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Zentrum für Marine und Atmosphärische Wissenschaften

Max-Planck-Institut für Meteorologie

GKSS Forschungszentrum Geesthacht

Deutsches Klimarechenzentrum

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Adaptive triangular meshes for inundation modeling

19.10.2010, University of Maryland, College Park

Jörn Behrens

KlimaCampus, Universität Hamburg

Acknowledging: Widodo Pranowo, Nicole Beisiegel

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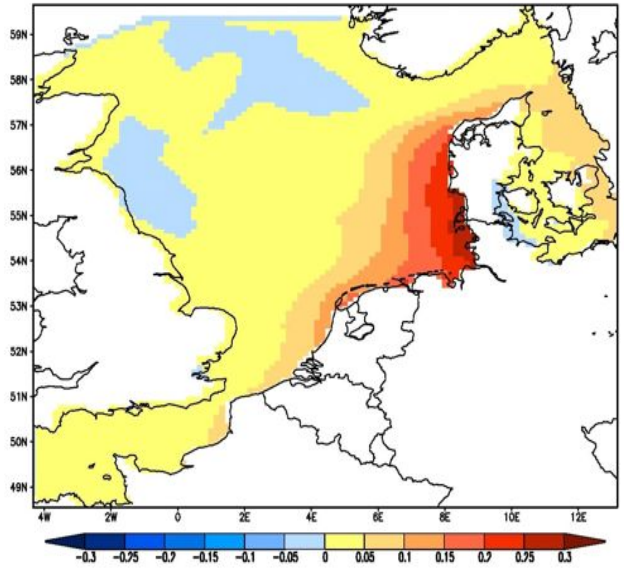
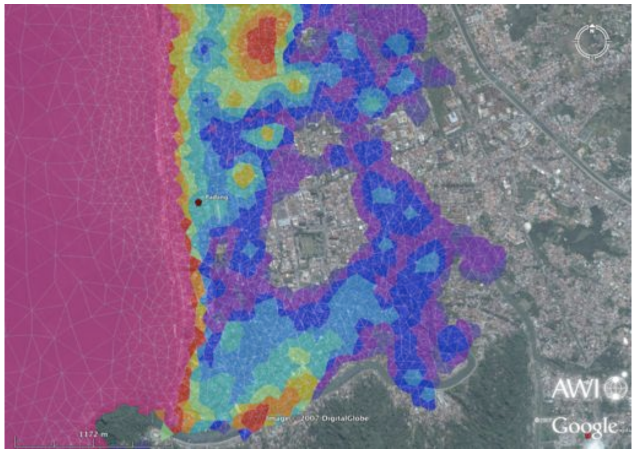
Multiple scales

Spatial Scales:

- Tides:
  - Ocean-wide/ global range
  - O(10000) km
- Surge:
  - Typical Length scale
  - O(100) km
- Near-shore waves:
  - Shoaling effect
  - O(1-10) km
- Inundation (Parameterized):
  - No resolution of buildings, etc.
  - O(0.01-0.1) km





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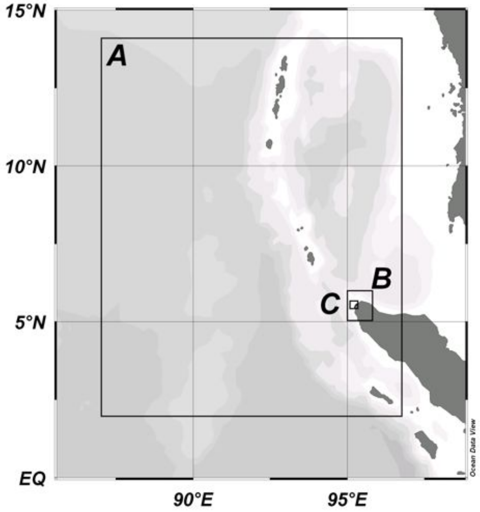
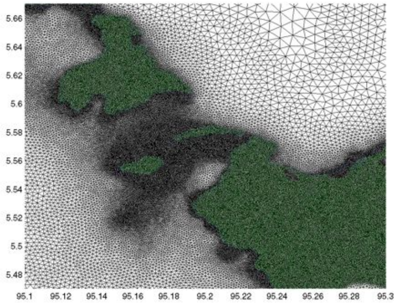
Multiple scales – Computational consequences

Computational cost uniform refinement

- 10^5 grid points per dimension
- $(10^5)^2 = 10^{10}$ grid points
- That does not resolve streets, etc.





Variable resolution!

Intelligent grid control – adaptive mesh refinement

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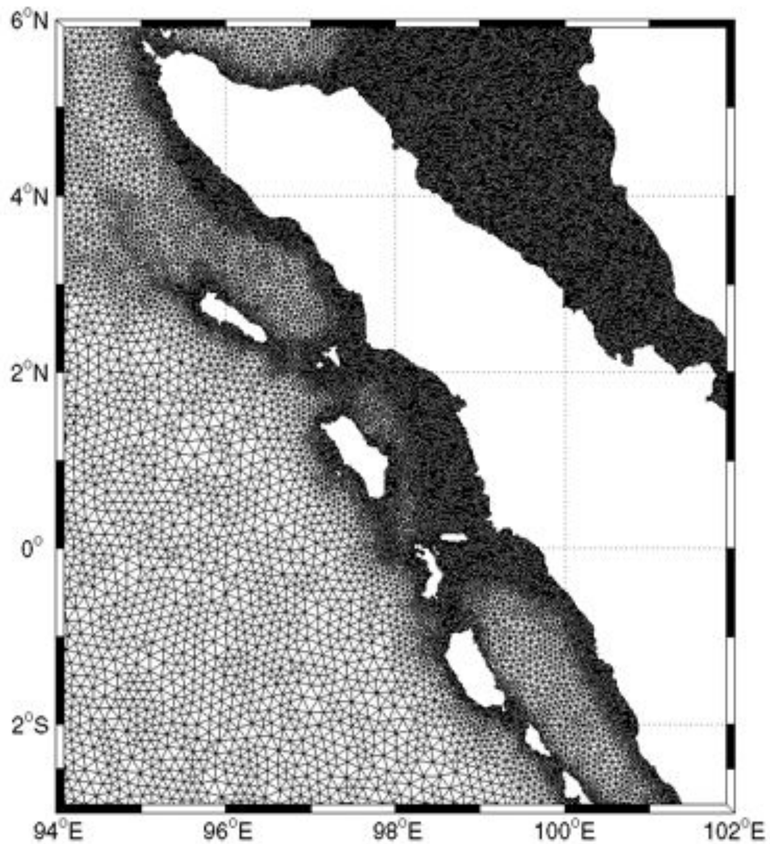
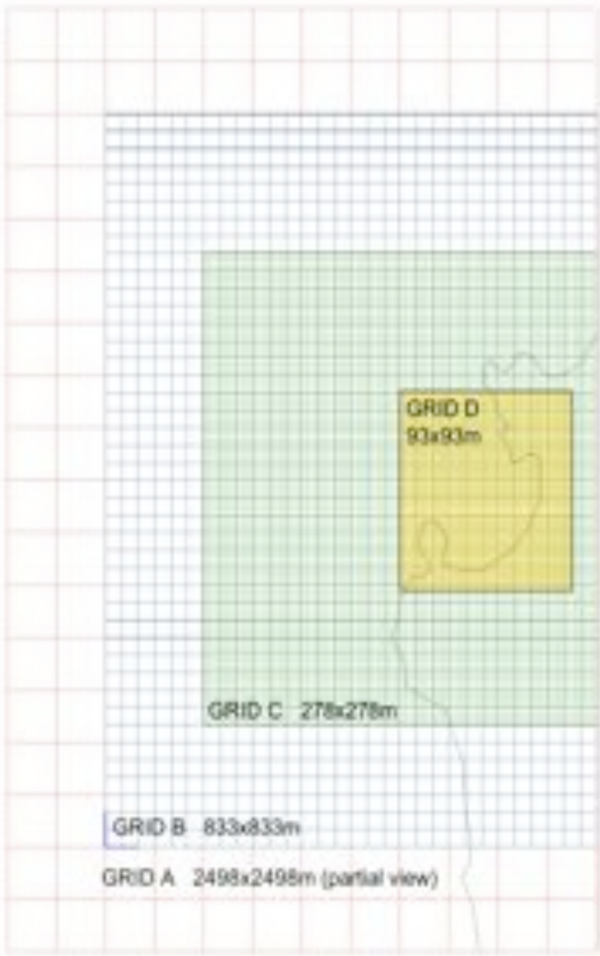
Bridging the scale gap: multi-resolution

TSUNAWI



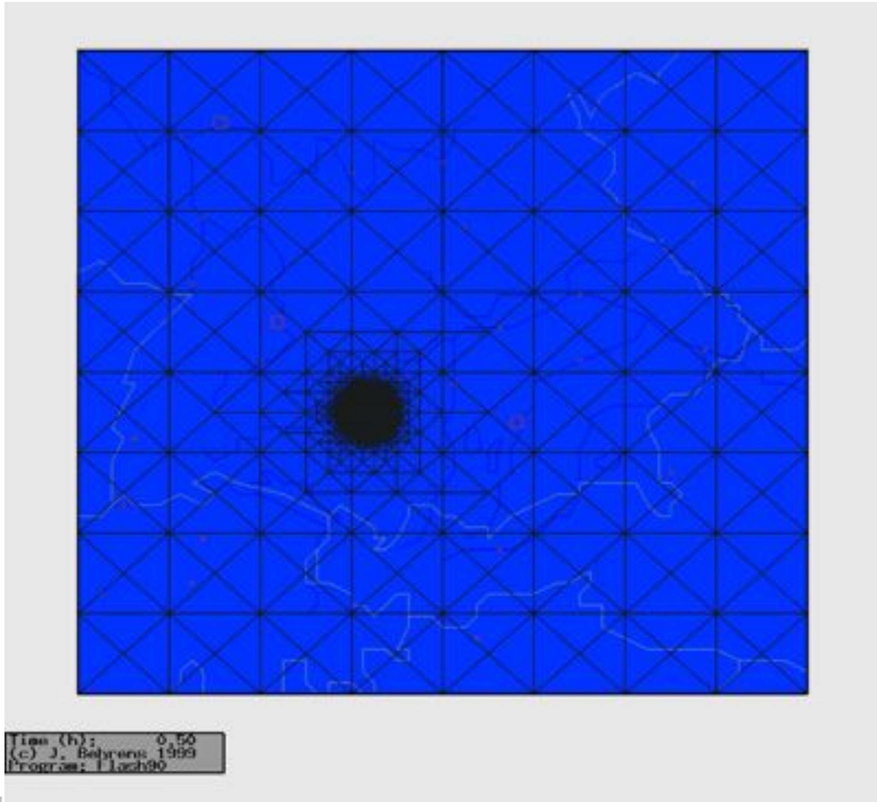
Resolution:
generally 200 m-10 km
In priority areas: 80 m

Tunami-N3

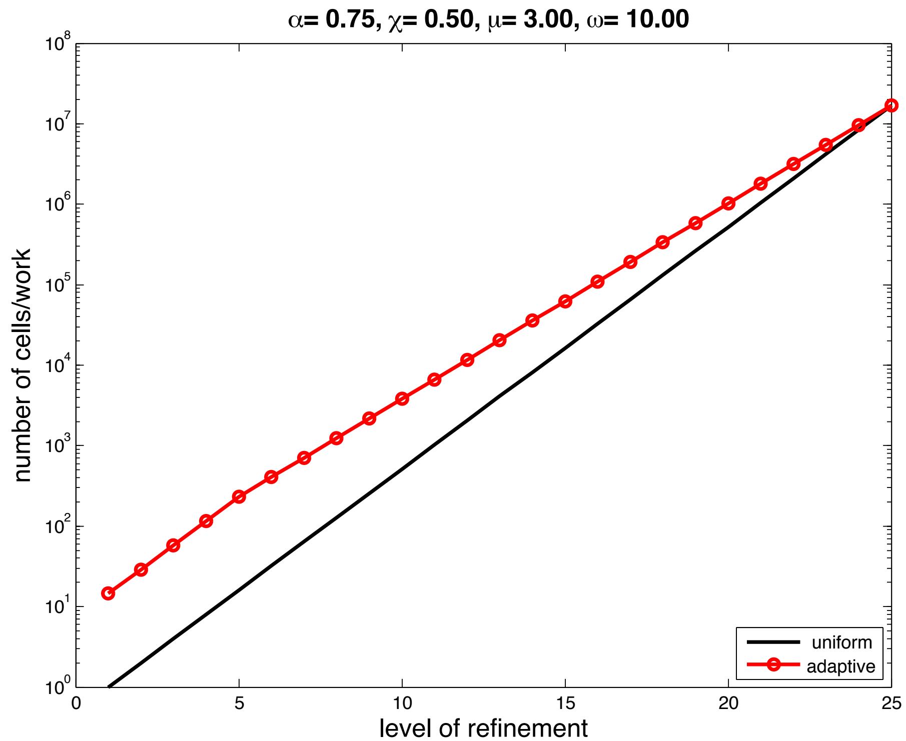


Adaptive Methods – Key Questions

- When to apply an adaptive method?
- What is needed for numerical efficiency?
- What is needed for computational efficiency?



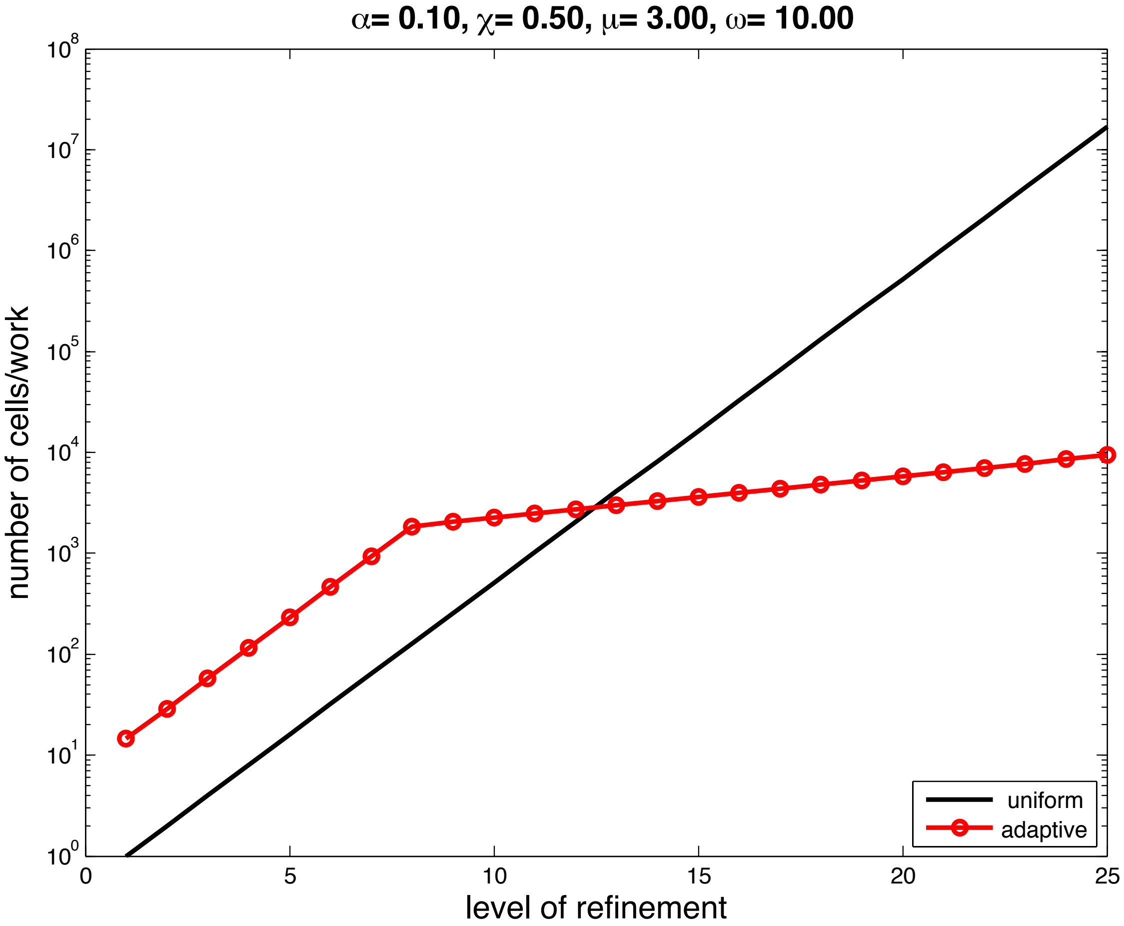
Model for adaptive efficiency



- Assumptions:
  - Tree refinement (binary tree here)
  - Overheads for
    - refinement criterion
    - Less efficient numerics
    - Mesh management
  - Refinement area as fraction
  - Computational work = no. of unknowns
- Example:
  - Area of refinement large (α=3/4)
  - Overheads large:
    - Refinement criterion (χ=1/2 of a computational step)
    - Numerics slow (ω= 10 times slower as uniform comp.)
    - Mesh management demanding (μ= 3 times a comp. step)

Model for adaptive efficiency – small adaptive area

- Now:
  - Area of refinement small (α=1/10)
  - Rest unchanged



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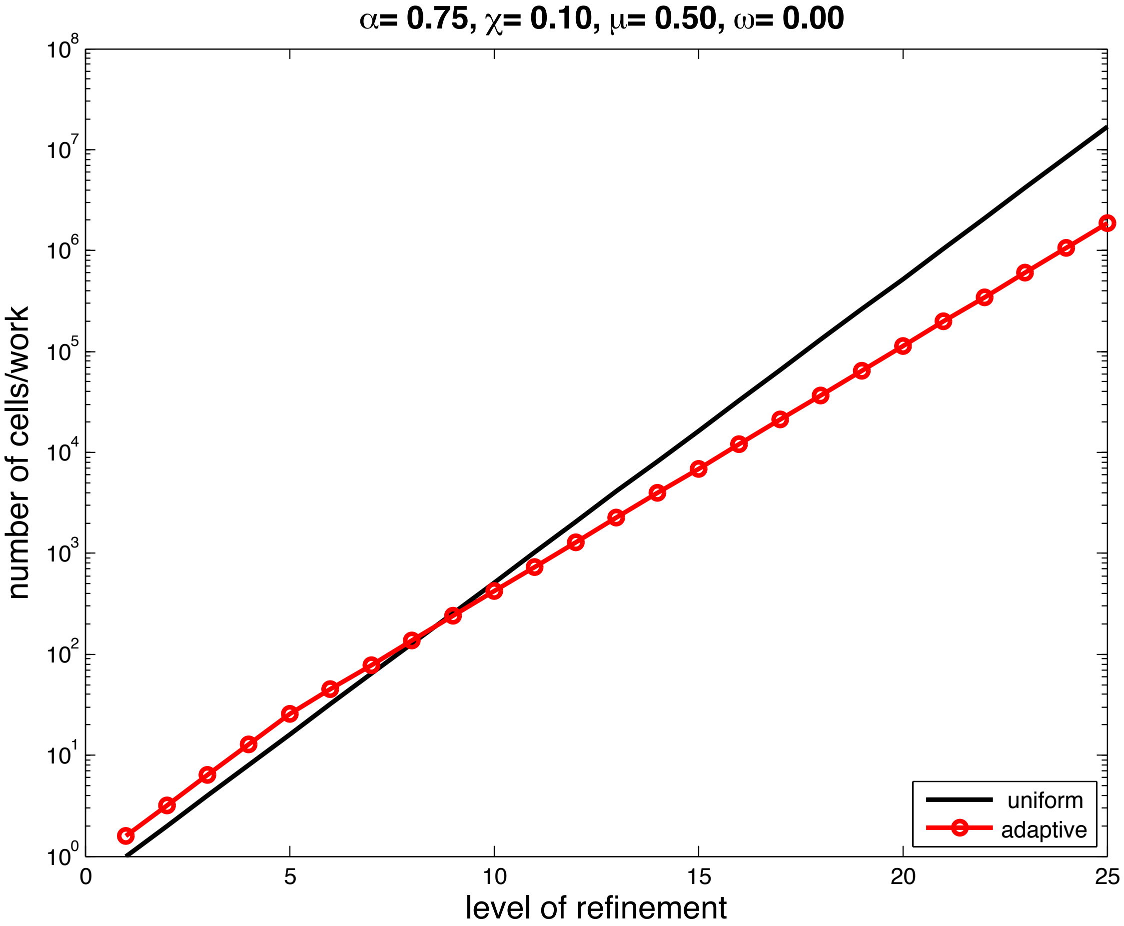
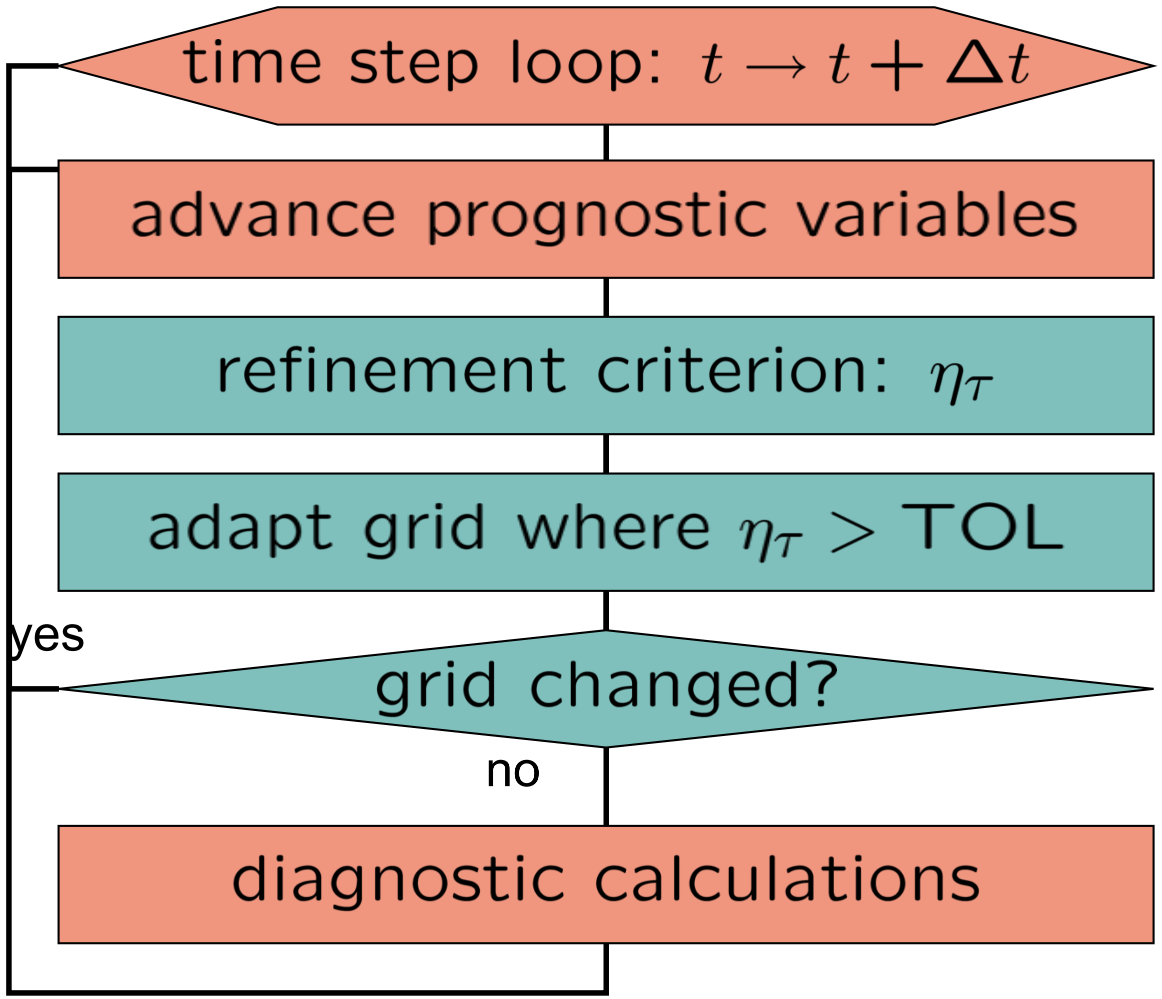
Model for adaptive efficiency – small adaptive area

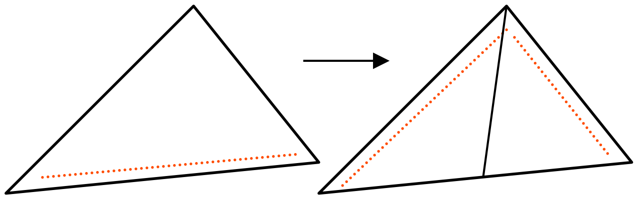
- Now:
  - Area of refinement large (α=3/4)
  - But efficient grid management and numerics!



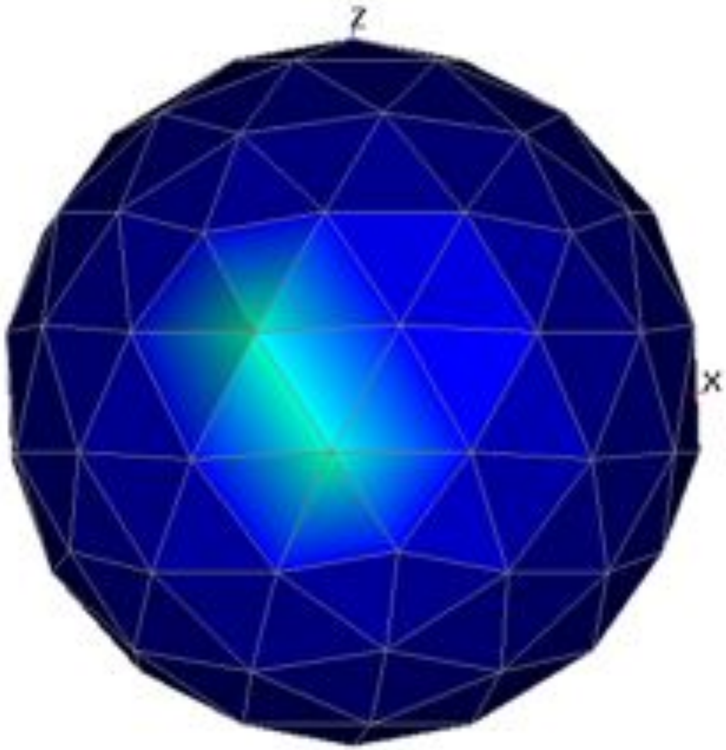
Adaptive algorithm



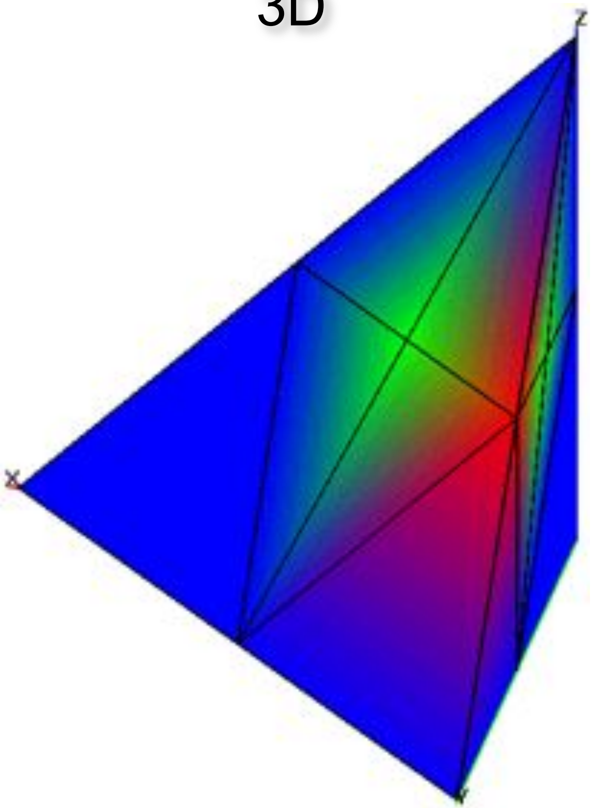
1. Efficient Mesh Refinement Strategy



2D



3D



Rivara (1984), Bänsch (1991), Grids created with amatos

Refinement control: gradient based or averaging

Gradient based

$$\text{refine if} \quad \nabla h|_\tau > \theta_{\text{ref}} \max(\nabla h|_\tau)$$
$$\theta_{\text{ref}} \text{ threshold value}$$

Averaging

$$\begin{aligned} \eta &= \| h|_\tau - \bar{h} \|, \quad \text{with } \bar{h} = \frac{1}{A} \sum_{\text{neighbors t}} h|_t, \\ \text{refine if} \quad \eta &> \theta_{\text{ref}} \bar{\eta}, \quad \text{with } \bar{\eta} \text{ mean error} \\ & \quad\; \theta_{\text{ref}} \text{ threshold value} \end{aligned}$$

- ☑ Simple
- ☑ Easy to compute
- ☑ Robust

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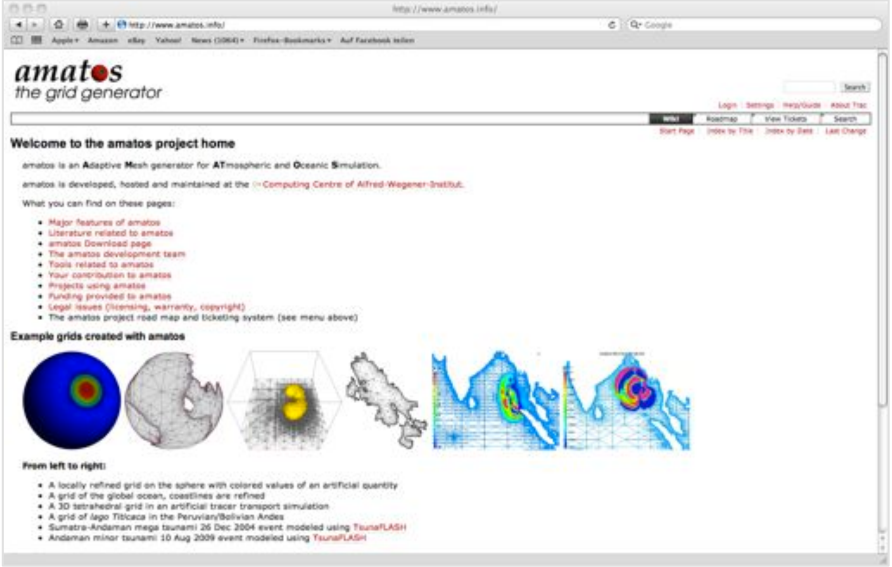
3. Efficient Programming Framework

- Triangular grids with boundary
- Object oriented + gather/scatter
- Built in SFC ordering
- Simple programming interface
- Generic FEM/SEM support
- Coupling capability
- Parallel
- 2D plane and spherical geometries
- Documentation, open source
- http://www.amatos.info



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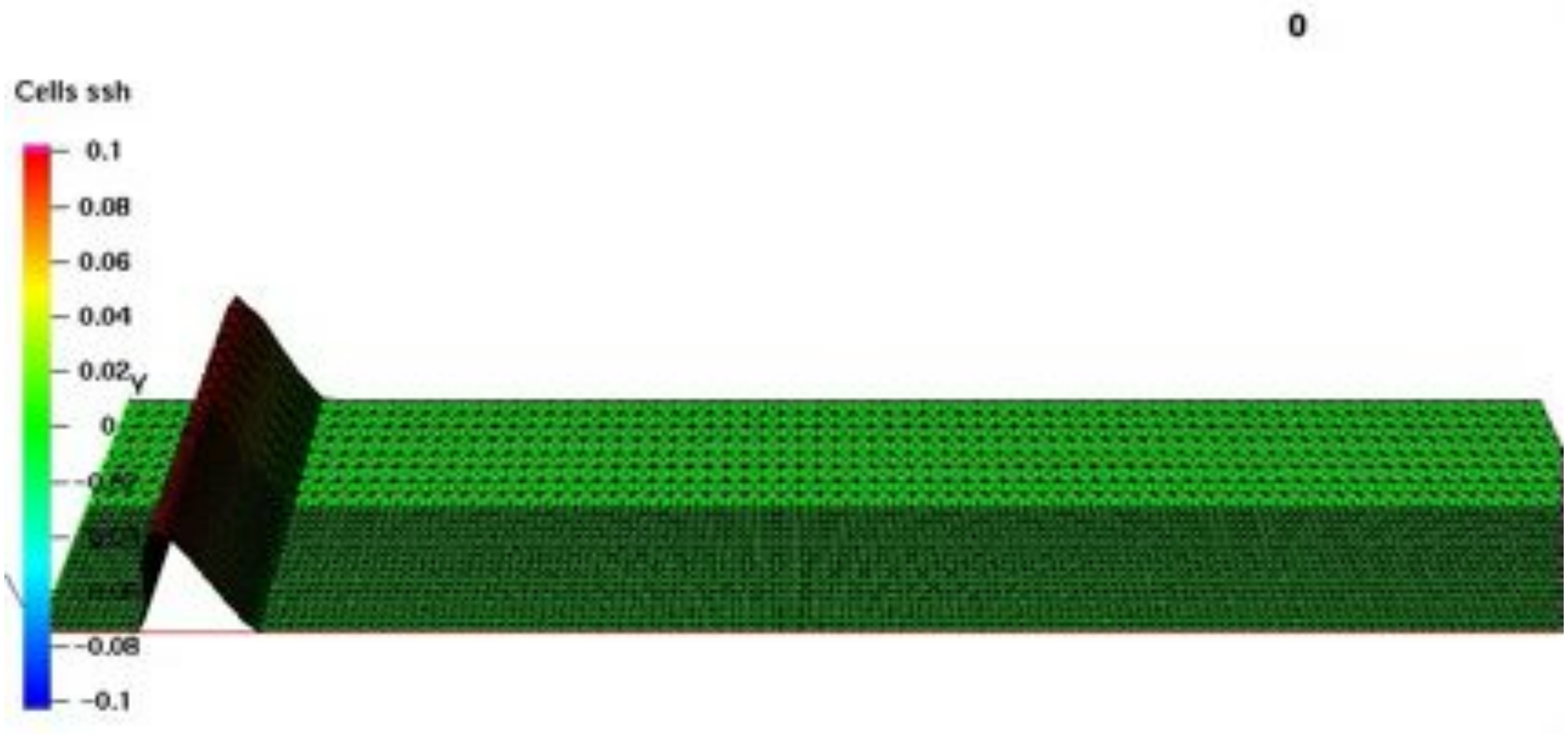
4. Efficient and robust (!) numerical scheme

Independence of grid resolution!



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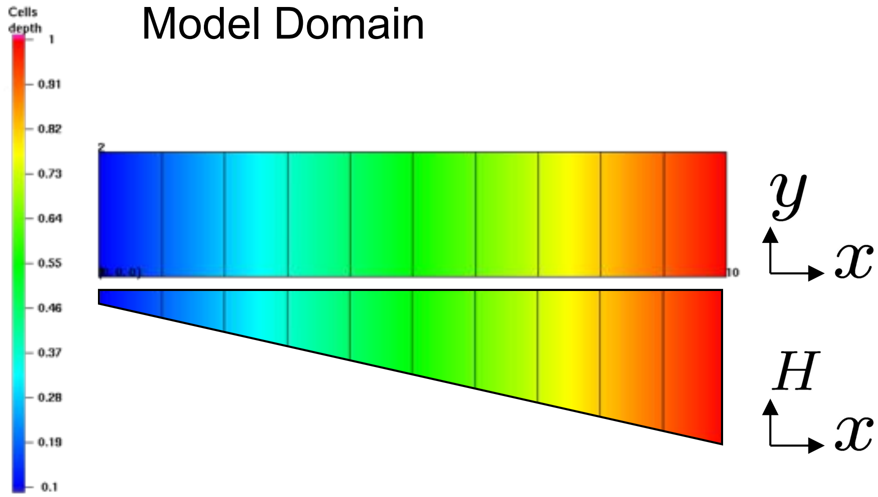
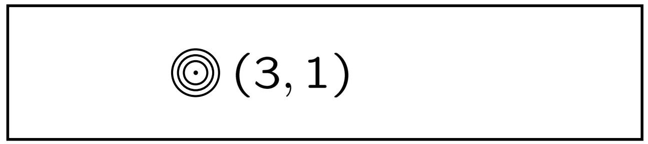
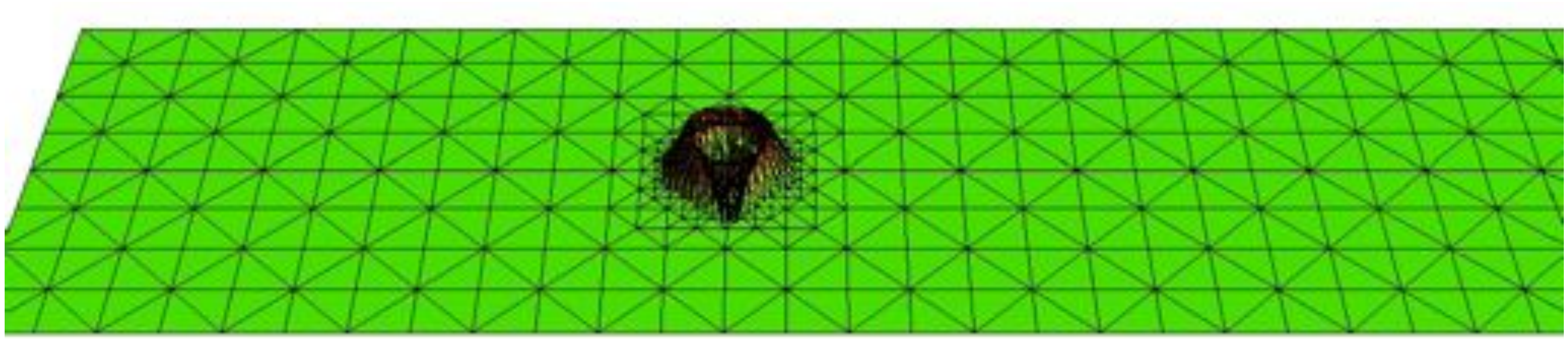
Adaptive mesh refinement – simple example

Model Domain



Initial values





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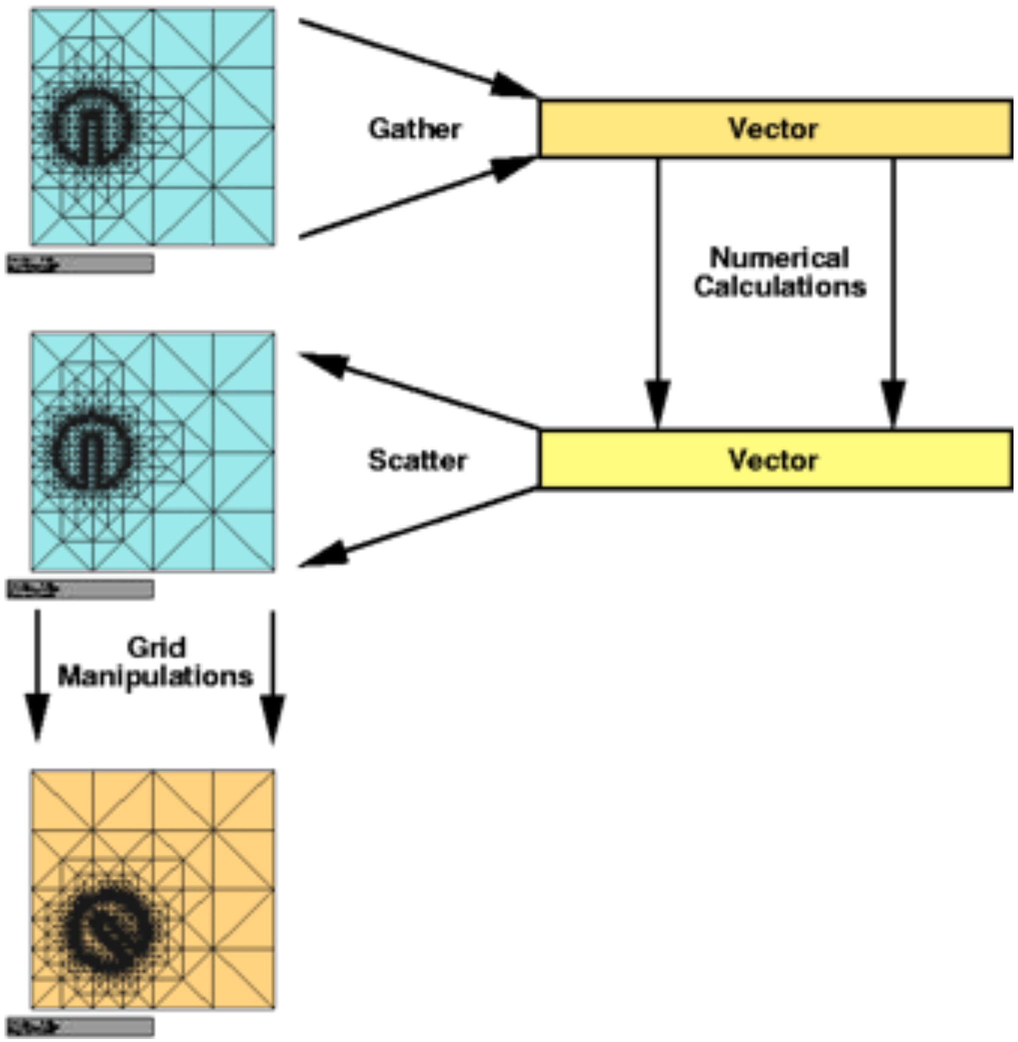
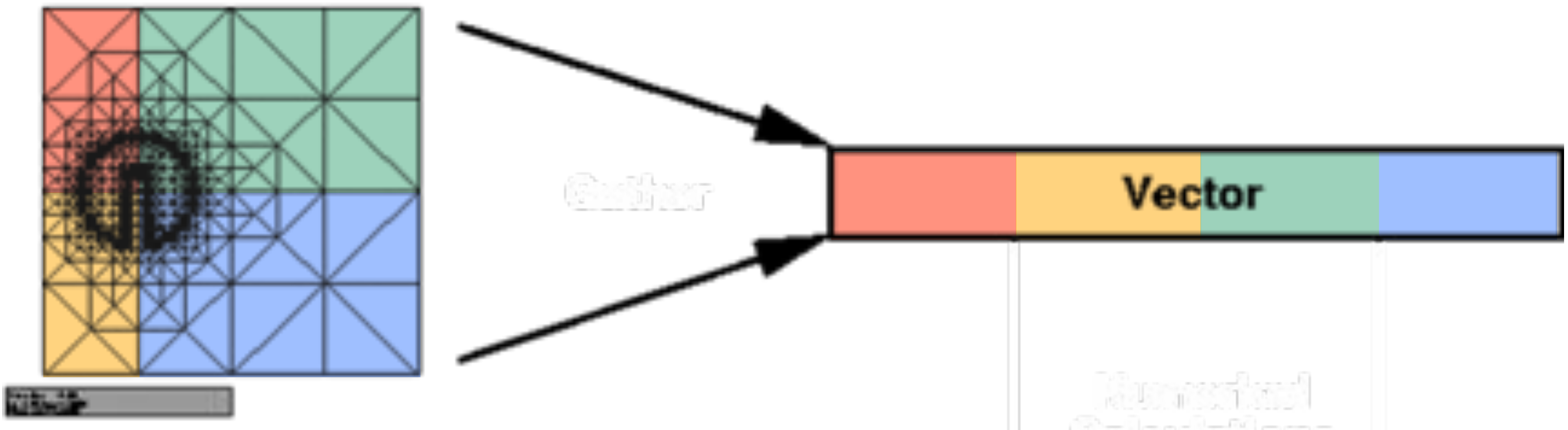
5. Efficient Data Management



Data Locality

We need data to stay local!



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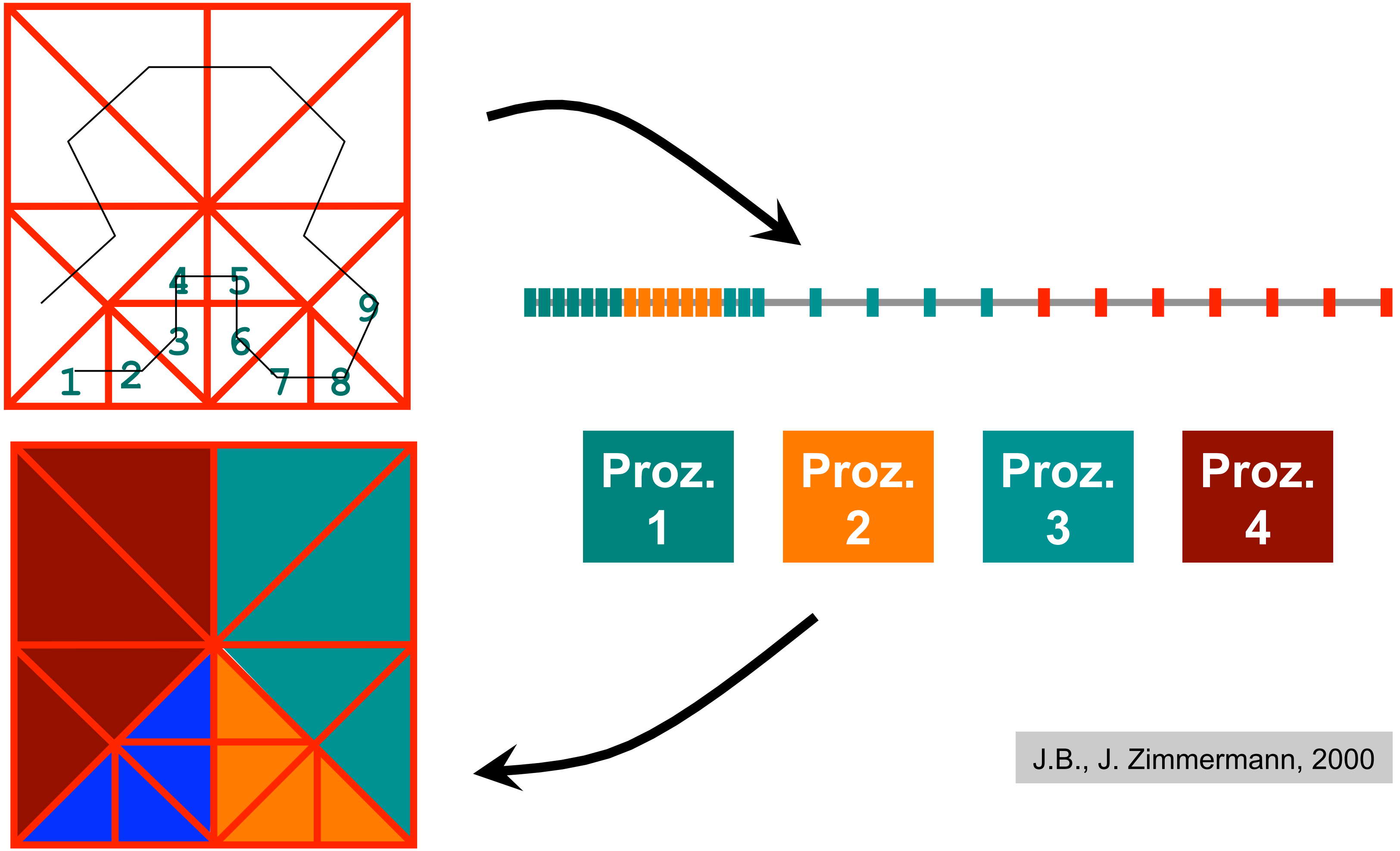
Optimization: Parallel Partitioning

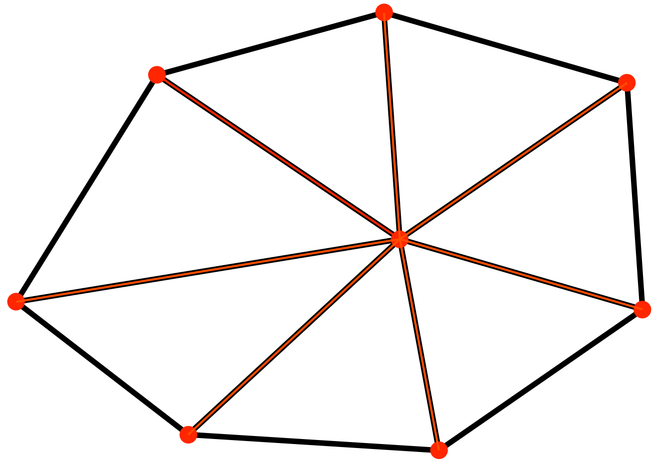
Space-filling curve (Sierpinski)



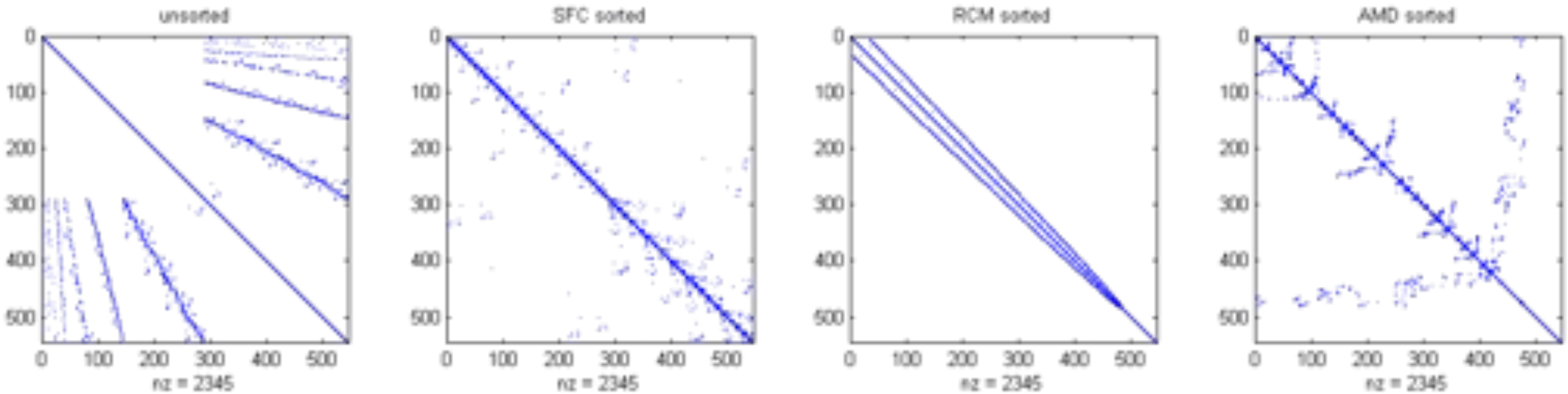
J.B., J. Zimmermann, 2000

Optimization: Matrix Ordering and Cache Optimization

Nearest neighbor communication (vertex-wise)

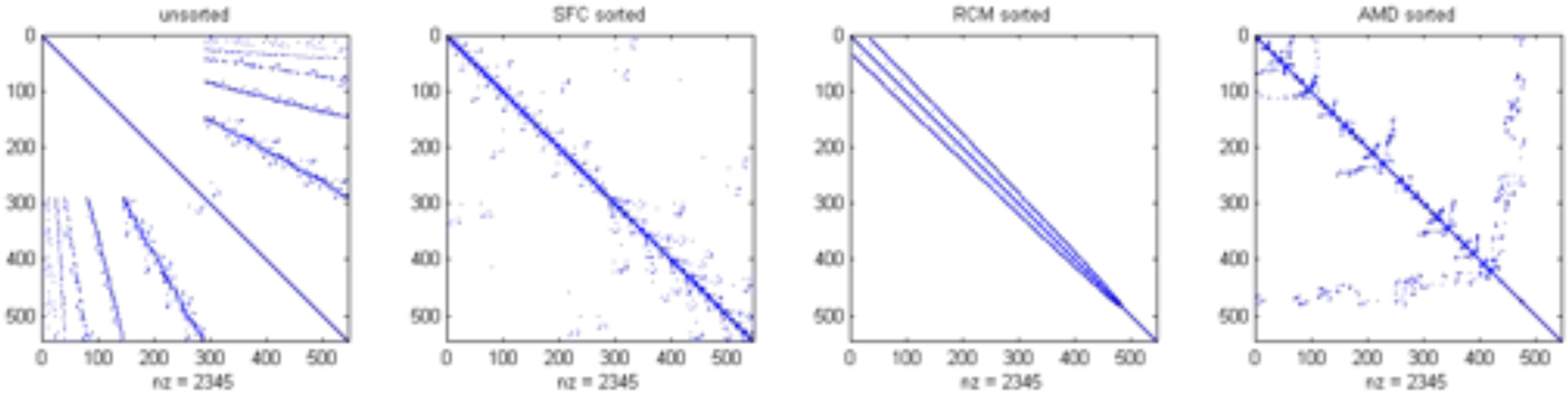


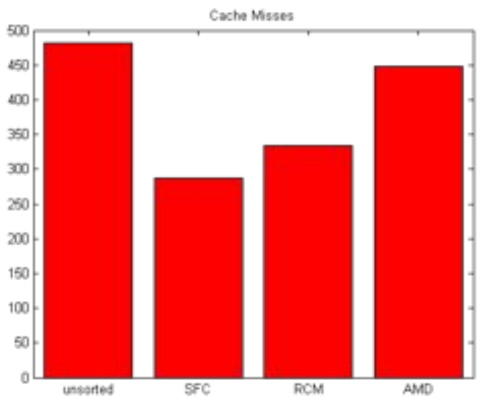
Connectivity matrix with different orderings



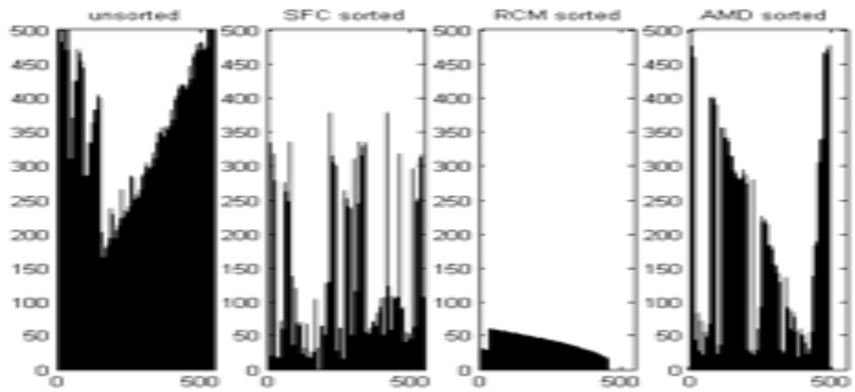
Optimization: Cache Optimization

Connectivity matrix with different orderings





Cache misses



Distance structure

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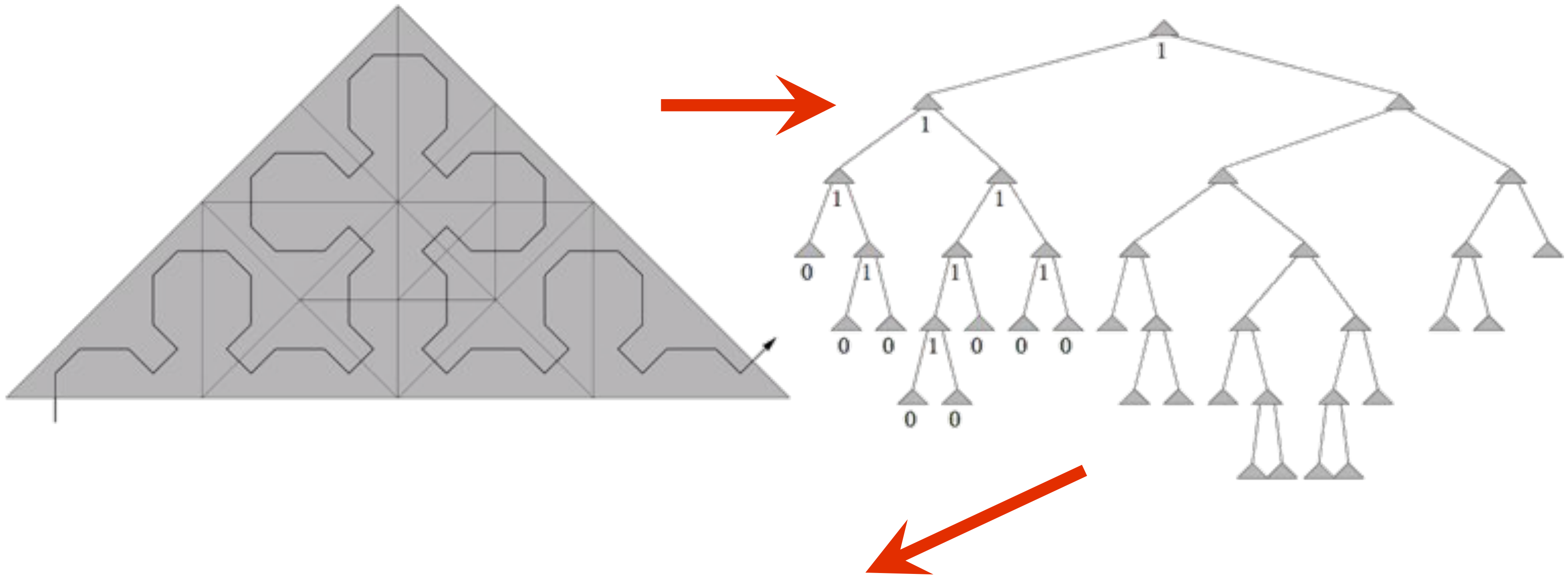
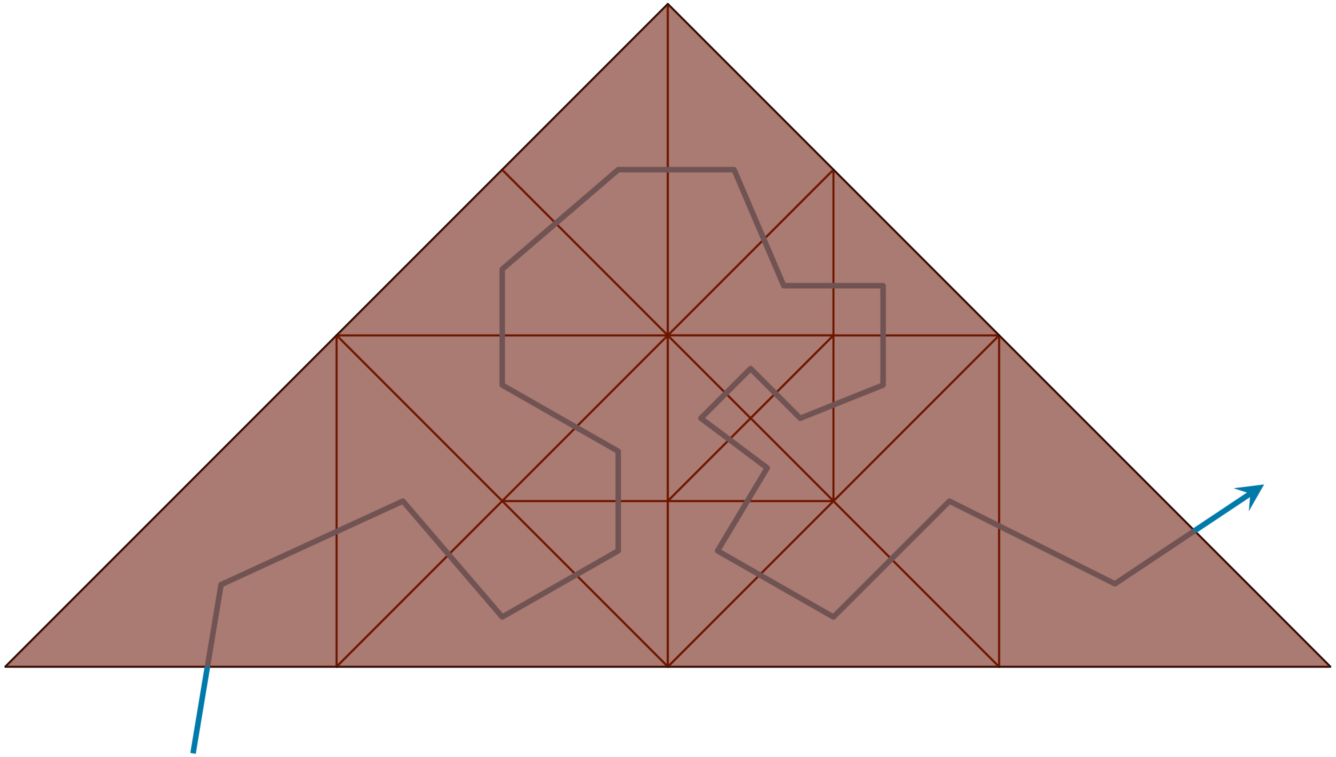
Observation 1: Refinement tree



1110100111000100...

Refinement tree as bitstream

Observation 2: Element-wise computation



- Depth first traversal induces Sierpinski curve
- Element by element computation

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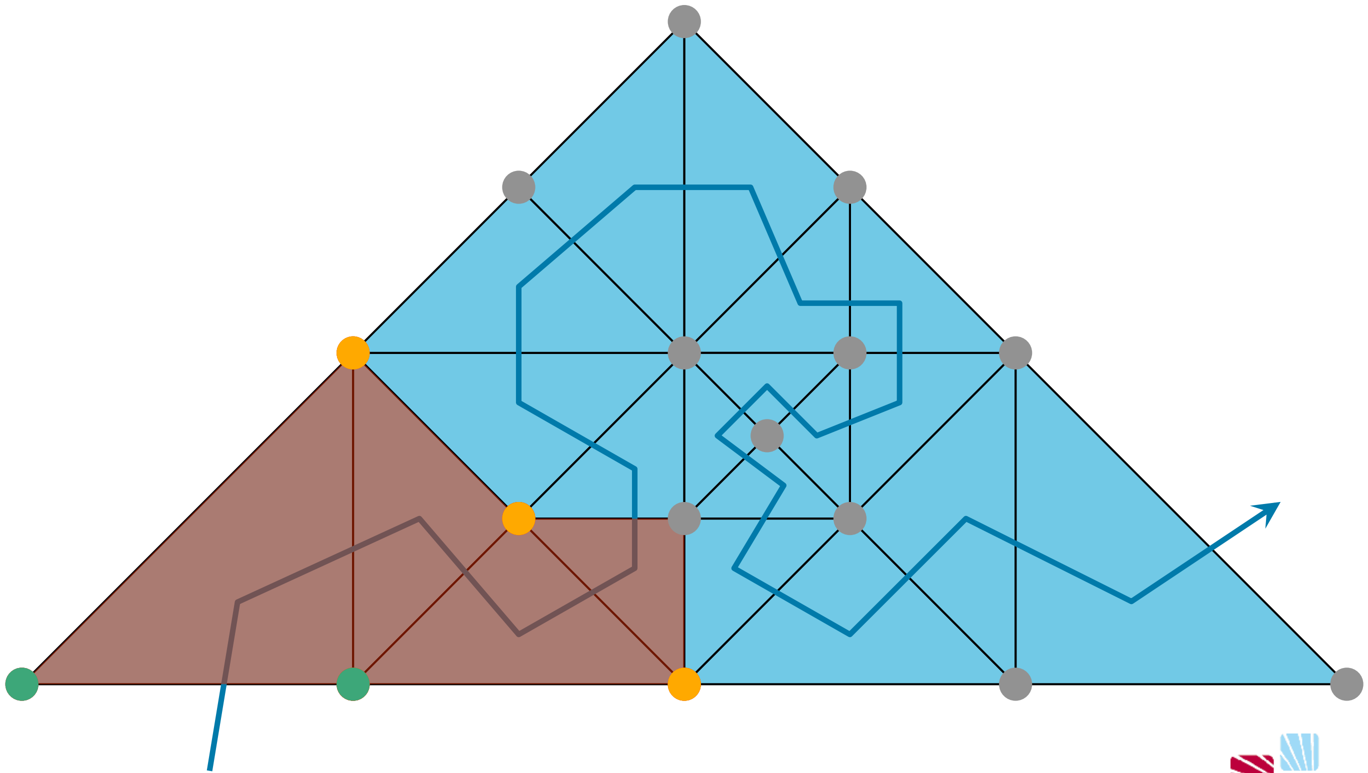
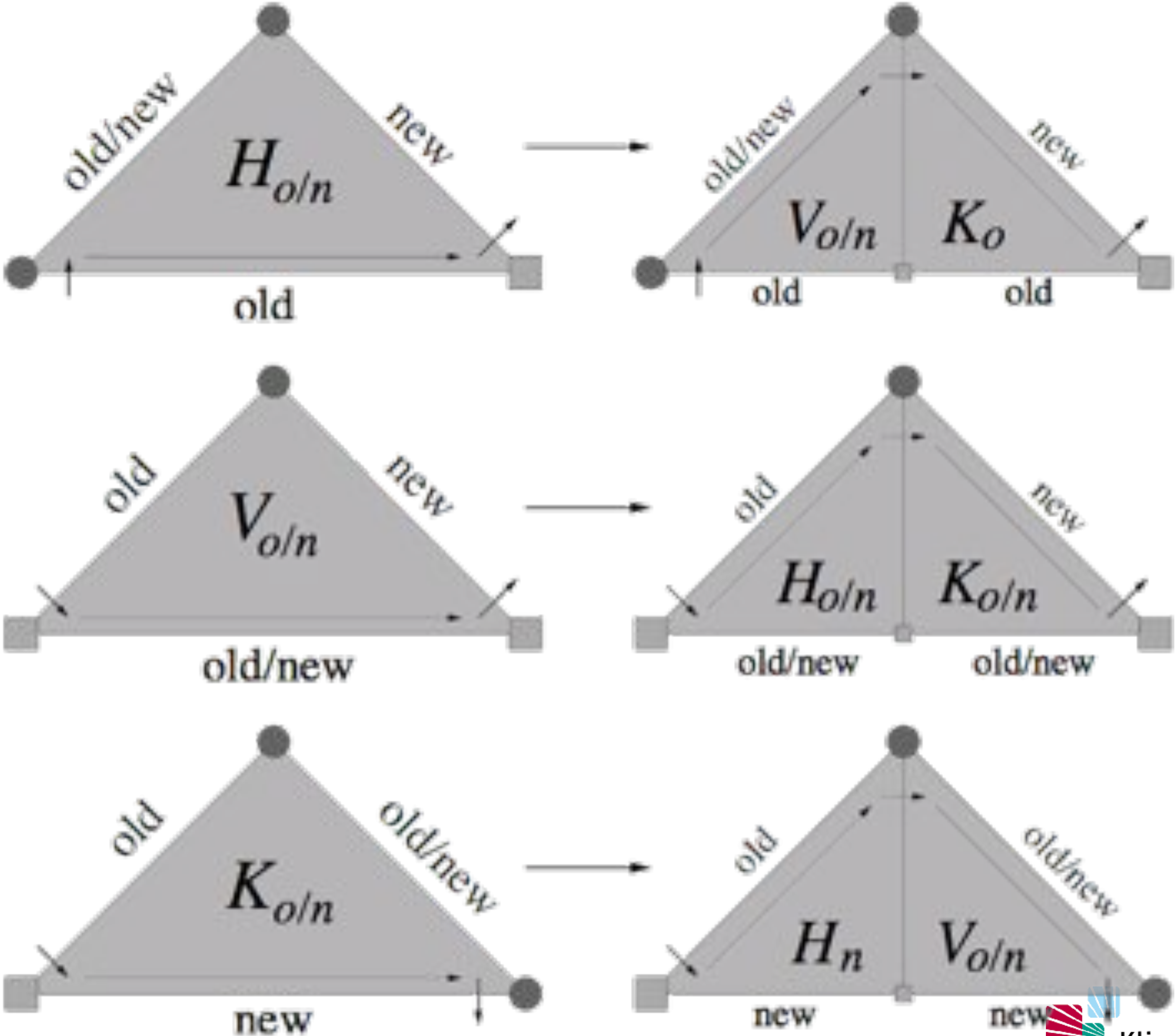
Observation 3: Nodal computations

- not yet accessed
- just being computed
- waiting for further computation
- computations finished

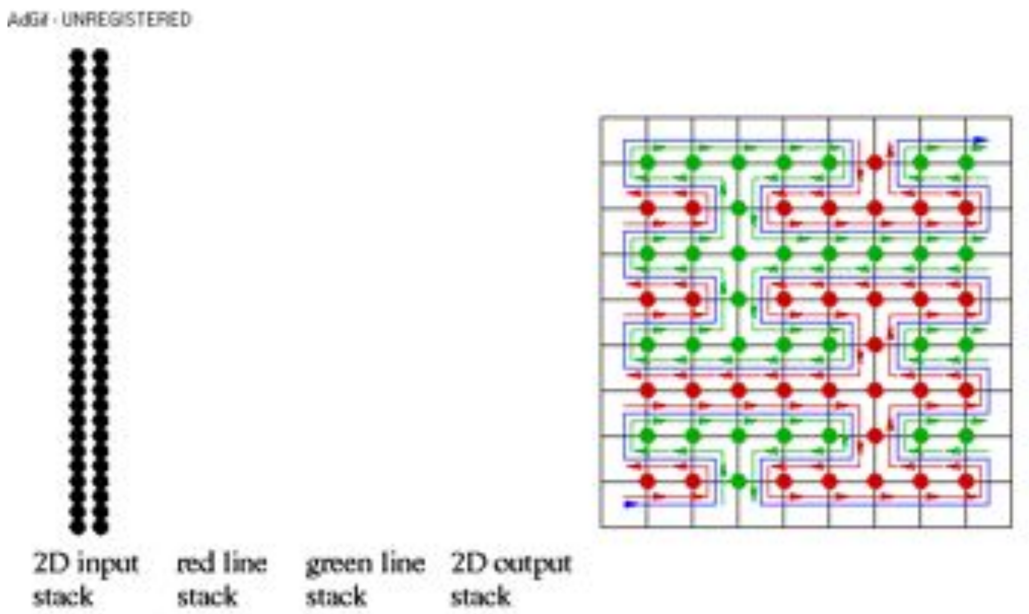


Observation 4: Tabulated element types



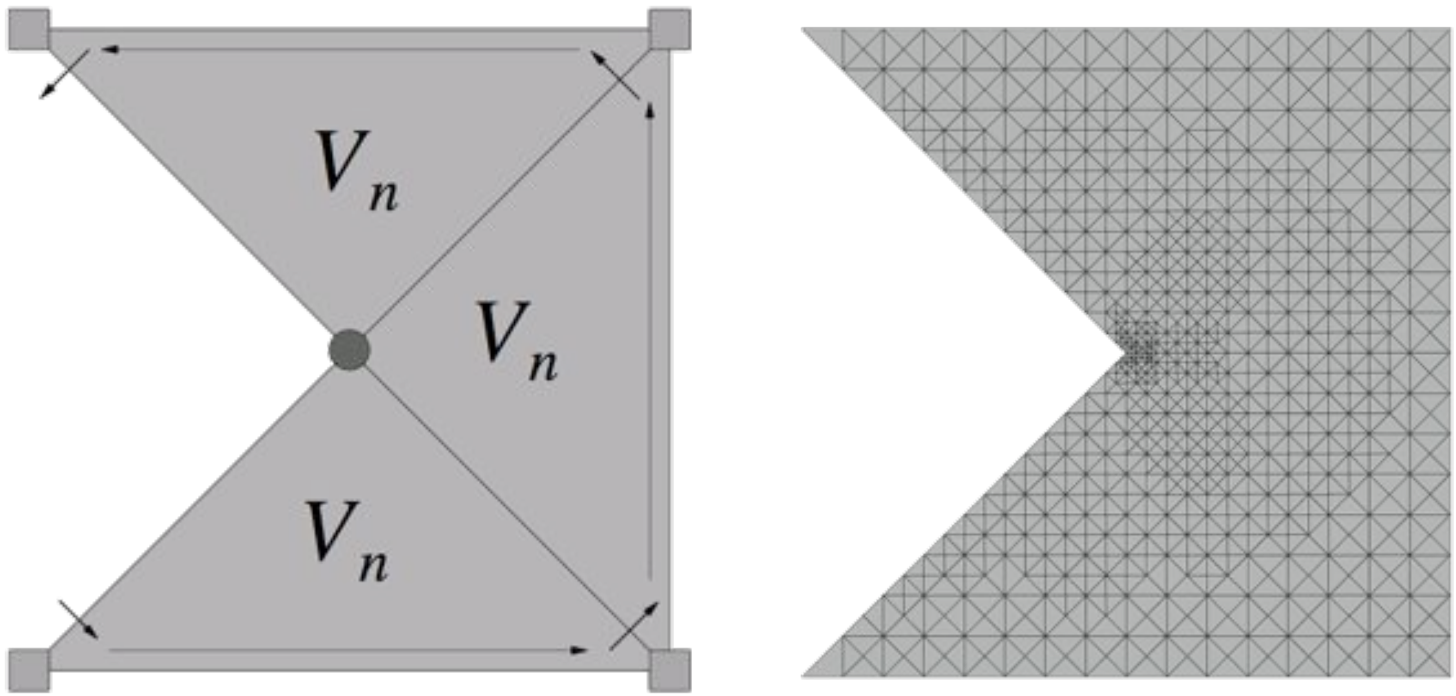
Idea

- Linearize grid structure by depth-first-traversal/Sierpinski curve
- Take element types for computing order
- Use heap data structures for storing intermediate values
- Use input/output stream



Mehl, Zenger (2004)

Preliminary result Reentrant corner



- Poisson's eq. with local refinement
- Solver: MG preconditioned CG
- Refinement criterion: hierarchical basis based (Deuflhard)

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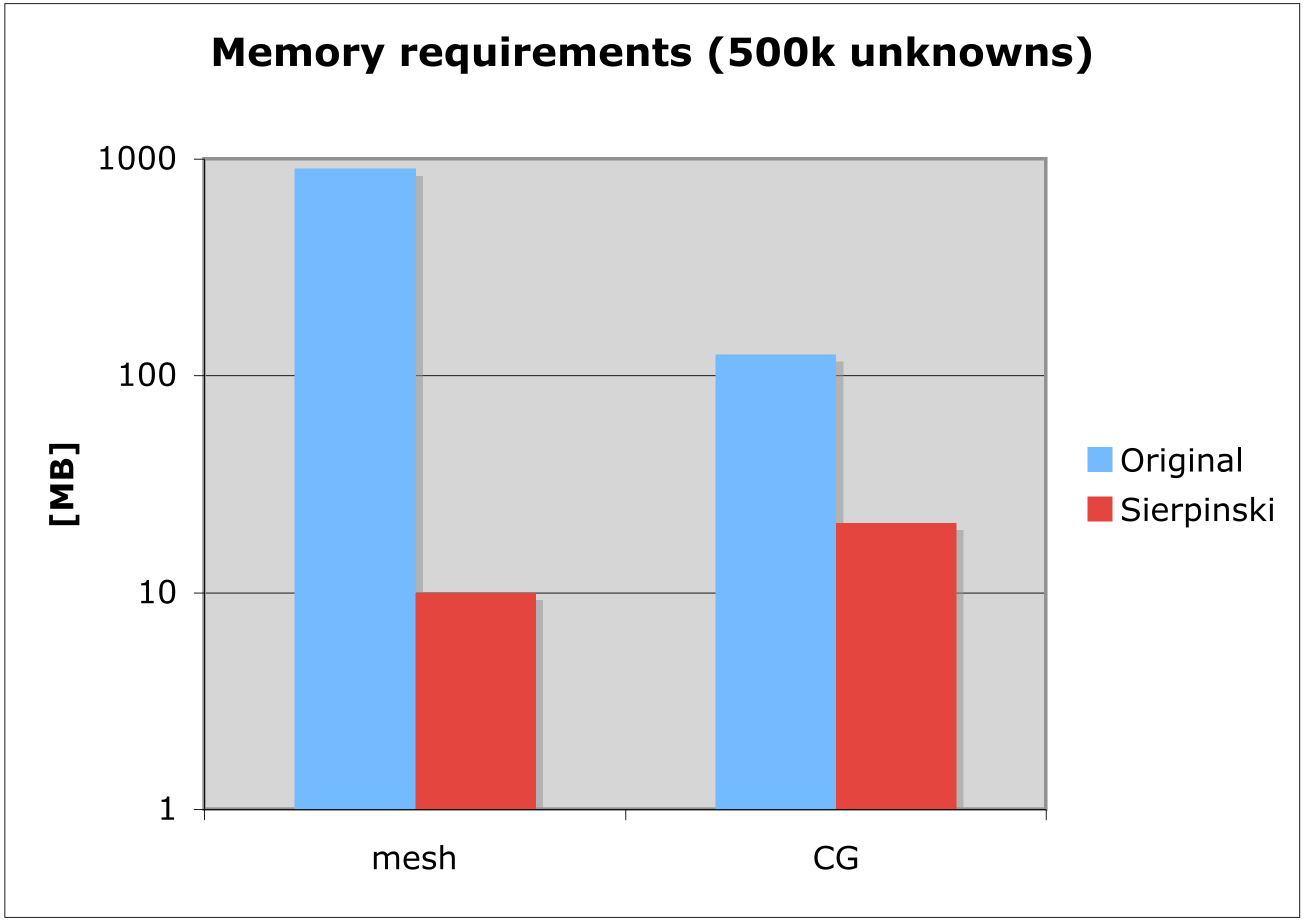
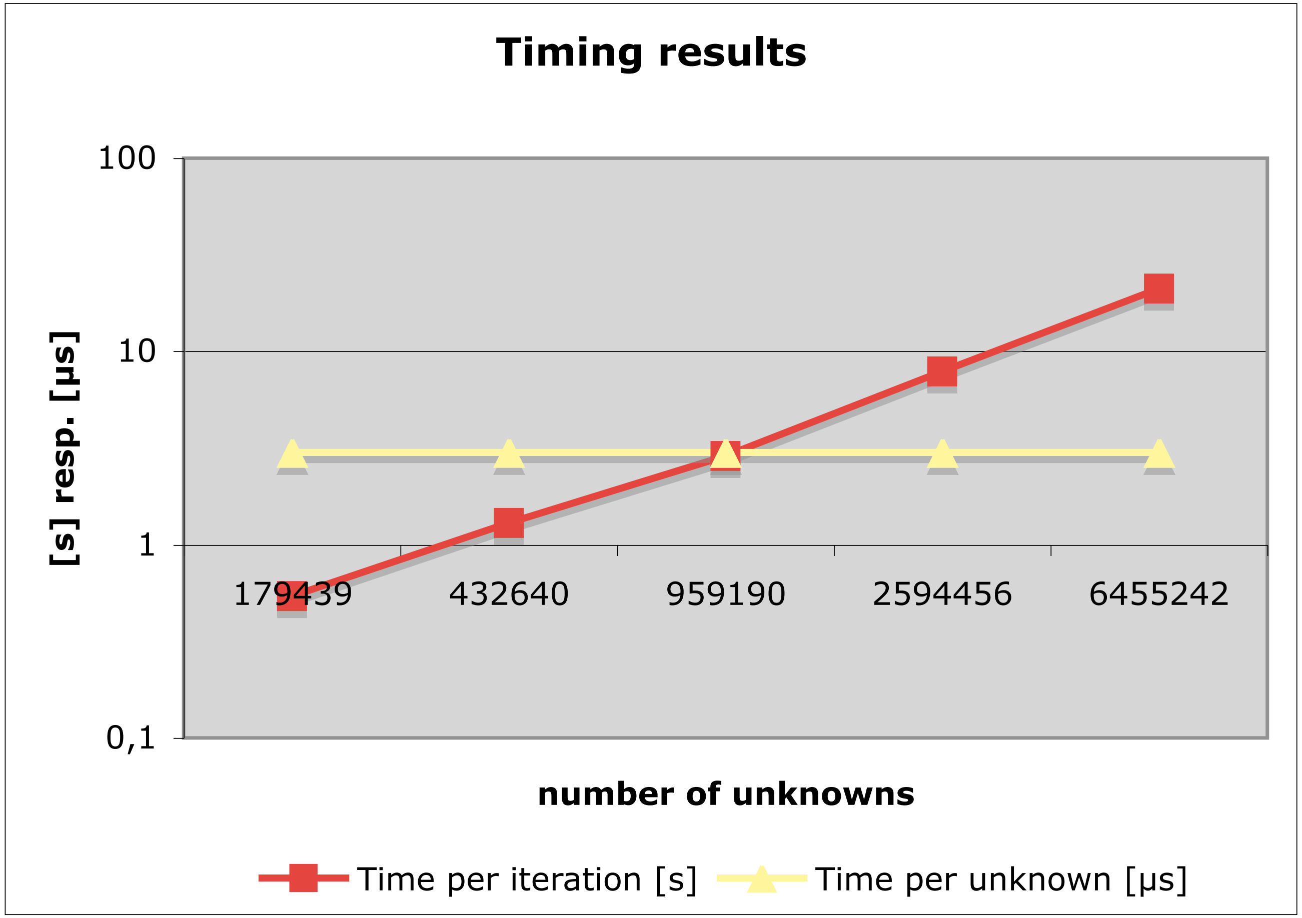
Preliminary result Reentrant corner



J.B., M. Bader (2009)

Preliminary result Reentrant corner



- 99% cache hits

J.B., M. Bader (2009)

Basic equations

Shallow Water Equations

Continuity equation (flux form)

$$\frac{\partial \zeta}{\partial t} + \nabla \cdot (\mathbf{v}(\zeta + H)) = 0$$

Momentum equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} + g\nabla\zeta + \boxed{f \times \mathbf{v}} - \boxed{\frac{\mathbf{n}^2\mathbf{v}|\mathbf{v}|}{\sqrt[3]{H}}} - \boxed{\nabla \cdot (K_h \nabla \mathbf{v})} = 0$$

Coriolis term   Bottom friction   Viscosity term

Boundary conditions

Radiation boundary condition (open/liquid boundary)

$$\mathbf{v} \cdot \mathbf{n} = \sqrt{\frac{g}{H + \zeta}}\, \zeta$$

No-slip boundary condition (solid boundary)

$$\mathbf{v} \cdot \mathbf{n} = 0$$

Note: inundation boundary conditions according to Lynett/Wu/Liu (2002)

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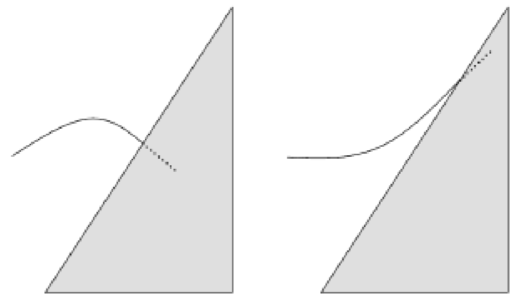
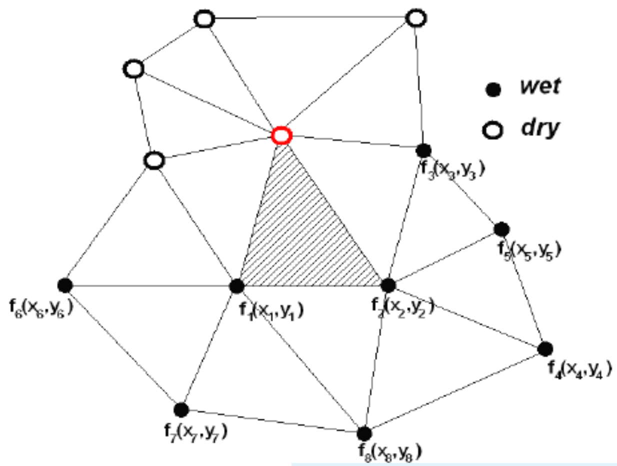
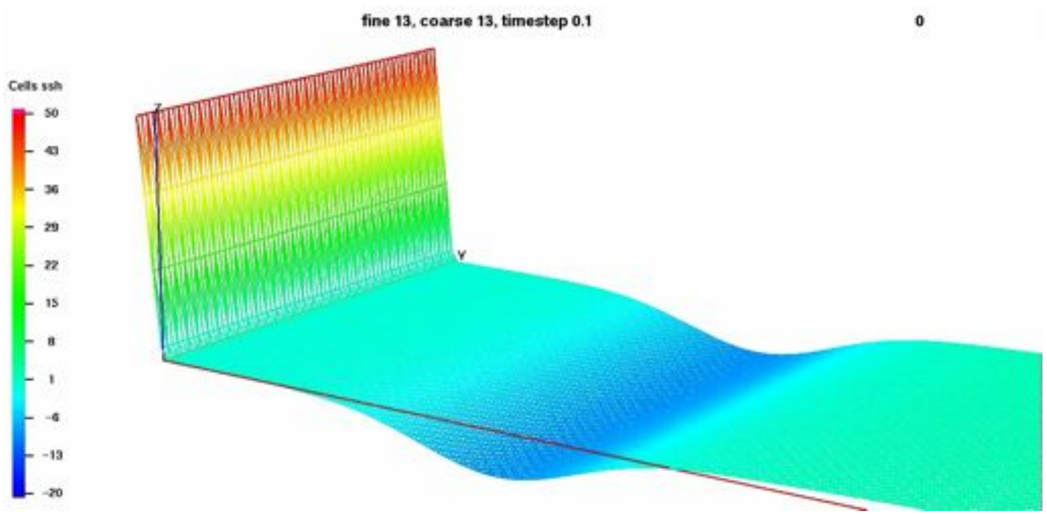
Inundation boundary conditions

Extrapolation of wave height



Extrapolation based on neighbors



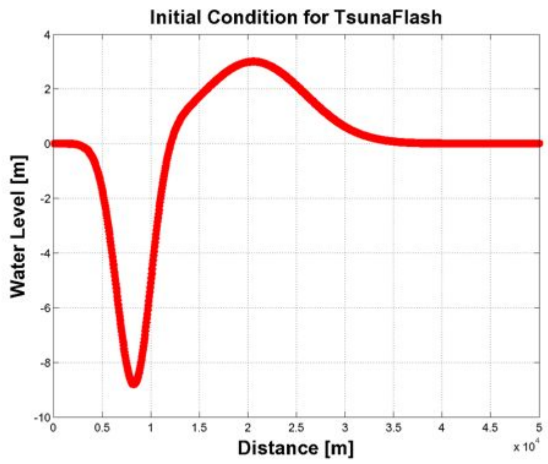


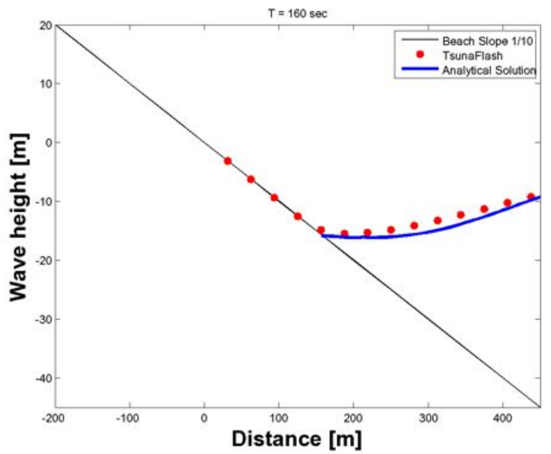
Lynnet, Wu, Liu, 2002

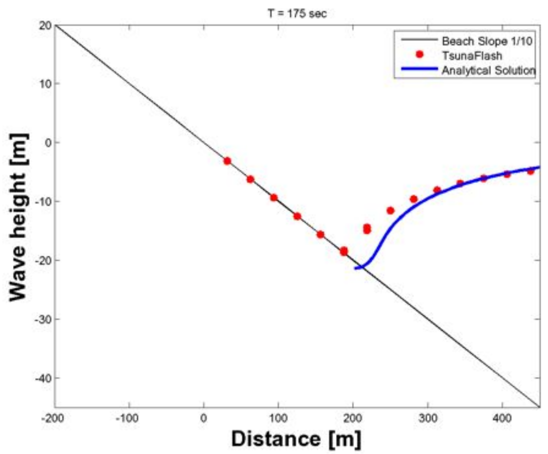
Validation: Inundation test case

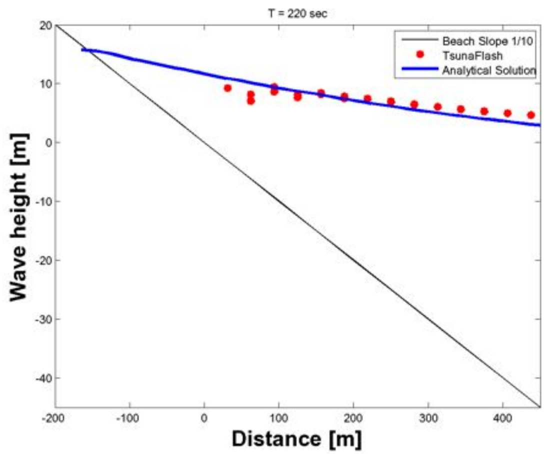
Sloping Beach test case [Liu et al., 2008]

- Pre-scribed initial condition
- Analytical solution
- Simplified quasi-1D test case



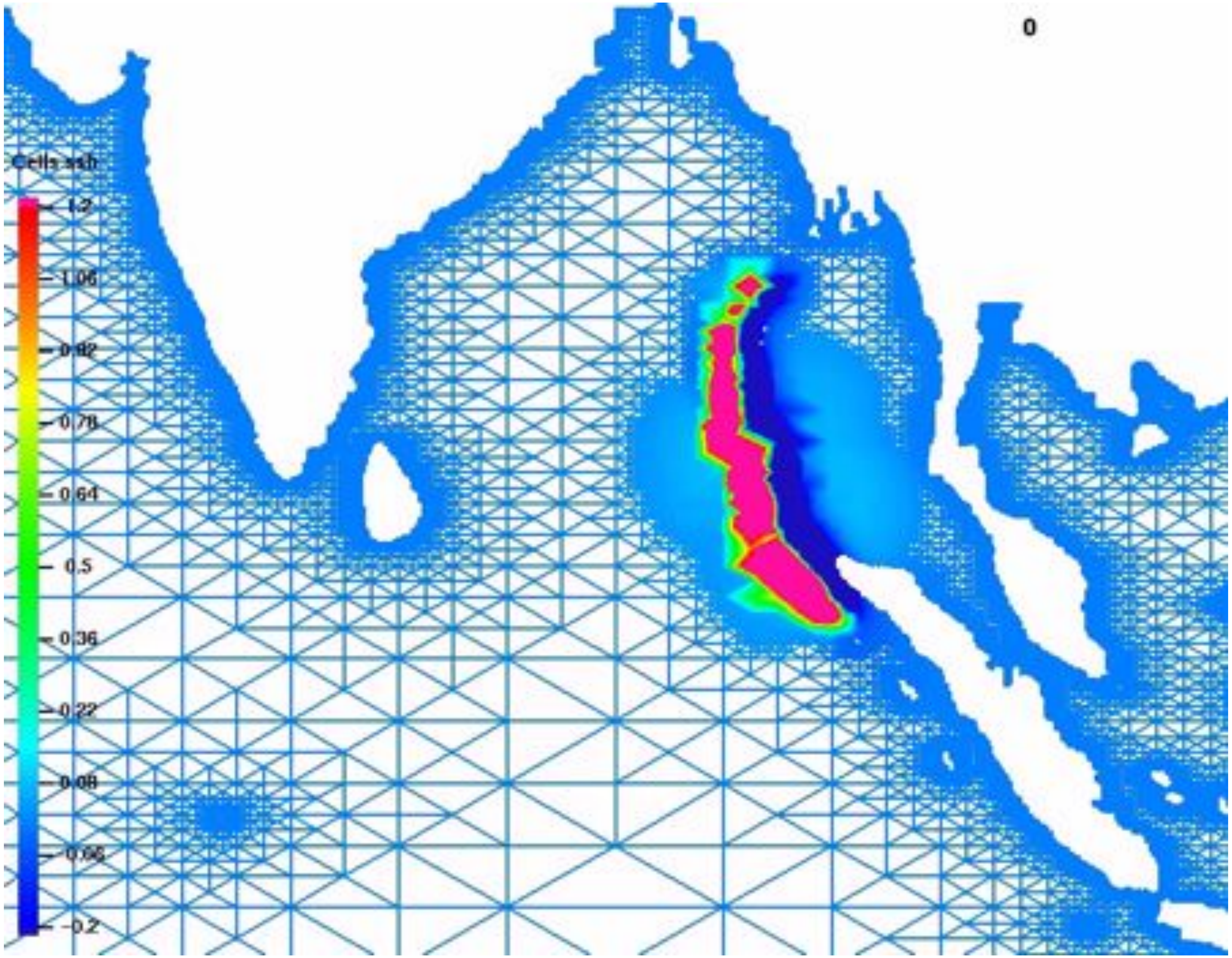




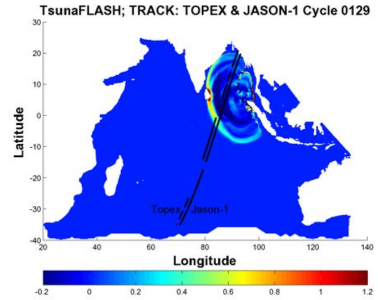


W.S. Pranowo (2010)

Andaman-Sumatra Rupture

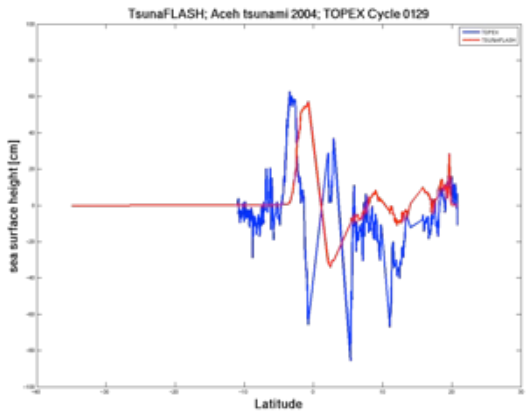


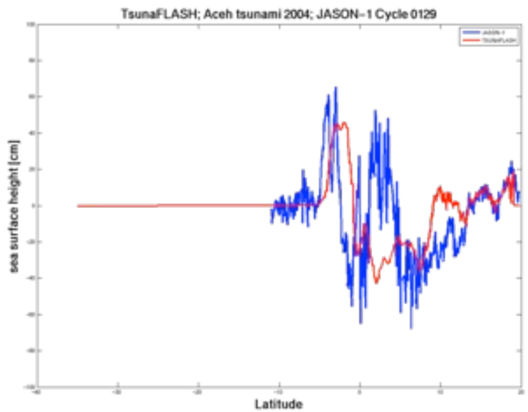
Validation: Comparison to field data

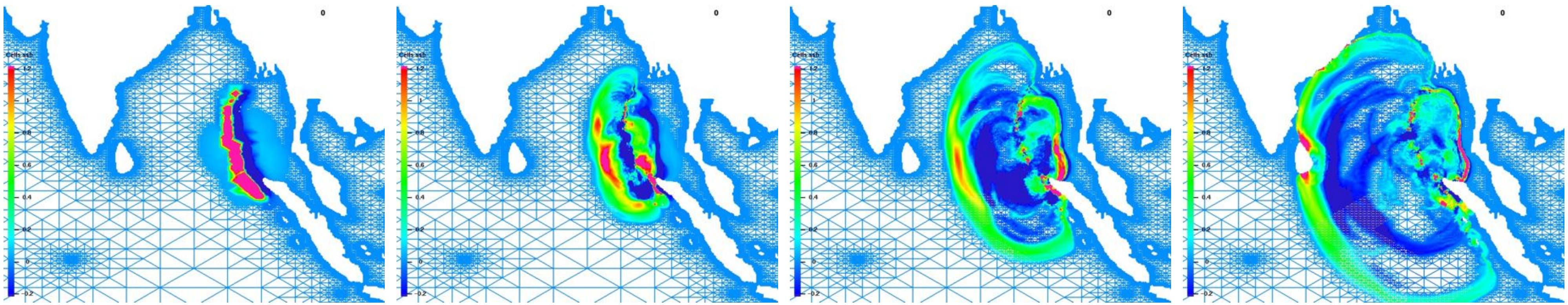


Field Data from 2004 Sumatra Tsunami

- Initial condition [Höchner et al., 2008]
- Sattelite track data PMEL/NOAA [Smith et al., 2005]







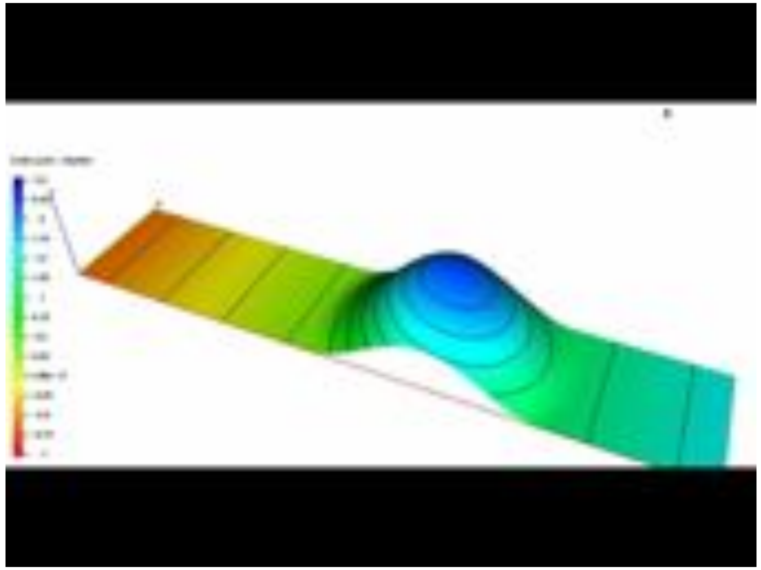
W.S. Pranowo (2010)

Conclusions and outlook

- Multiple scales need numerical representation
- Model for adaptive computation gain
- 5 Efficiency ingredients (meshing, refinement, programming, numerics, data managmnt.)
- Optimization

Challenges:

- Extend equations by surface wind stress term
- Extend equations by tidal forcing term
- Include (surface-) wave model?



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