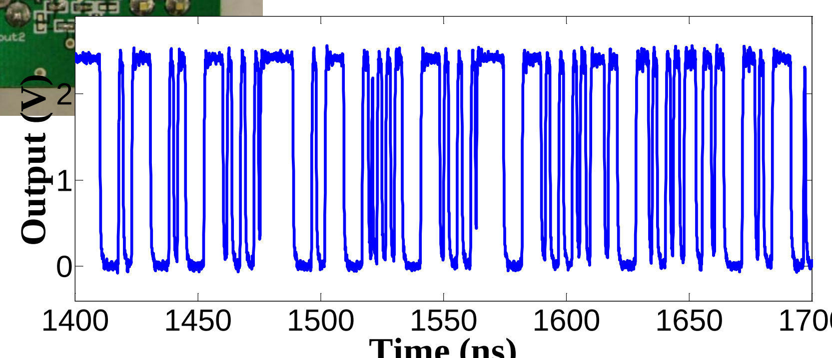




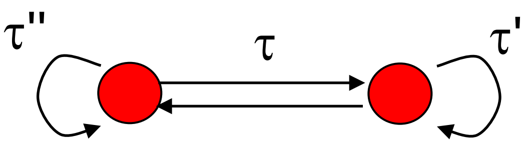


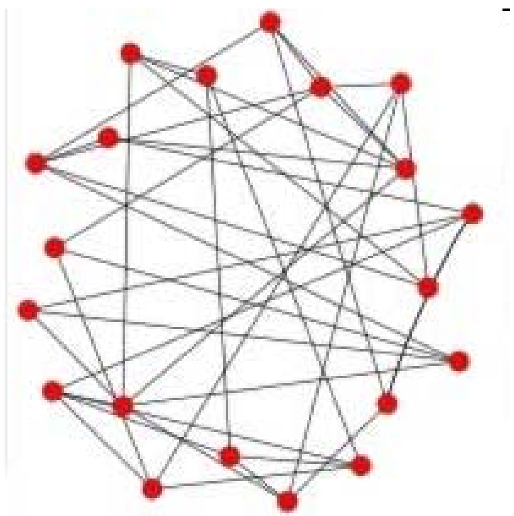
Coupled Networks

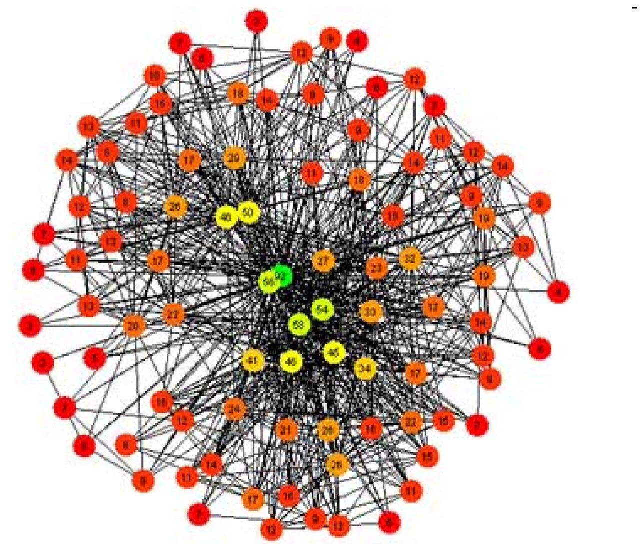
There exists a wide range of natural, "large" networks

How do we study experimentally "large" networks with well characterized nodes and links?

Nodes with fast dynamics: time delays are important



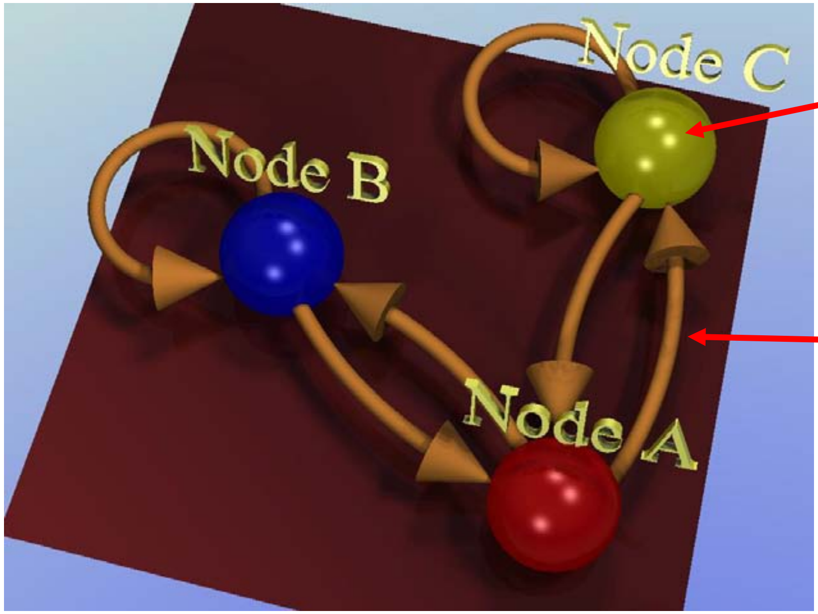




Measure here, learn about links?

Experimental Approach: Use a Newly Discovered Boolean-Like Network

Boolean Network



Nodes:
Only two states (Boolean)
Perform Boolean logic

Links:
Distinct, continuous time delays

There is NO clock in the system!

Autonomous Boolean Time-Delay System

Boolean Delay Equations

D. Dee and M. Ghil, *Boolean Difference Equations, I: Formulation and Dynamical Behavior*, SIAM J. Appl. Math. **44**, 111 (1984).

M. Ghil and A. Mullhaupt, *Boolean Delay Equations. II. Periodic and Aperiodic Solutions*, J. Stat. Phys. **41**, 125 (1985)

Boolean delay equations are *autonomous* evolution equations for a vector Boolean variables ***x(t)***.

$$x_i(t) = f_i[x_1(t-\tau_{i1}),\ldots,x(t-\tau_{in})]$$

Boolean function

Continuous time!

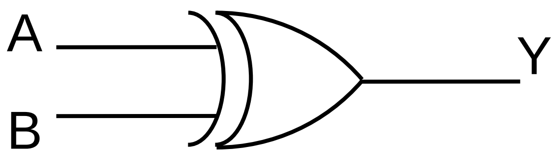
Properties of Boolean Delay Equations

A wide class of Boolean networks display "complexity"

- certainly works when every node changes state when there is a change in the input state (probably overly restrictive)

- Exclusive ORs (XORs) are favorable

$$Y = A \oplus B = A \cdot \overline{B} + \overline{A} \cdot B$$

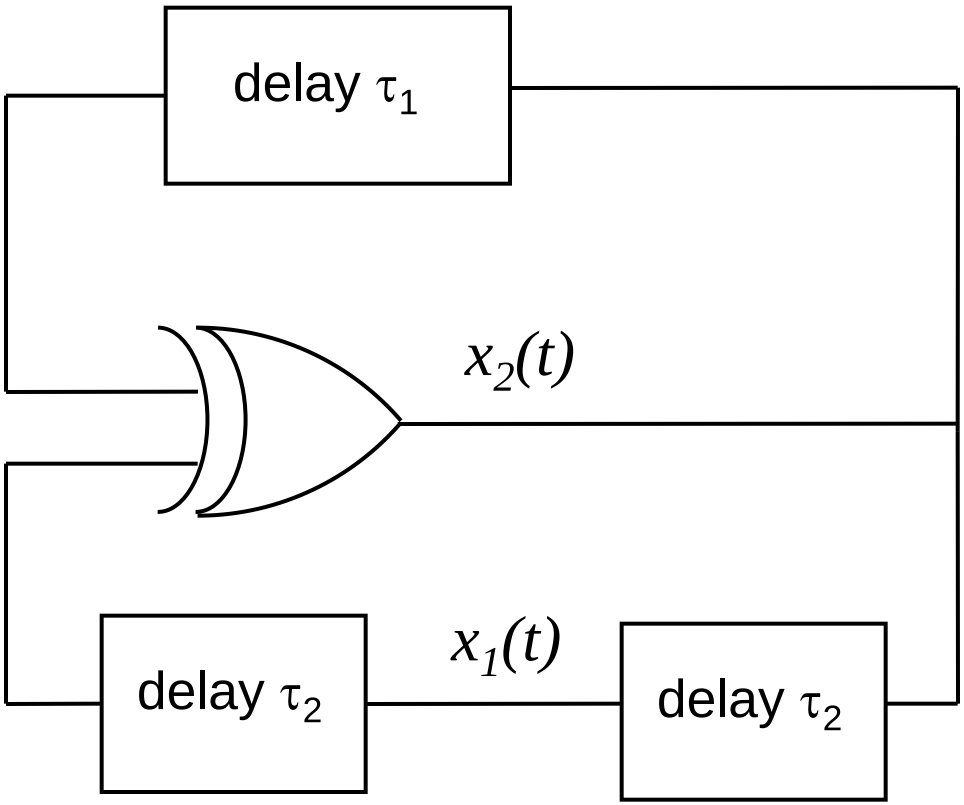


XOR gate		
input1	input2	output
0	0	0
0	1	1
1	0	1
1	1	0

Ghil Complexity: Number of switching events in the network per unit time increases as a power law

Example Complex BDE (Dee, Mullhaupt, Ghil)

Irrationally related delays



$$x_1(t) = x_2(t - \tau_2)$$

$$x_2(t) = x_1(t - \tau_2) \oplus x_2(t - \tau_1)$$

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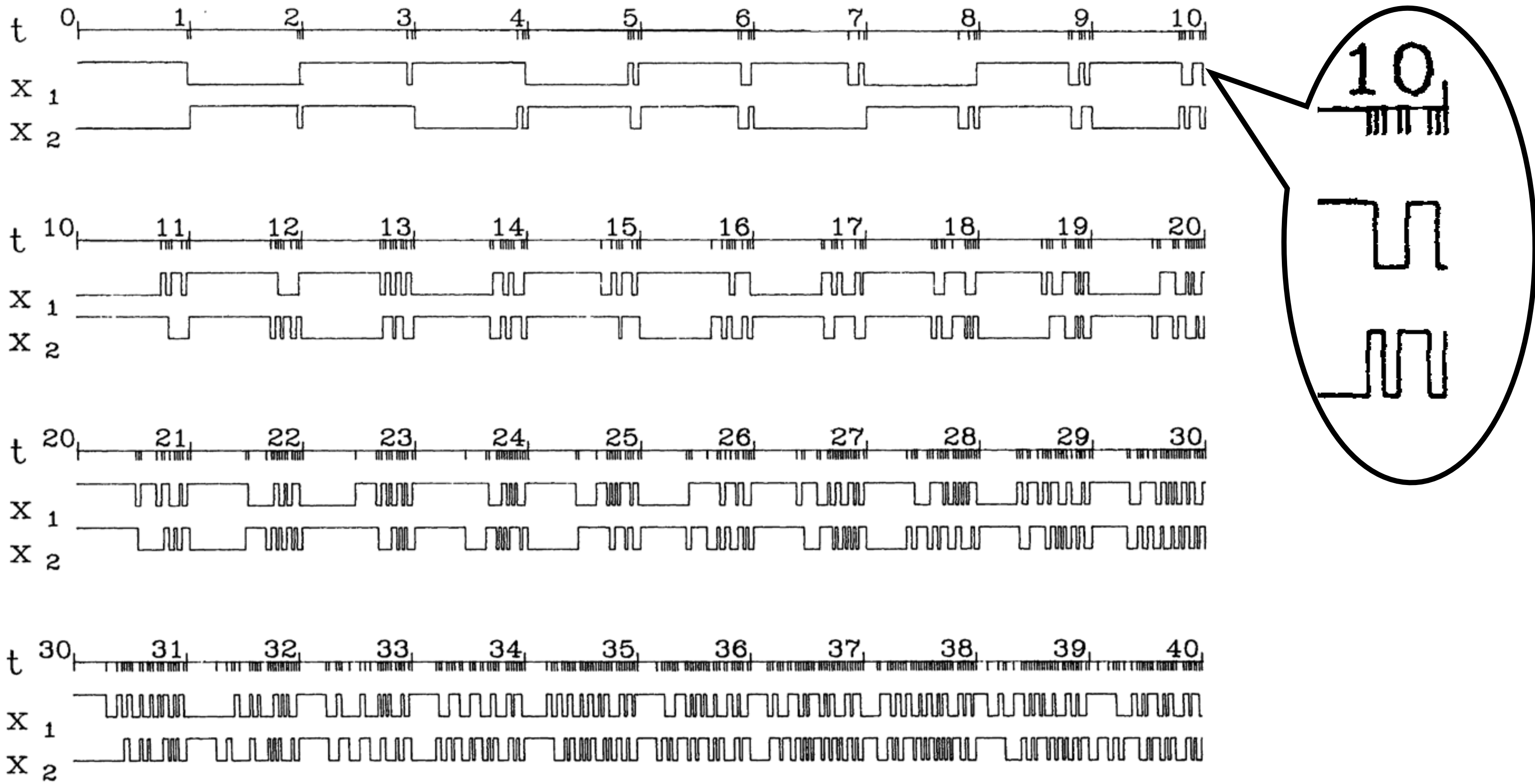
Simulated “Complex” Behavior



Power-law increase in switching rate:
Ultraviolet catastrophe

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Experimental Realization of a Complex Boolean Network

H. L. D. de S. Cavalcante, D. J. Gauthier, J. E. S. Socolar, and R. Zhang, `On the Origin of Chaos in Autonomous Boolean Networks,' Philos. Trans. Royal Soc. A **368**, 495 (2010).

R. Zhang, H.L.D. de S. Cavalcante, Z. Gao, D.J. Gauthier, J.E.S. Socolar, M.M. Adams, and D.P. Lathrop, ` Boolean chaos,' Phys. Rev. E. **80**, 045202(R) (2009).

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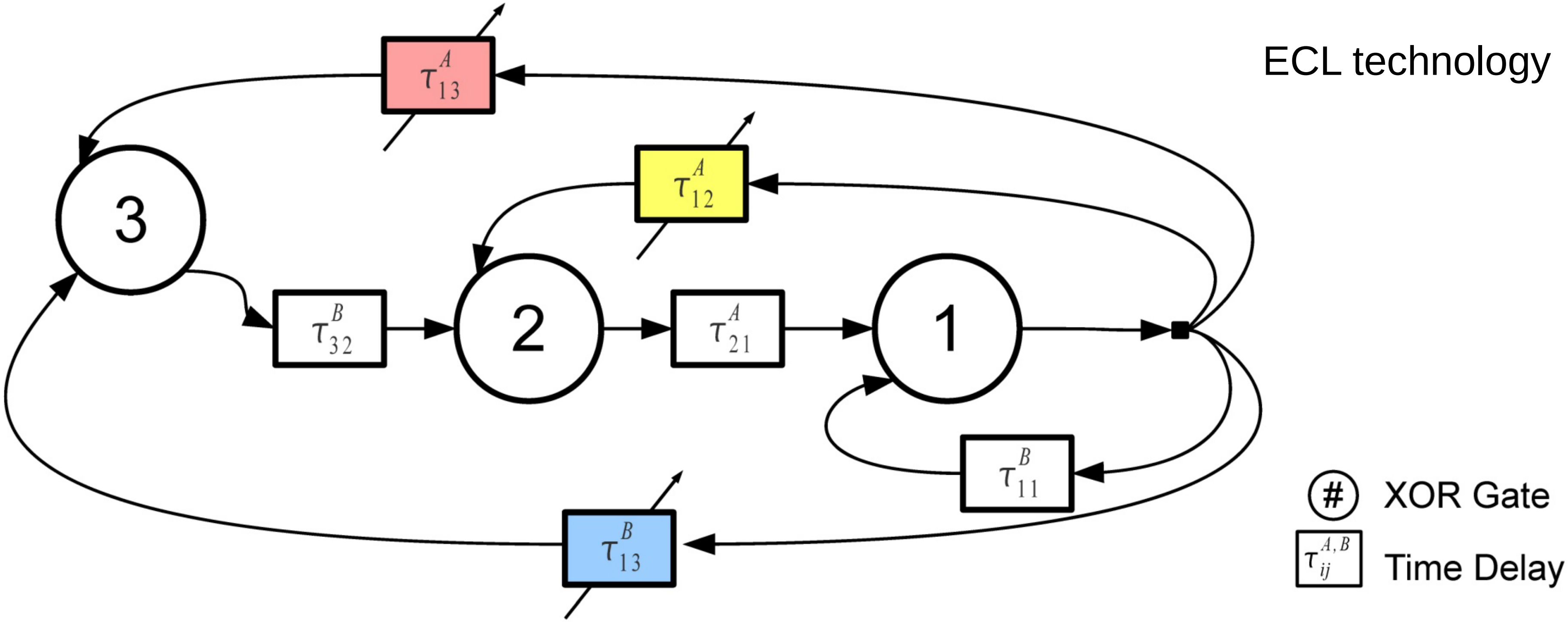
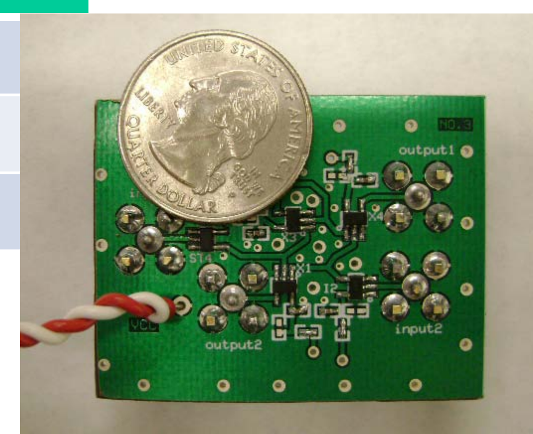
Three-Node Network Based on Electronic Logic Gates

ECL technology



Gate	Model	f (GHz)	Price ($)	Manufacture
XOR	MC100EP08	3	5	Onsemi
Buffer	NB6L16	6	6	Onsemi
Delay	SY89296	1.5	7	Micrel



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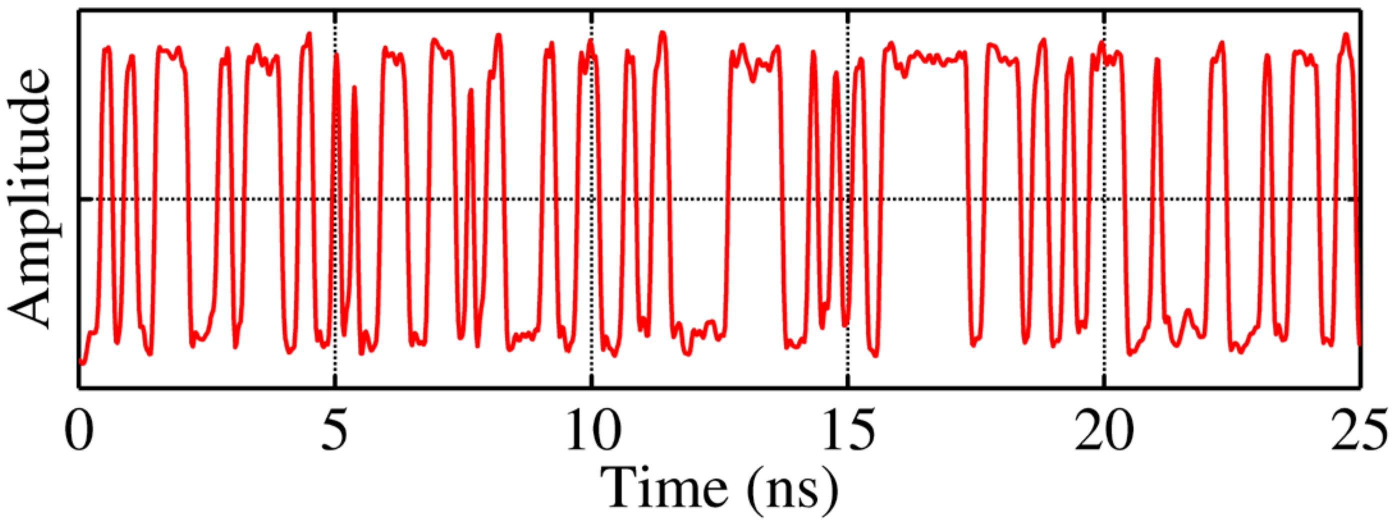
Boolean Network Temporal Evolution



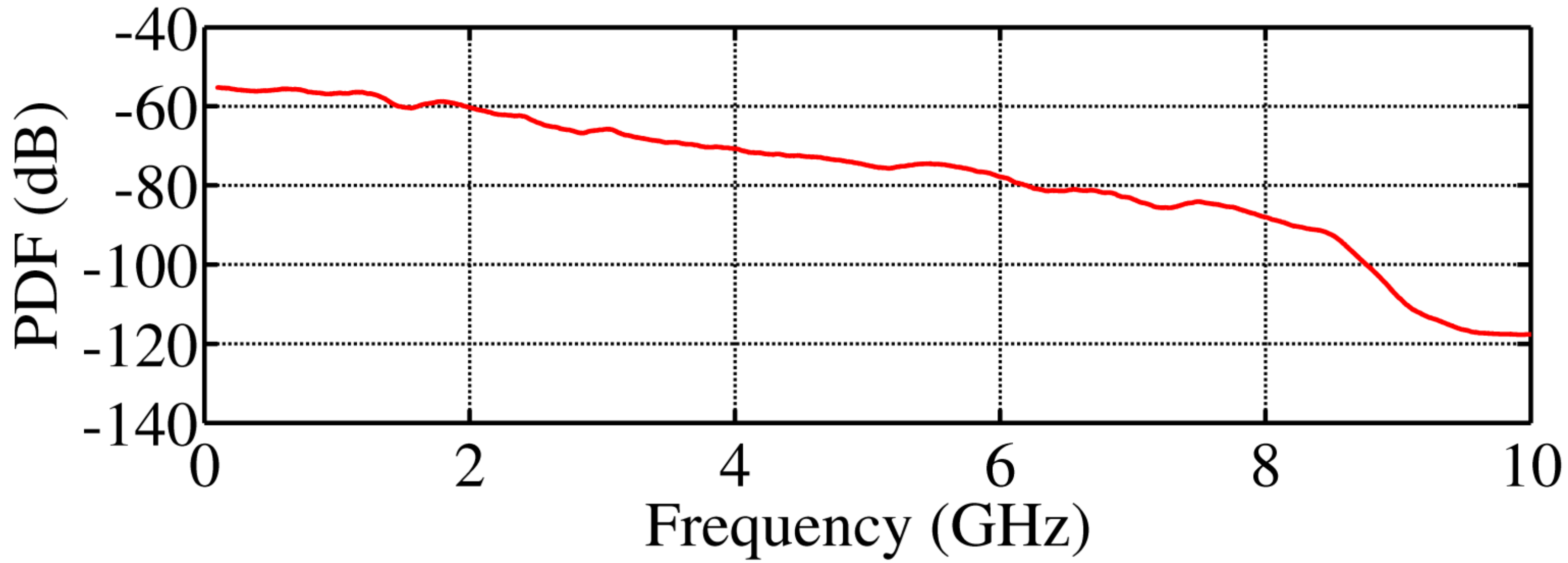
Transient ultra-violet catastrophe → system naturally fast (up to response time), non-ideal behaviors affect behavior

The temporal evolution of the voltage is complex and non-repeating

It has clearly defined high and low values, indicating Boolean-like behavior

Behavior can be understood from a perturbed Boolean model

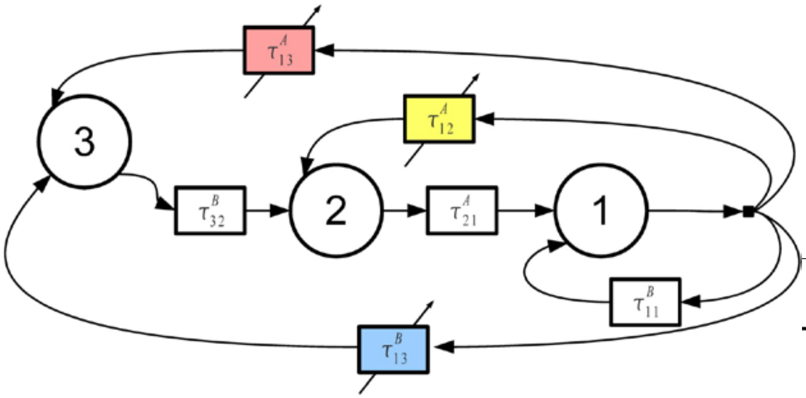
Power Spectrum of the Fluctuations

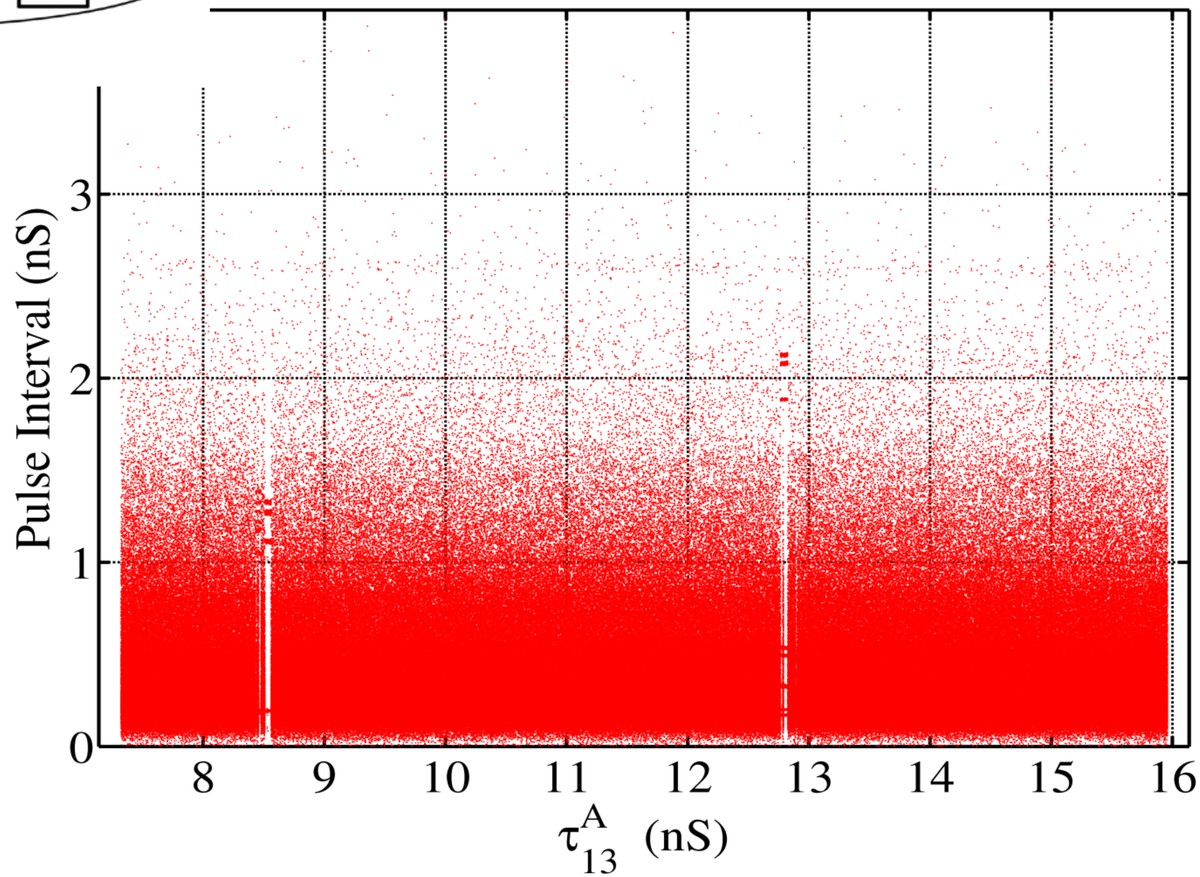


Ultra-wide-band device, extending from DC to beyond 3 GHz

(lots of applications!)

Scan One Time Delay





Chaos, periodic windows, wide range of operation

A New Paradigm for Fast Time-Delay Chaos

Ghil predicts a very large class of networks show "complexity" using assumption of ideal logic

Conjecture: Experimental version of most of these networks show chaos

VERY cheap!

Easy to adjust time delays

Future:

- Learn network behavior from limited measurements?
- Multi-function logic - programmable networks
- "Evolutionary" networks?

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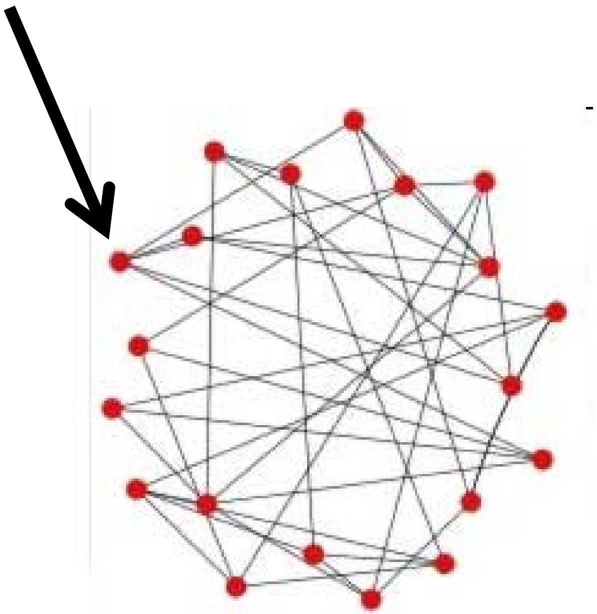
Can we learn network behavior from limited measurements?

Data Assimilation



Approach: Synchronize two networks

Adjust parameters to obtain best synchronization

Data assimilation: B. Ravoori *et al*. PRE **80**, 056205 (2009); A.B. Cohen *et al*., PRL **101**, 154102 (2008); T.E. Murphy *et al*. Phil. Trans. A **368**, 343 (2010); and F. Sorrentino , http://arxiv.org/abs/0912.4902/

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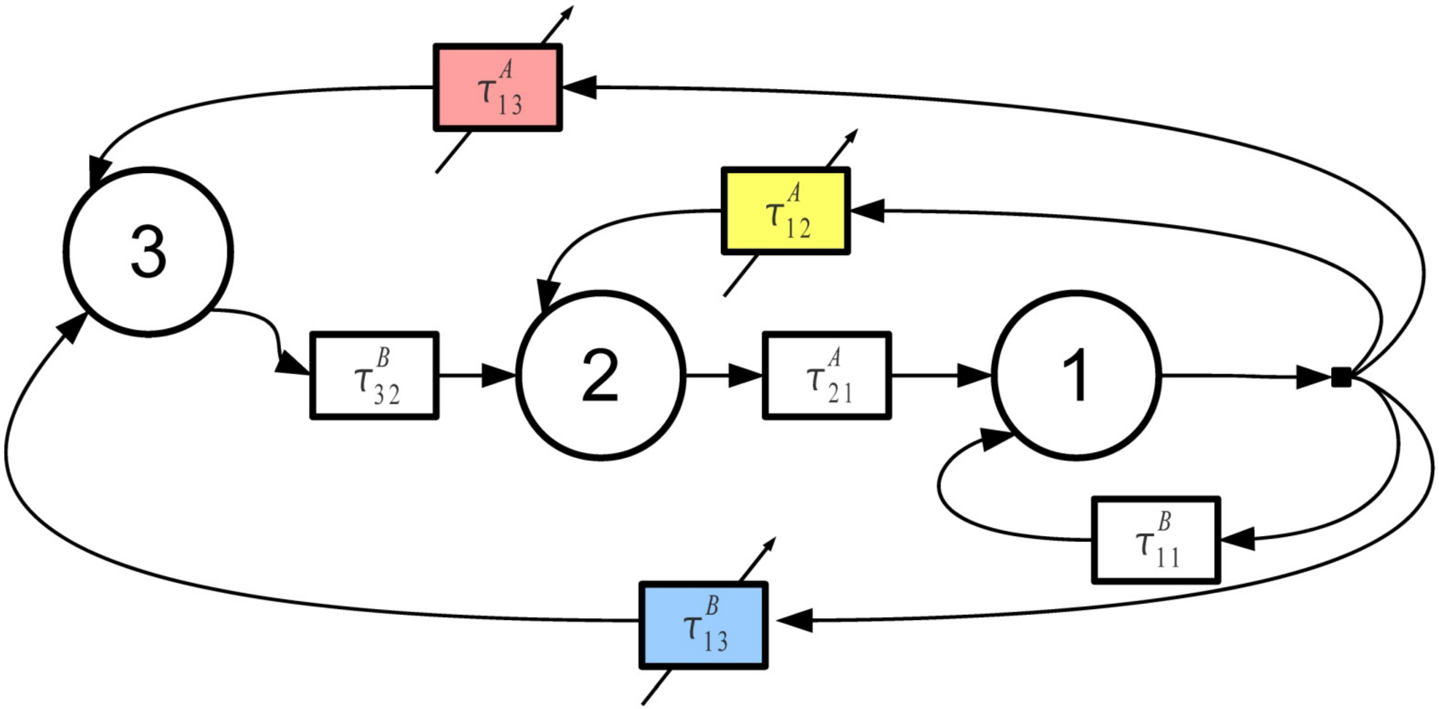
Statement of the Problem

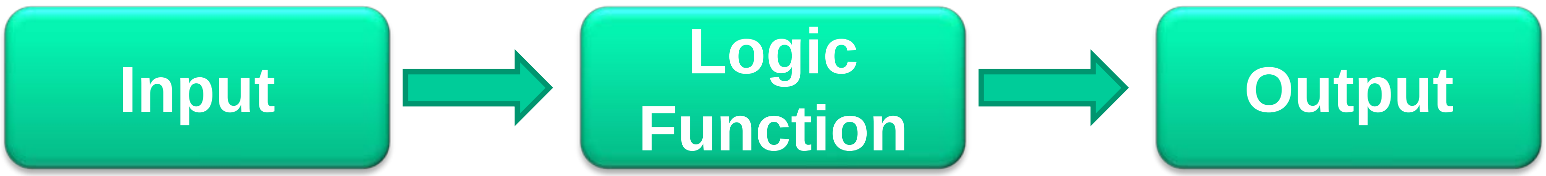


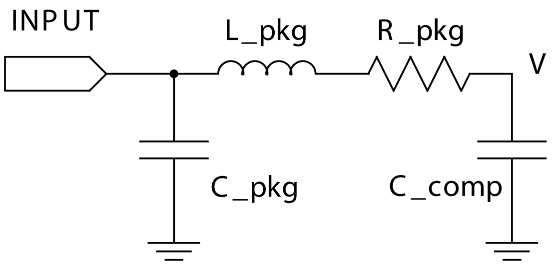
With full knowledge of node dynamics, can we determine the time delays of three links through data assimilation (both experiment to experiment and experiment to model)?

Dynamical Characteristics of Individual Nodes and Links

Node Dynamics: Logic Gates and Time Delay Buffer

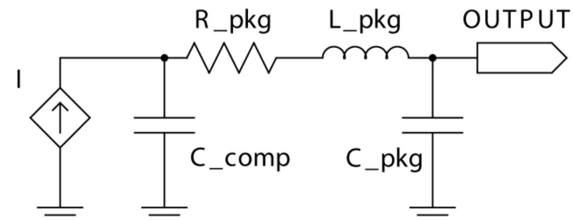
Buffer most important – slowest component



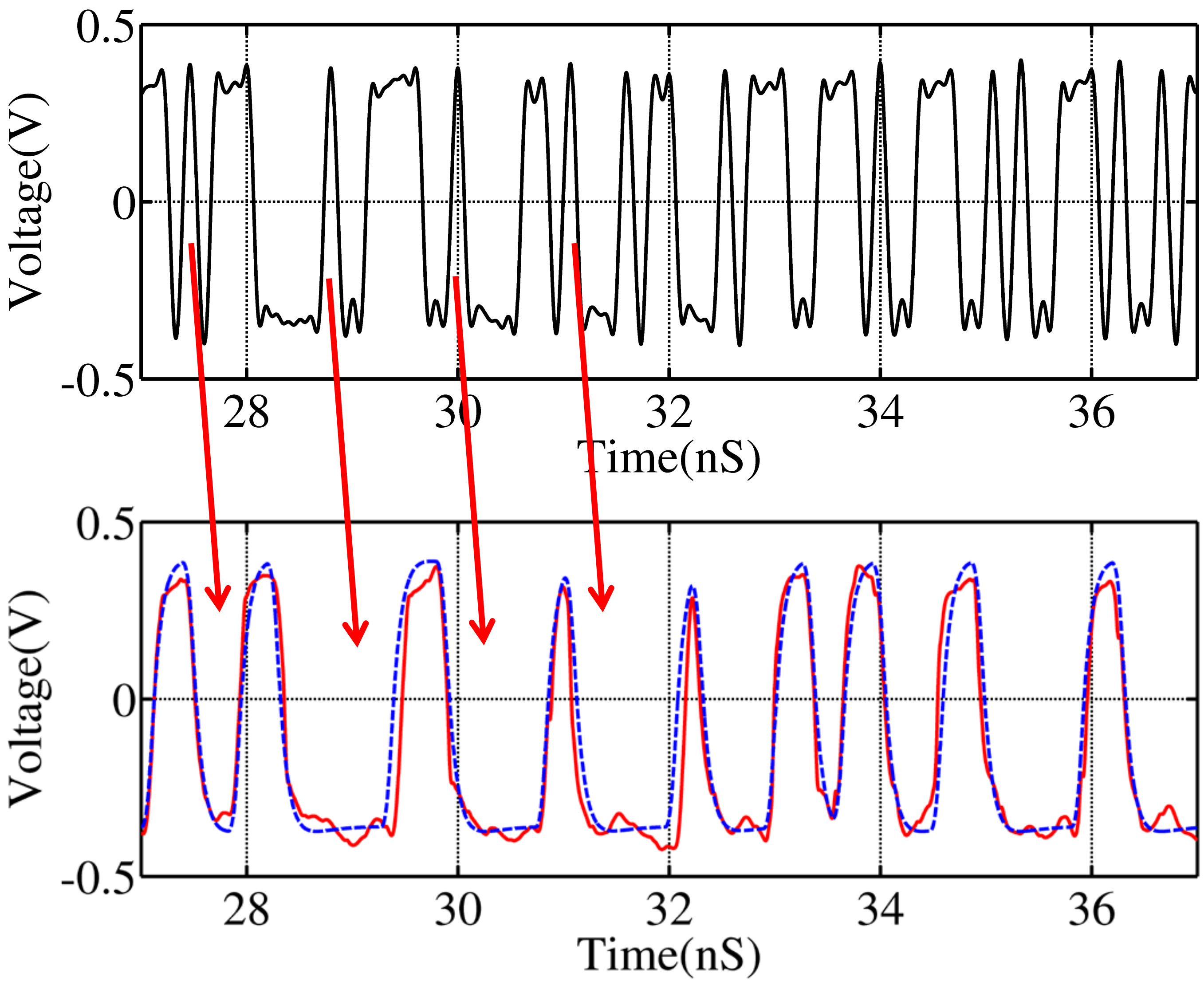


$$V_{in} = V_0 \cdot \tanh(\alpha \cdot V)$$
$$I = I_0 \cdot \sec h^2(\beta \cdot (V_{in} - V_{th}))$$

$\alpha, \beta, Vo, Io, Vth$ are constants



Inject Fast Signal Into Time Delay Unit (only)



SY89296
1.5 GHz
Delay Chip

7.5Gbps PRBS input

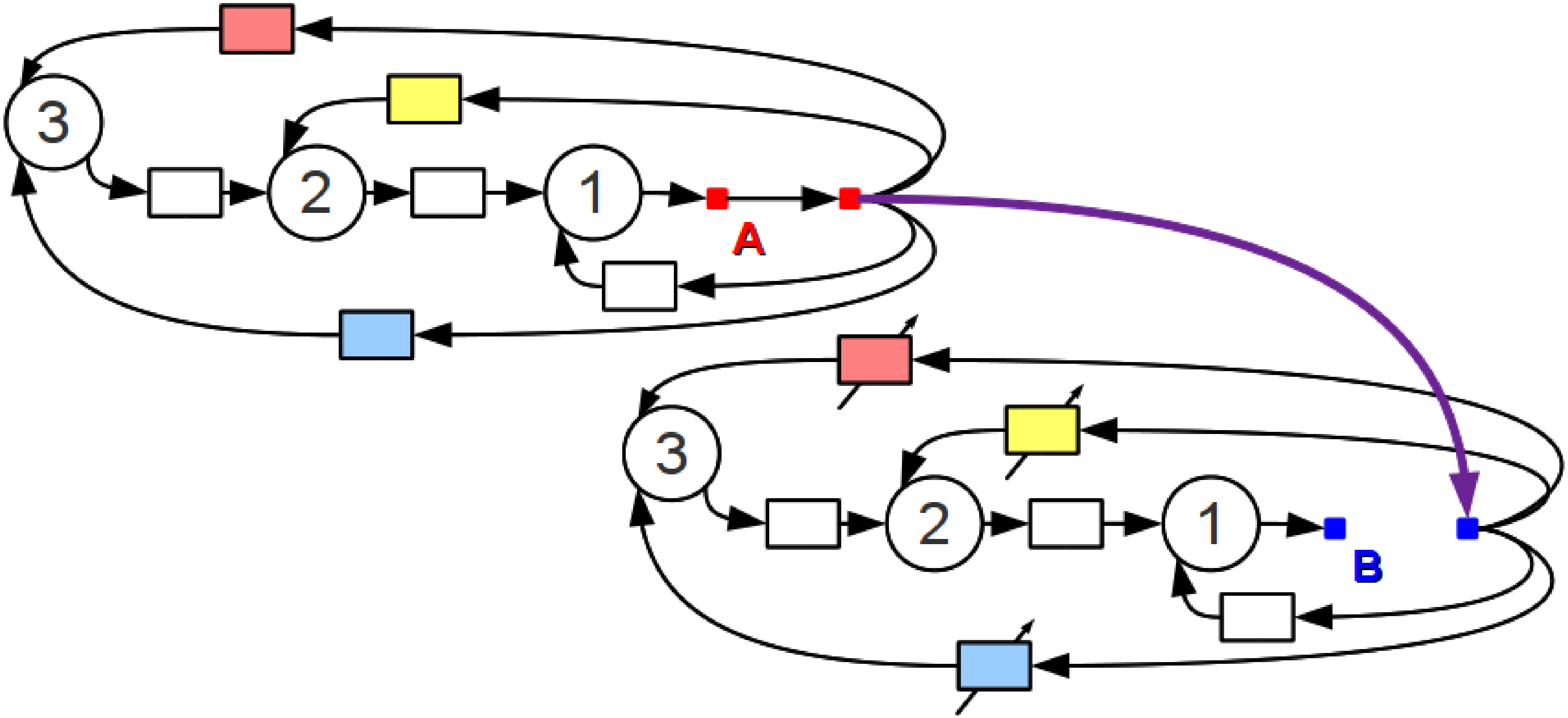
short pulses rejected

exp output

model output

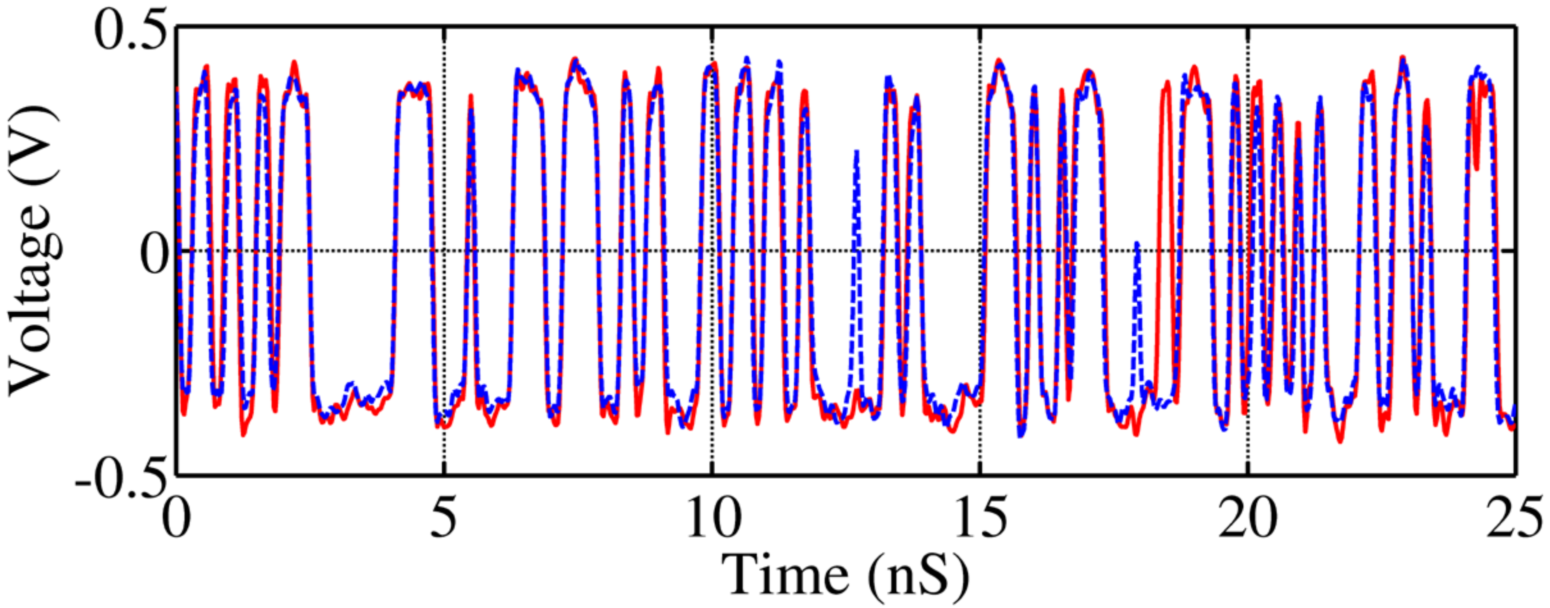
Synchronize Network to Network (both experimental)

"Open Loop" Synchronization



Adjust three time delays independently in slave oscillator, measure correlation

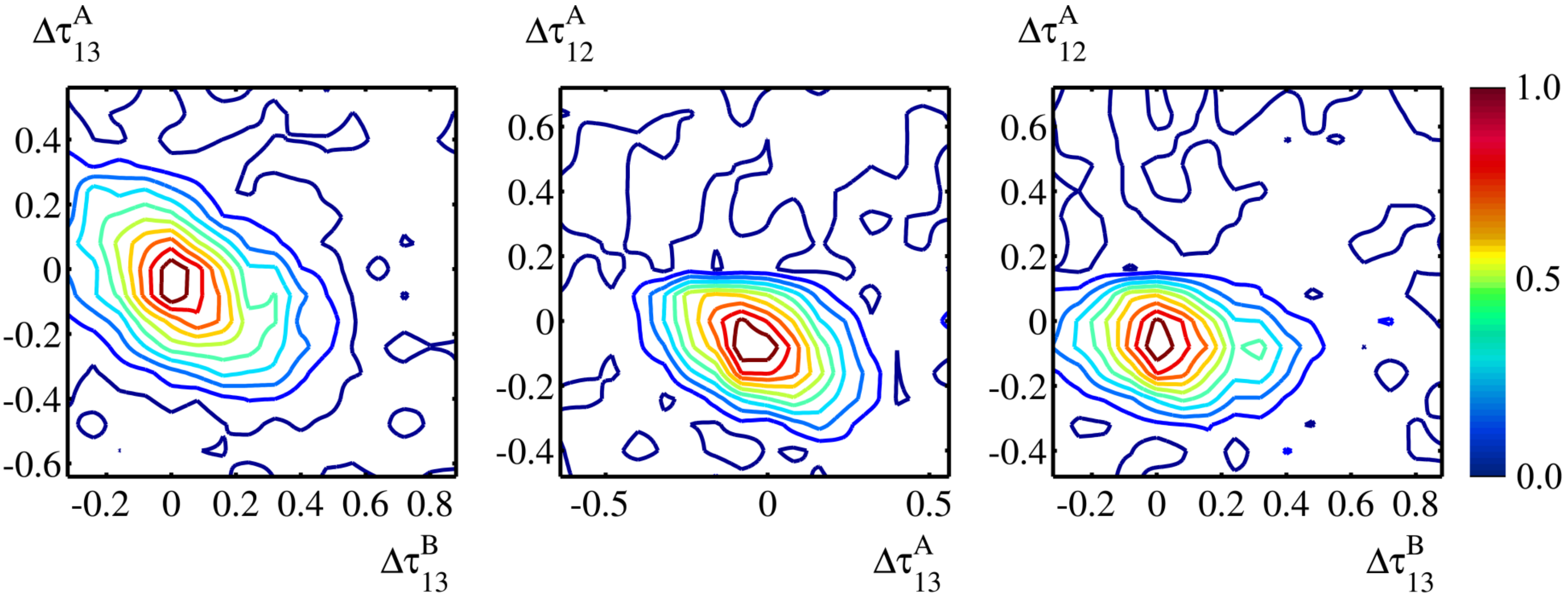
Best Synchronization at Optimum Time Delays



Correlation >0.8, as high as 0.95

Attractor Bubbling?

Explore Time Delay Space: Correlation vs. Time Delays

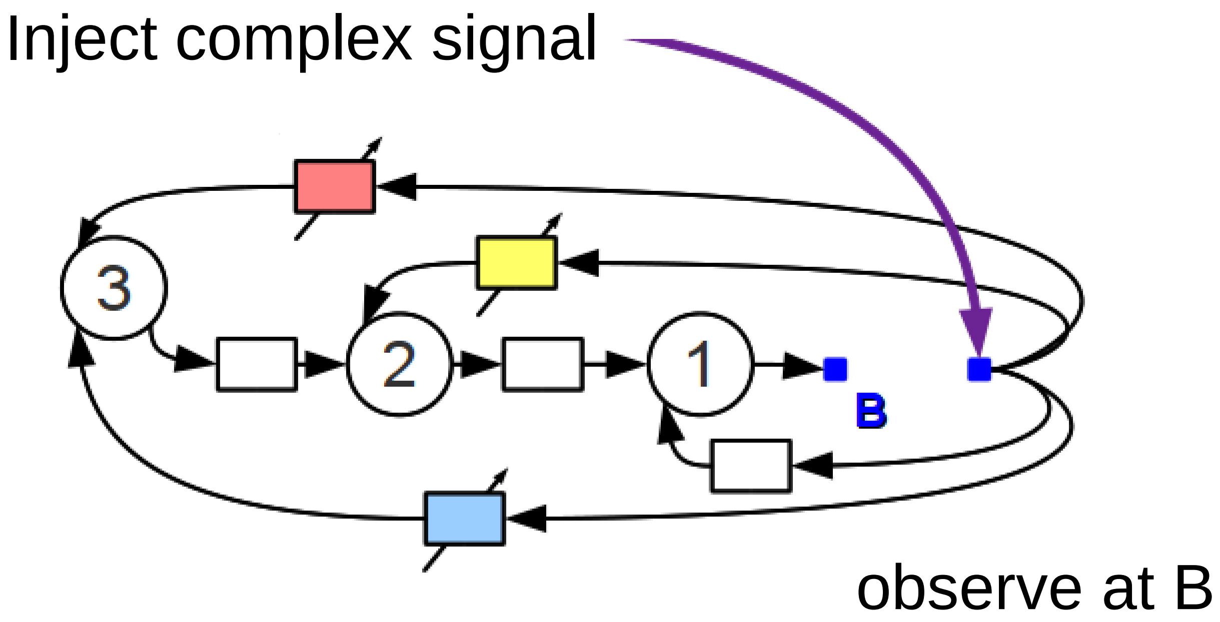


units: nanoseconds

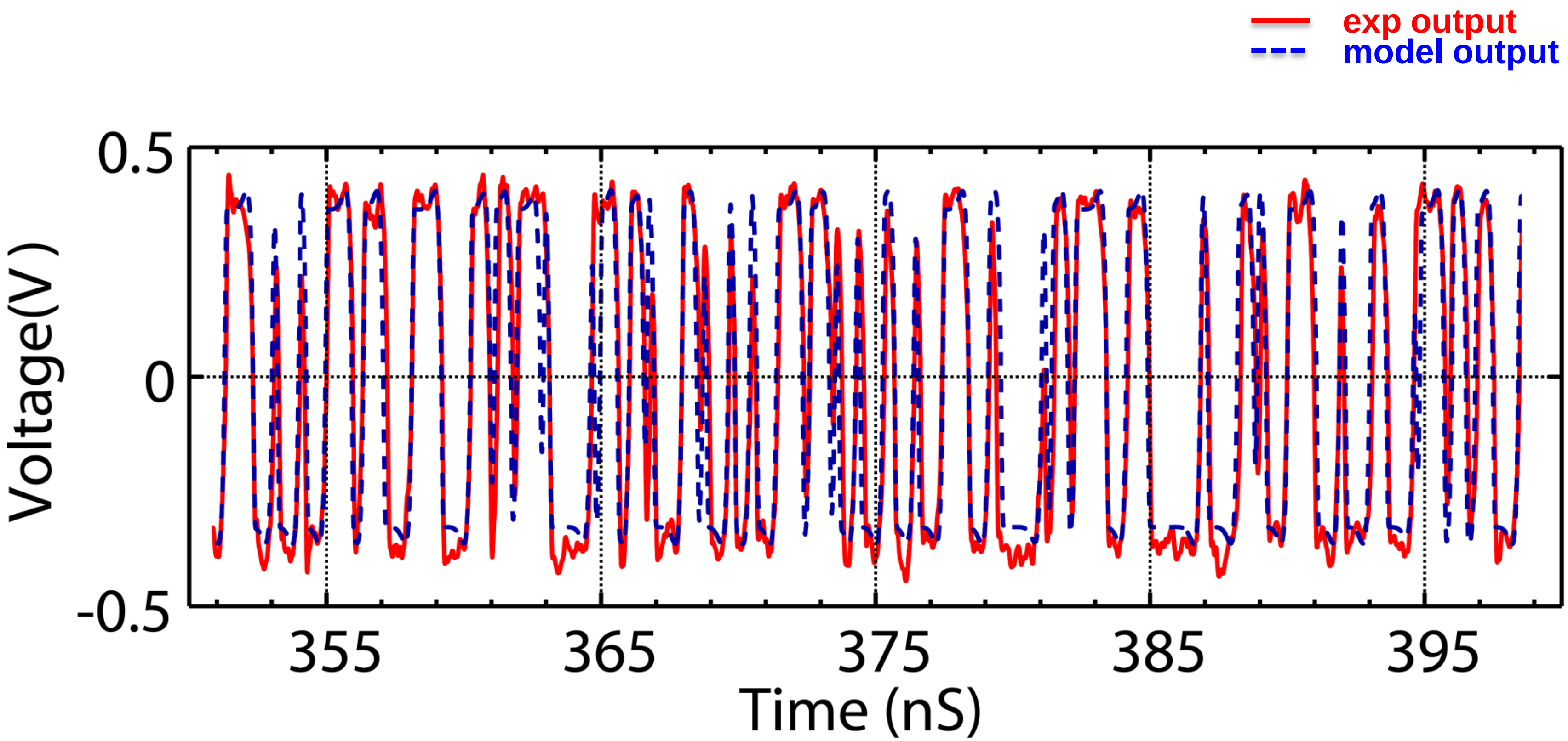
Can determine network delays to within ~ 100 ps
(shortest delay ~ 2 ns, longest ~ 10 ns)

Compare Predicted Network Behavior to Experimentally Observed Behavior

Data Assimilation: Synch Model to Experiment

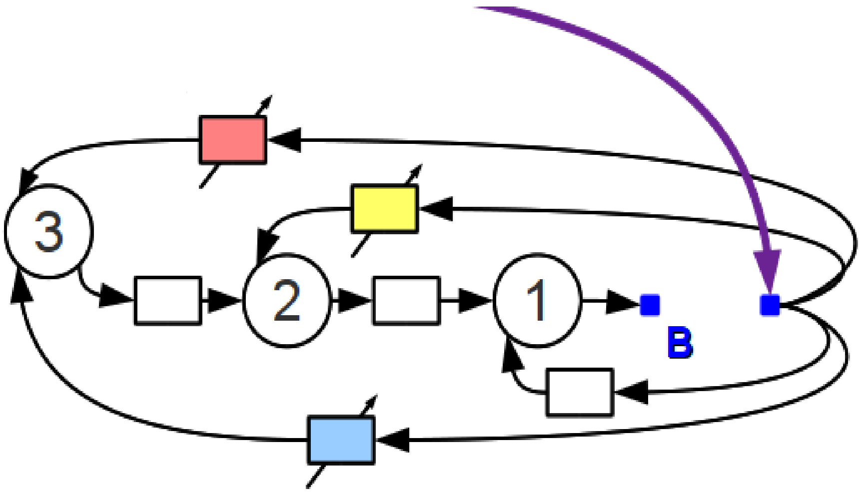


Best Synchronization: Model to Experiment

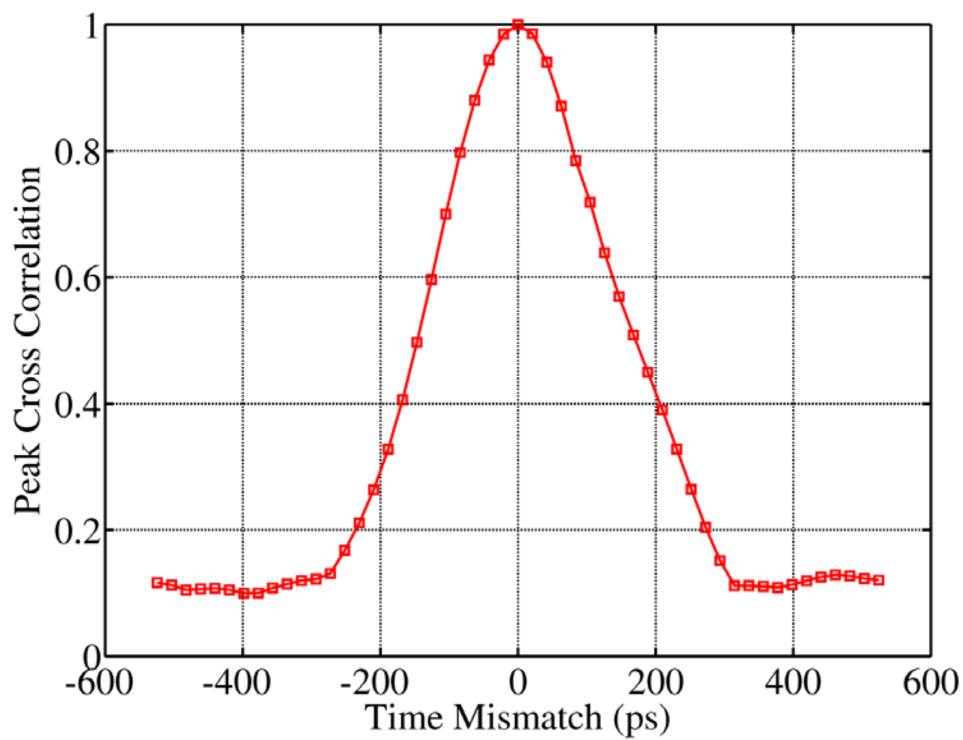


Cross correlation: 0.91

Adjust Time Delay in Model



Consistent with experiment-experiment sensitivity



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Summary

- Possible to infer link time delay in a autonomous Boolean network

- Broadband chaos likely important – sample wide range of behaviors

- Boolean network dynamics “orthogonal” with modest changes in time delays – several applications

- Electronic implementation of Boolean networks is cheap and easy – possible to scale up to much larger networks

- Is data assimilation possible with larger networks?

- Can a Boolean delay equation model behavior of our network?