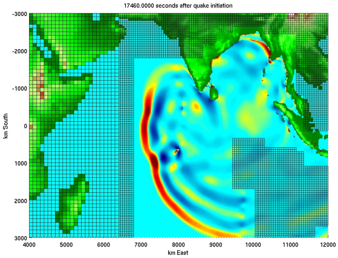
GeoClaw's Adaptive Mesh Refinement with Dry States and Well Balancing

Randall J. LeVeque
Department of Applied Mathematics
University of Washington



GeoClaw: **www.clawpack.org/geoclaw**

Collaborators

David George, University of Washington
Mendenhall postdoctoral Fellow at the
USGS Cascades Volcano Observatory (CVO)

Marsha Berger, Courant Institute, NYU

Roger Denlinger and Dick Iverson,
USGS Cascades Volcano Observatory (CVO)

David Alexander and William Johnstone,
Spatial Vision Group, Vancouver, BC
Barbara Lence, Civil Engineering, UBC

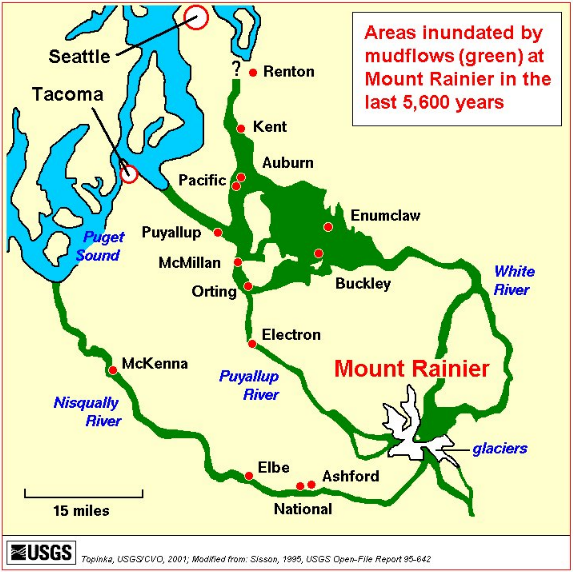
Harry Yeh, Civil Engineering, OSU

Numerous other students and colleagues

Supported in part by NSF, ONR

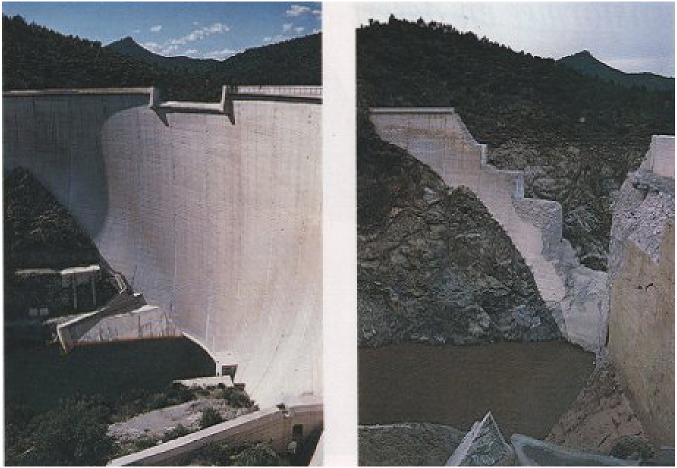
Mt. Rainier and Tacoma

Mud and debris flows from Mt. Rainier



Malpasset Dam Failure

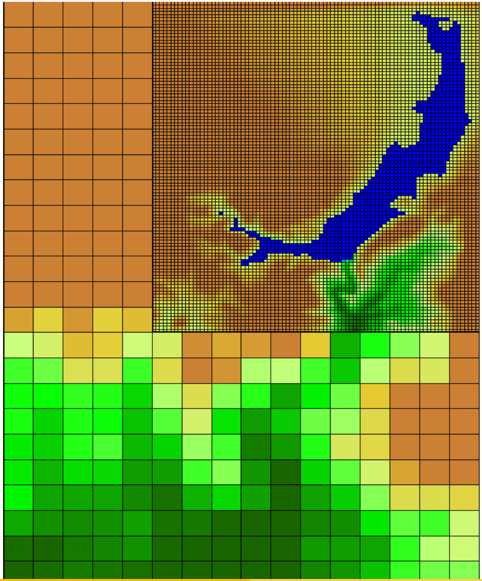
Catastrophic failure in 1959



Malpasset Dam Failure

Modeling work by David George, using GeoClaw

Coarse: 400m cell side, Level 2: 50m, Level 3: 12m, Level 4: 3m



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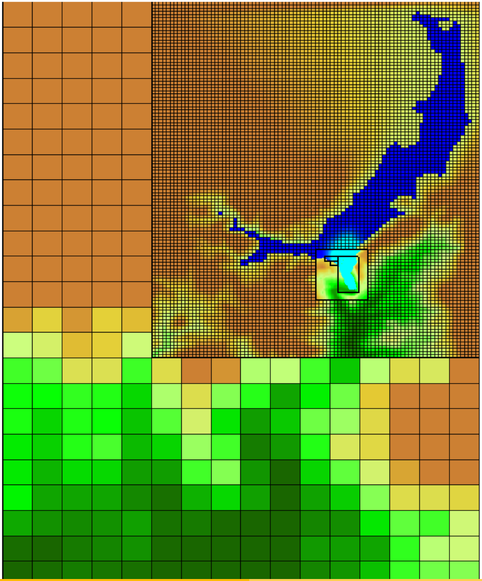

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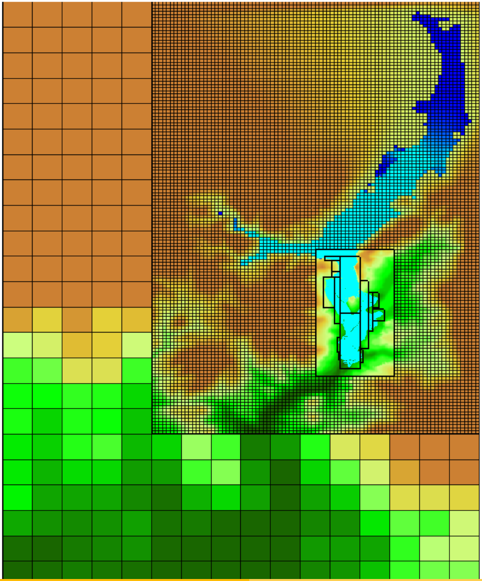

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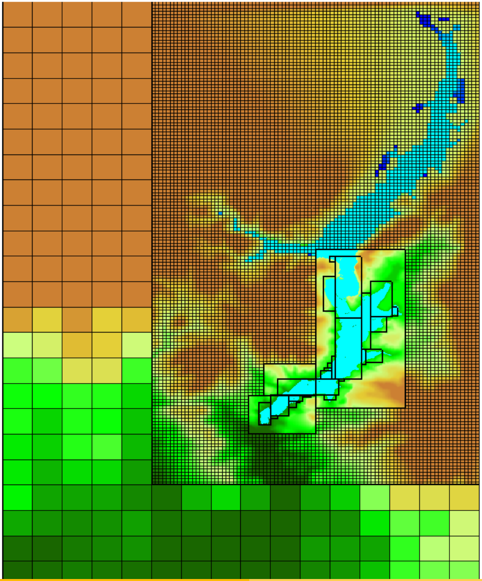

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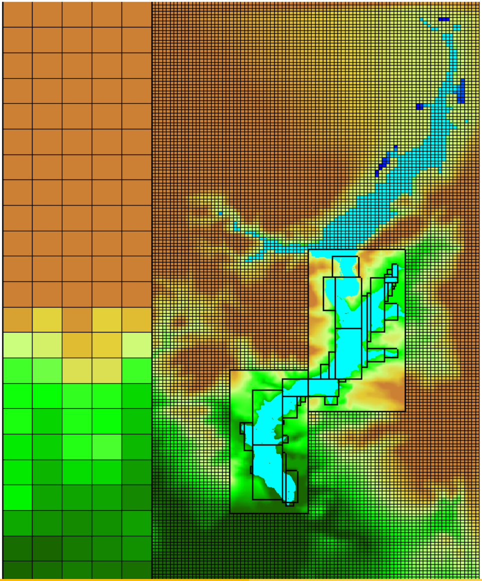

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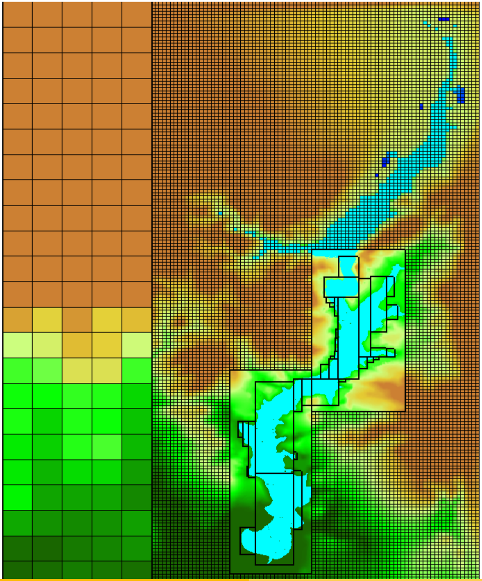

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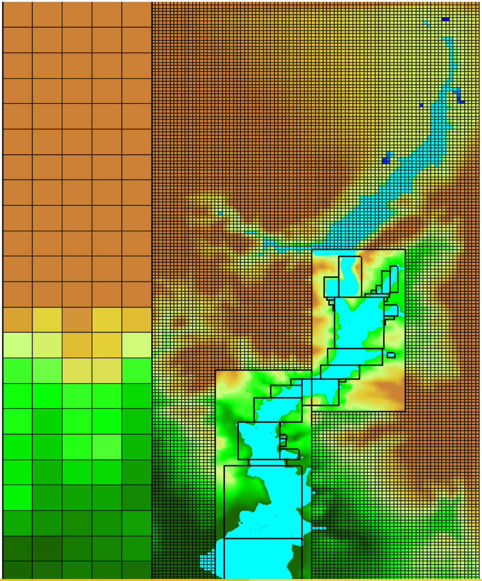
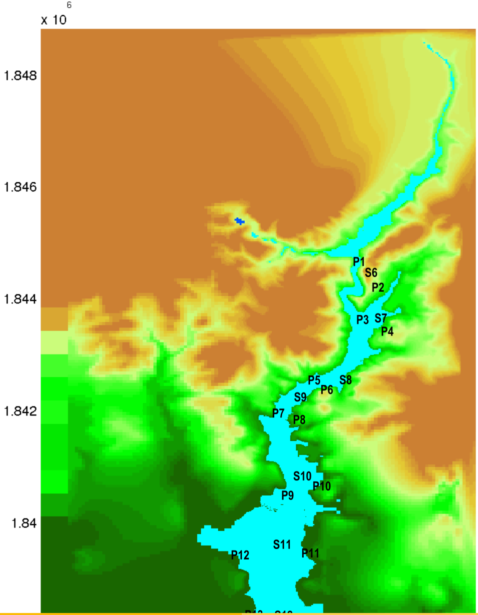


Malpasset survey locations



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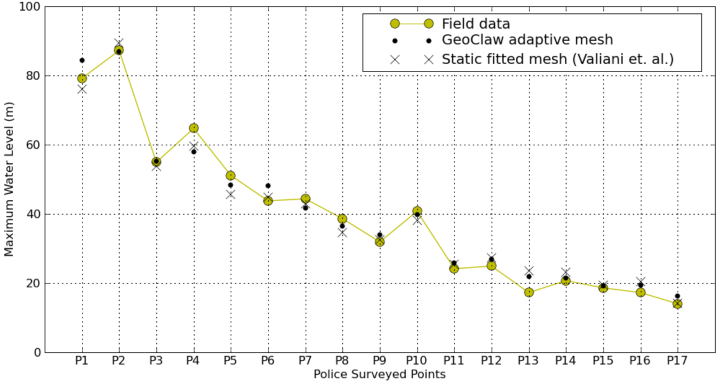
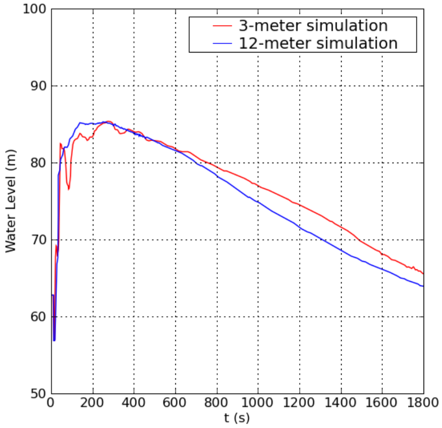


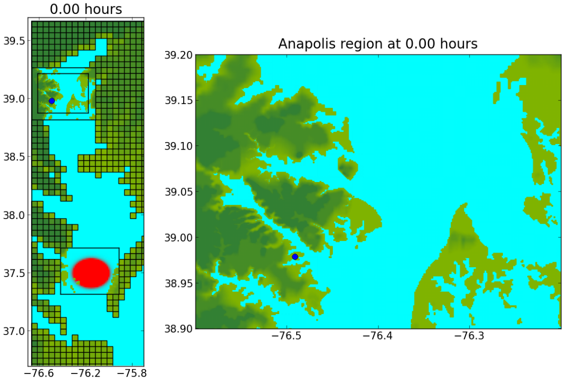
Grid convergence study

Water depth gauge at location P2 computed with two different resolutions (using 4 levels or only 3):



Chesapeake Bay and Anapolis

Quick test after yesterday's discussion...



Data from **Design-a-Grid** NOAA National Geophysical Data Center (NGDC)

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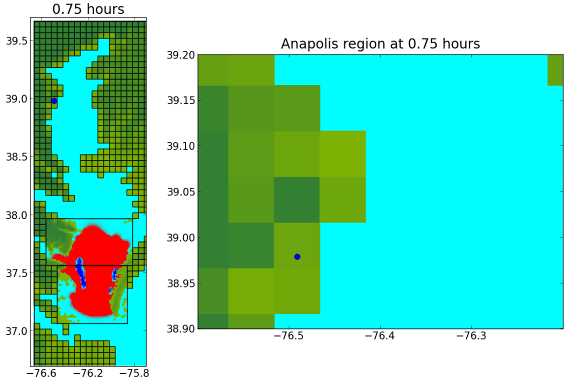

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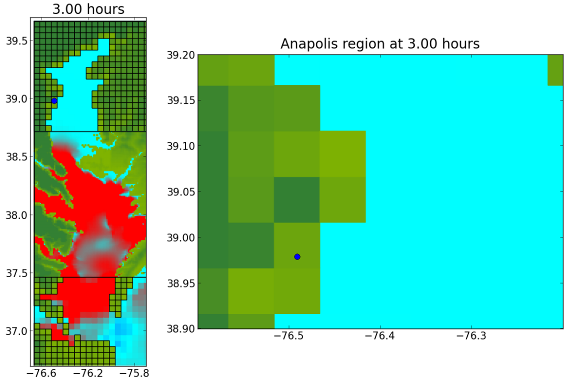

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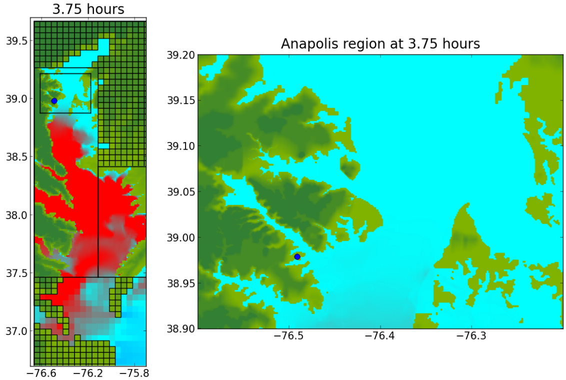

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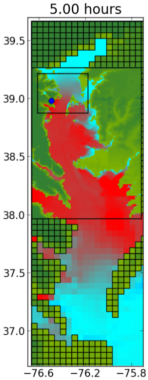
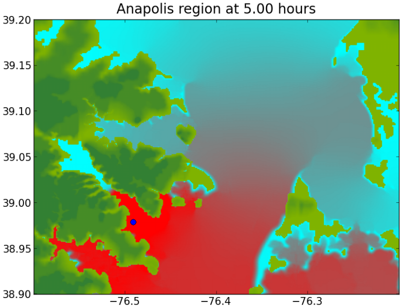




Data from **Design-a-Grid** NOAA National Geophysical Data Center (NGDC)

Chesapeake Bay and Anapolis

Timeline:

View Google Earth, download bathymetry: $\approx$ 30 minutes

GeoClaw implementation started: 5:40pm

The run just shown...
Started: 6:16pm
Ended: 6:56pm

Went to dinner...

Making plots of 29 frames and movie:
Started: 7:58pm
Ended: 8:16pm

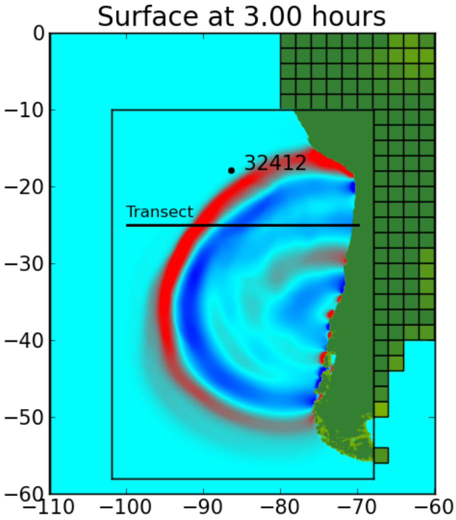
Shallow water equations with bathymetry $B(x, y)$

$$
\begin{aligned}
h_t + (hu)_x + (hv)_y &= 0 \\
(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x + (huv)_y &= -ghB_x(x,y) \\
(hv)_t + (huv)_x + \left(hv^2 + \frac{1}{2}gh^2\right)_y &= -ghB_y(x,y)
\end{aligned}
$$

Some issues:

- Delicate balance between flux divergence and bathymetry:
  h varies on order of 4000m, rapid variations in ocean
  Waves have magnitude 1m or less.
- Cartesian grid used, with $h = 0$ in dry cells:
  Cells become wet/dry as wave advances on shore
  Robust Riemann solvers needed.
- Adaptive mesh refinement crucial
  Interaction of AMR with source terms, dry states

Cross section of Atlantic Ocean & tsunami



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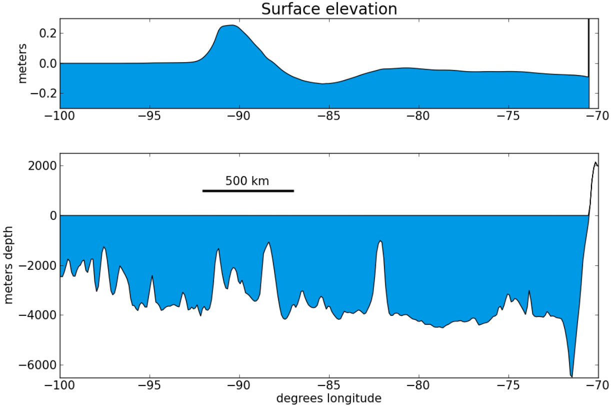
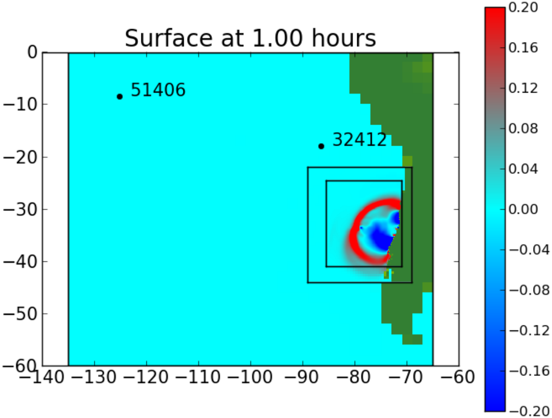


DART buoy data

Deep-ocean Assessment and Reporting of Tsunamis

NOAA's Network of pressure gauges on the ocean floor



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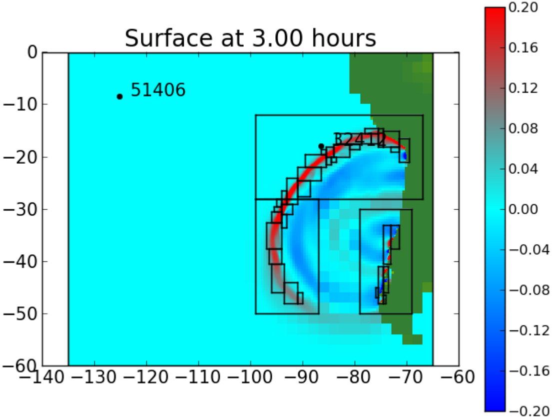

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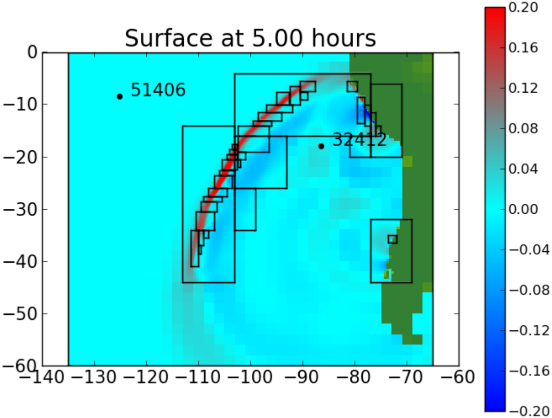

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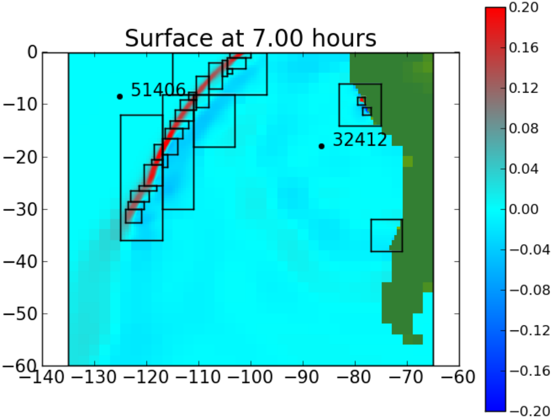

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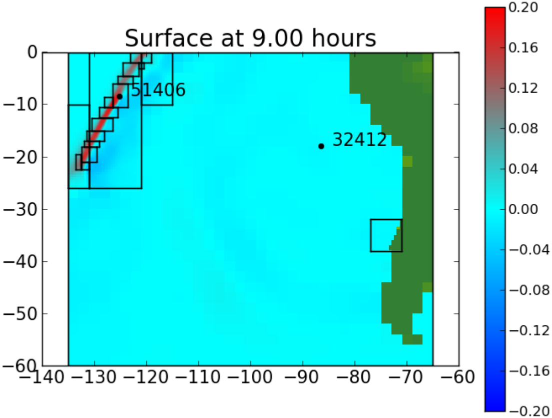

National Oceanic and Atmospheric Administration's

National Data Buoy Center

Center of Excellence in Marine Technology

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Storm Special! View the latest observations near Atlantic HURRICANE IGOR as of INTERMEDIATE ADVISORY NUMBER 53A @ 800 AM AST TUE SEP 21 2010, Atlantic TROPICAL STORM LISA as of ADVISORY NUMBER 2 @ 500 AM EDT TUE SEP 21 2010 and East Pacific TROPICAL STORM GEORGETTE of SPECIAL ADVISORY NUMBER 1 @ 500 AM PDT TUE SEP 21 2010.

Station 32412 - 630 NM Southwest of Lima, Peru

Owned and maintained by National Data Buoy Center
2.6-meter discus buoy
DART II payload
17.975 S 86.392 W (17°58'30" S 86°23'30" W)

Important Notice to Mariners

Meteorological Observations from Nearby Stations and Ships

Station 32412 - 630 NM Southwest of Lima, Peru

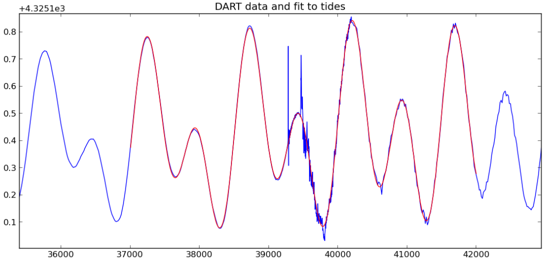
Owned and maintained by National Data Buoy Center
17.975 S 86.392 W (17°58'30" S 86°23'30" W)

Available historical data for station 32412 include:

- **Quality controlled data for 2010** (*data descriptions*)
  - **Water column height (Tsunami) (DART) data:** Jan Feb Mar Apr May Jun Jul
- **Historical data** (*data descriptions*)
  - **Water column height (Tsunami) (DART) data:** 2007 2008 2009

www.ndbc.noaa.gov/station_page.php?station=32412

DART buoy data



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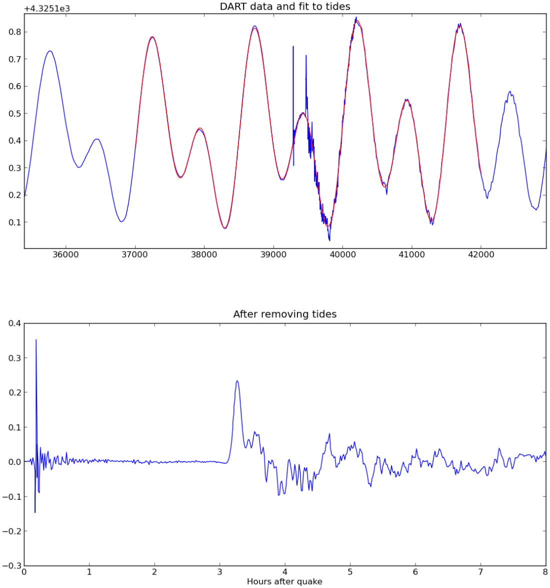


NOAA unit sources for subduction zone

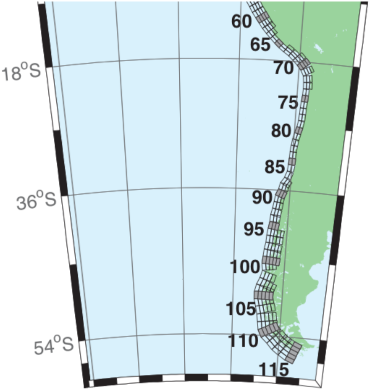
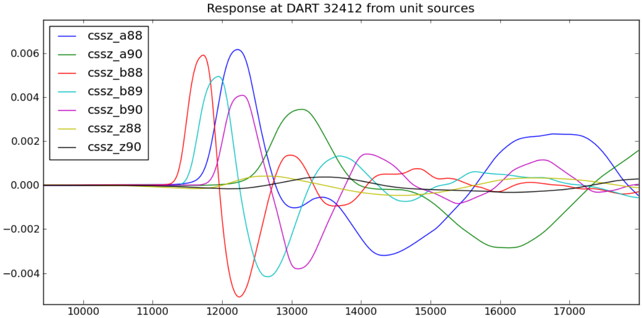


Figure B2: Central and South America Subduction Zone unit sources.

From: Tang, L., V.V. Titov, and C.D. Chamberlin (2010): A Tsunami Forecast Model for Hilo, Hawaii. NOAA OAR Special Report, PMEL Tsunami Forecast Series: Vol. 1, 94

http://nctr.pmel.noaa.gov/pubs.html

Response at DART buoy from unit earthquakes



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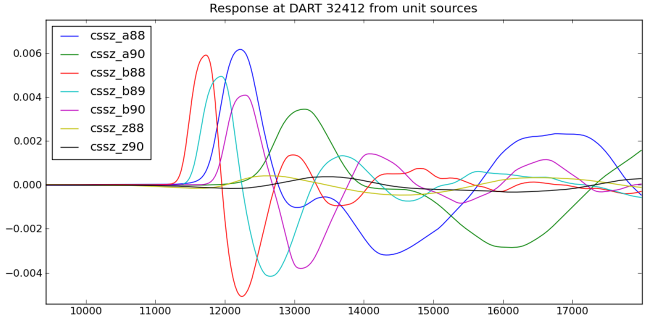
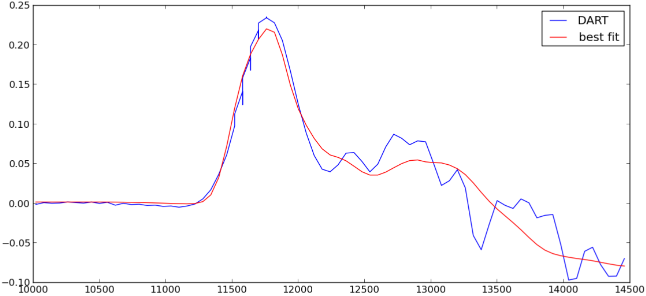


Propagation in deep water is essentially linear...

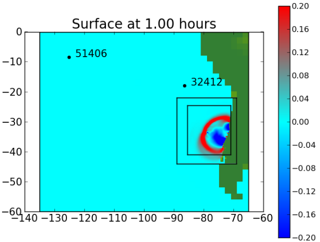
Fit linear combination of these responses to DART data.

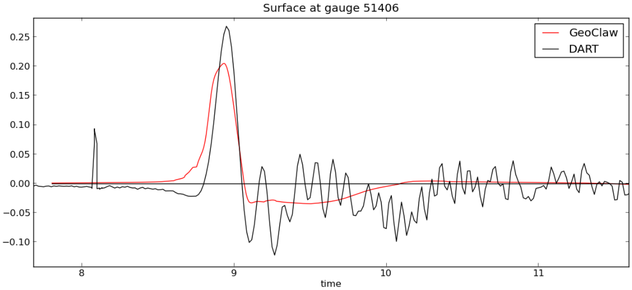
Best fit from unit earthquakes



Best fit with constraint that all coefficients (dislocations) positive.

Response at DART 51406





CLAWPACK — www.clawpack.org

- Open source, 1d, 2d, 3d
- Originally f77 with Matlab graphics.
- Moving to f95 with Python.
- Adaptive mesh refinement.
- OpenMP and MPI.

User supplies:

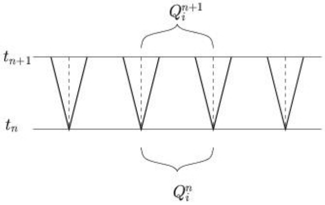
- Riemann solver, splitting data into waves and speeds
  (Need not be in conservation form)
- Boundary condition routine to extend data to ghost cells
  Standard `bc1.f` routine includes many standard BC's
- Initial conditions — `qinit.f`
- Source terms — `src1.f`

Options for using Clawpack

1. Install from tar file or Subversion: **Instructions**.
   Requires some **prerequisites:** Fortran, Python modules.
2. Use the **VirtualClaw** virtual machine.
3. For some applications, use **EagleClaw**
   (Easy Access Graphical Laboratory for Exploring Conservation Laws)

Also perhaps useful:
Class notes on Python, Fortran, version control, etc.

Godunov's Method for $q_t + f(q)_x = 0$



1. Solve Riemann problems at all interfaces, yielding waves $\mathcal{W}_{i-1/2}^p$ and speeds $s_{i-1/2}^p$, for $p = 1,\ 2,\ \ldots,\ m$.

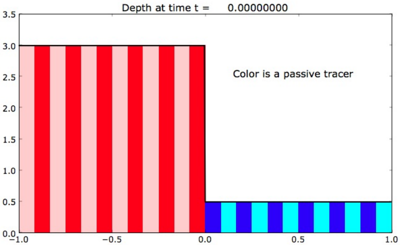
Riemann problem: Original equation with piecewise constant data.

The Riemann problem

Dam break problem for shallow water equations

$$h_t + (hu)_x = 0$$

$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x = 0$$



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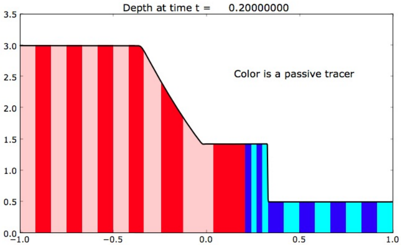

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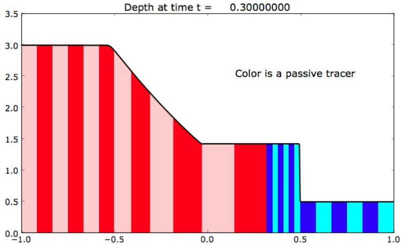

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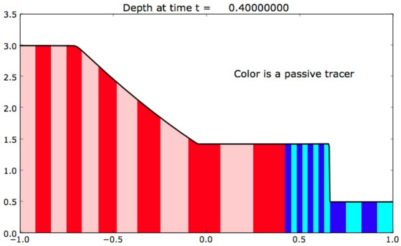

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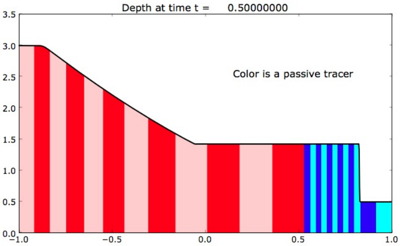
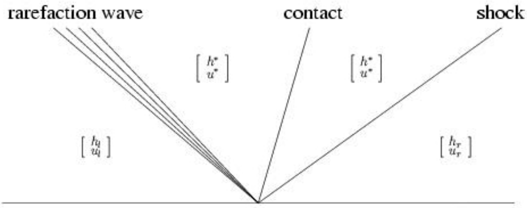


Riemann solution for the SW equations



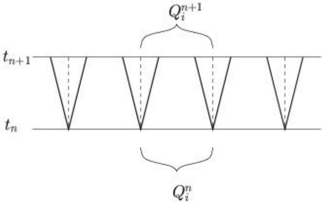
The Roe solver uses the solution to a linear system

$$q_t + \hat{A}_{i-1/2} q_x = 0, \qquad \hat{A}_{i-1/2} = f'(q_{\text{ave}}).$$

All waves are simply discontinuities.

Typically a fine approximation if jumps are approximately correct.

Godunov's Method for $q_t + f(q)_x = 0$



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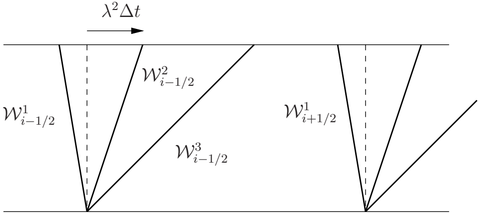
Riemann problem: Original equation with piecewise constant data.

Wave-propagation viewpoint

For linear system $q_t + Aq_x = 0$, the Riemann solution consists of

waves $\mathcal{W}^p$ propagating at constant speed λ^p.



$$Q_i - Q_{i-1} = \sum_{p=1}^{m} \alpha^p_{i-1/2} r^p \equiv \sum_{p=1}^{m} \mathcal{W}^p_{i-1/2}.$$

$$Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x}\left[\lambda^2\mathcal{W}^2_{i-1/2} + \lambda^3\mathcal{W}^3_{i-1/2} + \lambda^1\mathcal{W}^1_{i+1/2}\right].$$

Upwind wave-propagation algorithm

$$Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x}\left[\sum_{p=1}^{m}(s_{i-1/2}^p)^+\mathcal{W}_{i-1/2}^p + \sum_{p=1}^{m}(s_{i+1/2}^p)^-\mathcal{W}_{i+1/2}^p\right]$$

where

$$s^+ = \max(s,0), \qquad s^- = \min(s,0).$$

Note: Requires only waves and speeds.

Applicable also to hyperbolic problems not in conservation form.

For $q_t + f(q)_x = 0$, conservative if waves chosen properly,
e.g. using Roe-average of Jacobians.

Great for general software, but only first-order accurate (upwind method for linear systems).

Wave-propagation form of high-resolution method

$$Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x}\left[\sum_{p=1}^{m}(s_{i-1/2}^p)^+\mathcal{W}_{i-1/2}^p + \sum_{p=1}^{m}(s_{i+1/2}^p)^-\mathcal{W}_{i+1/2}^p\right]$$
$$- \frac{\Delta t}{\Delta x}(\tilde{F}_{i+1/2} - \tilde{F}_{i-1/2})$$

Correction flux:

$$\tilde{F}_{i-1/2} = \frac{1}{2}\sum_{p=1}^{M_w}|s_{i-1/2}^p|\left(1 - \frac{\Delta t}{\Delta x}|s_{i-1/2}^p|\right)\widetilde{\mathcal{W}}_{i-1/2}^p$$

where $\widetilde{\mathcal{W}}_{i-1/2}^p$ is a limited version of $\mathcal{W}_{i-1/2}^p$ to avoid oscillations.

(Unlimited waves $\widetilde{\mathcal{W}}^p = \mathcal{W}^p \implies$ Lax-Wendroff for a linear system $\implies$ nonphysical oscillations near shocks.)

Summary of wave propagation algorithms

For $q_t + f(q)_x = 0$, the flux difference

$$\mathcal{A}\Delta Q_{i-1/2} = f(Q_i) - f(Q_{i-1})$$

is split into:

left-going fluctuation: $\mathcal{A}^-\Delta Q_{i-1/2}$, updates Q_{i-1},
right-going fluctuation: $\mathcal{A}^+\Delta Q_{i-1/2}$, updates Q_i,
Waves: $Q_i - Q_{i-1} = \sum \alpha^p r^p = \sum \mathcal{W}^p$
Often take $\mathcal{A}^\pm\Delta Q_{i-1/2} = \sum (s^p)^\pm \mathcal{W}^p$.

Summary of wave propagation algorithms

For $q_t + f(q)_x = 0$, the flux difference

$$\mathcal{A}\Delta Q_{i-1/2} = f(Q_i) - f(Q_{i-1})$$

is split into:

left-going fluctuation: $\mathcal{A}^-\Delta Q_{i-1/2}$, updates Q_{i-1},
right-going fluctuation: $\mathcal{A}^+\Delta Q_{i-1/2}$, updates Q_i,
Waves: $Q_i - Q_{i-1} = \sum \alpha^p r^p = \sum \mathcal{W}^p$
Often take $\mathcal{A}^\pm\Delta Q_{i-1/2} = \sum (s^p)^\pm \mathcal{W}^p$.

f-wave formulation: Bale, RJL, Mitran, Rossmanith, SISC 2002

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In either case, limiters are applied to waves or f-waves for use in high-resolution correction terms.

Incorporating source term in f-waves

$q_t + f(q)_x = \psi(q)\sigma_x(x)$

Concentrate source at interfaces: $\Psi_{i-1/2}(\sigma_i - \sigma_{i-1})$

Split $f(Q_i) - f(Q_{i-1}) - (\sigma_i - \sigma_{i-1})\Psi_{i-1/2} = \sum_p \mathcal{Z}^p_{i-1/2}$

Use these waves in wave-propagation algorithm.

Steady state maintained:

If $\frac{f(Q_i)-f(Q_{i-1})}{\Delta x} = \Psi_{i-1/2}\frac{(\sigma_i-\sigma_{i-1})}{\Delta x}$ then $\mathcal{Z}^p \equiv 0$

Near steady state:

Deviation from steady state is split into waves and limited.

Incorporating source term in f-waves

$$q_t + f(q)_x = \psi(q)\sigma_x(x) \implies \Psi_{i-1/2}(\sigma_i - \sigma_{i-1})$$

Question: How to average $\psi(q)$ between cells to get $\Psi_{i-1/2}$?

A Well-Balanced Path-Integral f-wave Method for Hyperbolic Problems with Source Terms , to appear in *J. Sci. Comput.*

For some problems (e.g. ocean-at-rest) can simply use arithmetic average.

$$\Psi_{i-1/2} = \frac{1}{2}(\psi(Q_{i-1}) + \psi(Q_i)).$$

Shallow water equations with bathymetry $B(x)$

$$h_t + (hu)_x = 0$$

$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x = -ghB_x(x)$$

Ocean-at-rest equilibrium:

$$u^e \equiv 0, \qquad h^e(x) + B(x) \equiv \bar{\eta} = \text{sea level.}$$

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$$\begin{aligned} f(Q_i) - f(Q_{i-1}) - \Psi_{i-1/2}(B_i - B_{i-1}) = \\ &= \left(\frac{1}{2}gh_i^2 - \frac{1}{2}gh_{i-1}^2\right) + \frac{g}{2}(h_{i-1} + h_i)(B_i - B_{i-1}) \\ &= \frac{g}{2}(h_{i-1} + h_i)((h_i + B_i) - (h_{i-1} + B_{i-1})) \\ &= 0 \quad \text{if } h_i + B_i = h_{i-1} + B_{i-1} = \bar{\eta}. \end{aligned}$$

Adaptive Mesh Refinement (AMR)

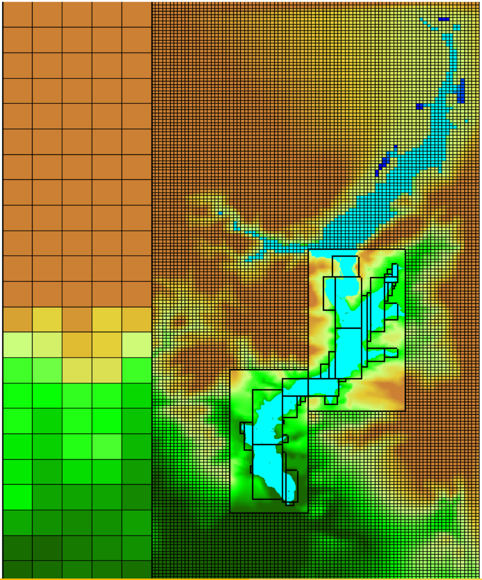
- Cluster grid points where needed
- Automatically adapt to solution
- Refined region moves in time-dependent problem

Basic approaches:

- Cell-by-cell refinement
  Quad-tree or Oct-tree data structure
  Structured or unstructured grid
- Refinement on "rectangular" patches
  Berger-Colella-Oliger style
  (AMRCLAW and CHOMBO-CLAW)

Nested AMR grids

Coarse: 400m cell side, Level 2: 50m, Level 3: 12m, Level 4: 3m

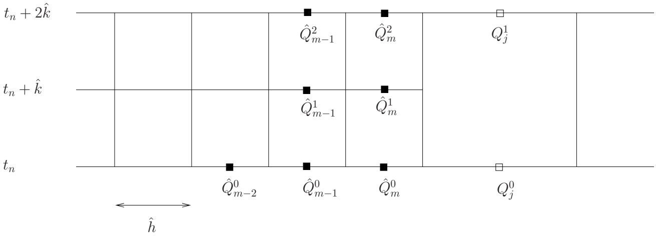


AMR Issues

- Refinement in time as well as space
- Conservation at grid interfaces
- Accuracy at interfaces, Spurious reflections?
- Refinement strategy, error estimation
- Clustering flagged points into rectangular patches

Time stepping algorithm for AMR

- Take 1 time step of length k on coarse grid with spacing h.
- Use space-time interpolation to set ghost cell values on fine grid near interface.
- Take L time steps on fine grid.
  $L =$ refinement ratio, $\hat{h} = h/L$, $\hat{k} = k/L$.
- Replace coarse grid value by average of fine grid values on regions of overlap — better approximation and consistent representations.
- Conservative fix-up near edges.



Flagging Cells for Refinement

Every `kcheck` time-steps at each level (except finest), check all grid cells and flag those needing refinement.

Use one or more of the following flagging criteria:

- Richardson estimation of truncation error.
  Compare result after last two time steps on this grid with one time step on a coarsened grid.
- Estimate spatial gradient of one or more components of solution.
- Check for regions where refinement is user-forced to some level.
- Problem-specific, e.g. near shore for tsunami simulation.
- Other user-supplied criterion set in `flag2refine.f`.

Clustering Flagged Cells for Refinement

Use Berger-Rigoutsos algorithm
[IEEE Trans. Sys. Man & Cyber.] 21(1991), p. 1278]

Clusters flagged points into a set of rectangular patches.

Tradeoff between:

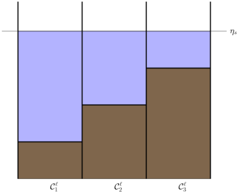
- Many small patches cover flagged points with minimal refinement of unflagged points.
- But.... increases overhead associated with each patch, e.g. boundary values: ghost cell values set by copying or interpolation from other grids,

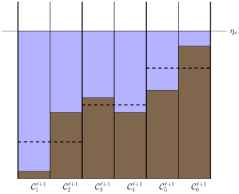
B-G algorithm has cut-off paramter: require that this fraction of refined cells be flagged (usually set to 0.7).

Refinement of topography

Topography should be consistent between different levels.

$$B_1^\ell = \frac{1}{2}(B_1^{\ell+1} + B_2^{\ell+1})$$





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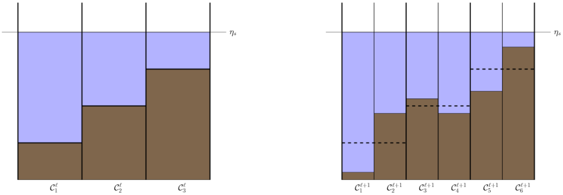
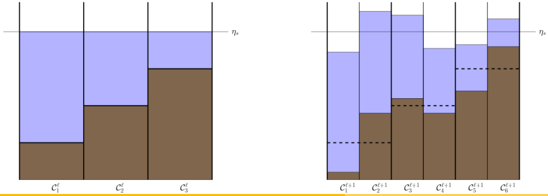


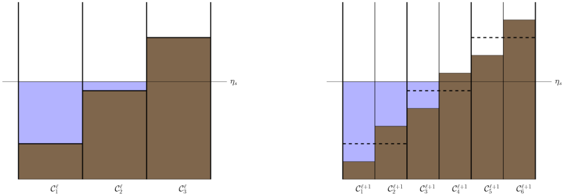
Important to interpolate surface, not depth, as in...



Inundation modeling

Refinement of topography near shore

Again need to maintain flat surface before wave arrives:



Mass cannot always be conserved!

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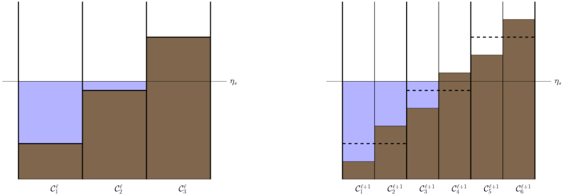

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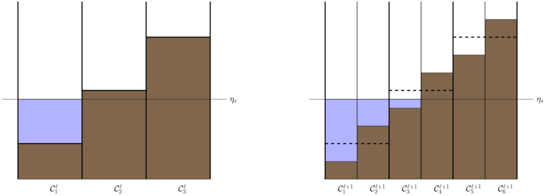
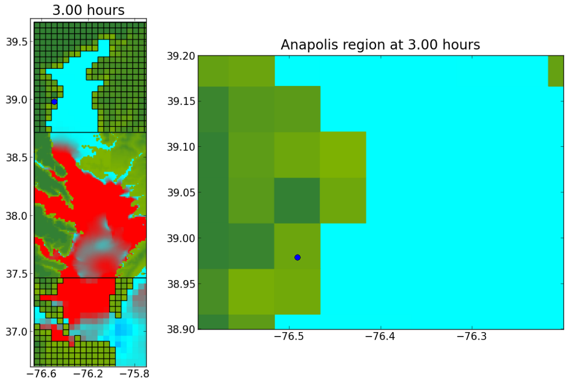


Chesapeake Bay and Anapolis

Cannot conserve mass when refining near shore!



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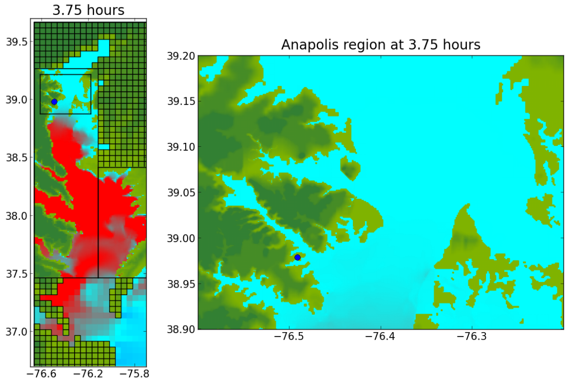
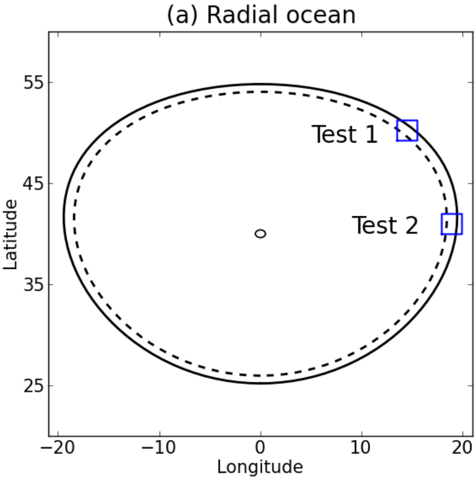


Radial ocean verification study



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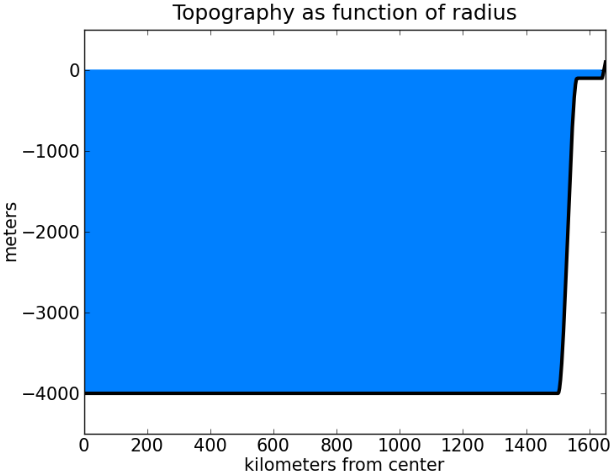

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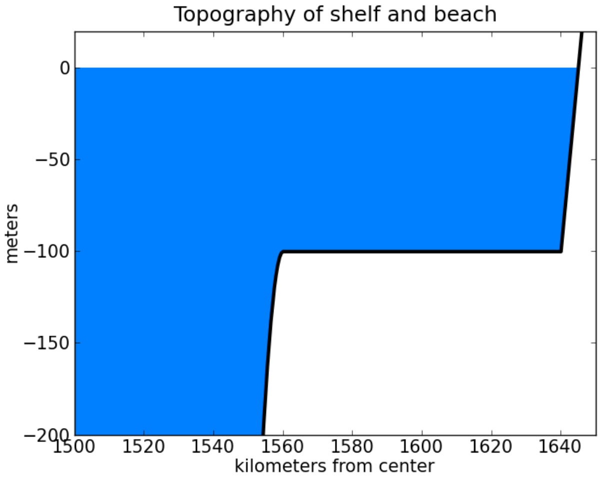

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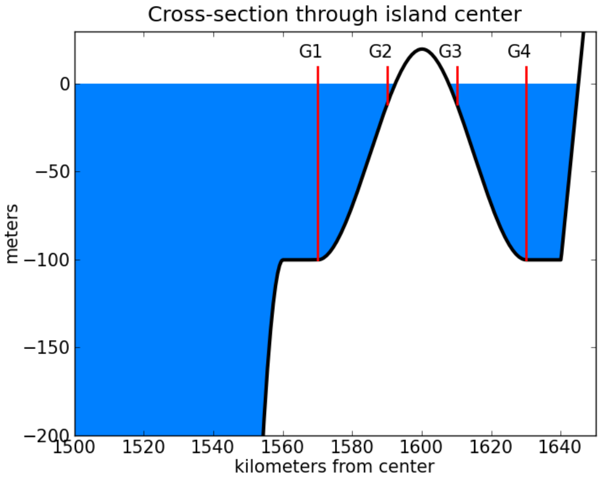

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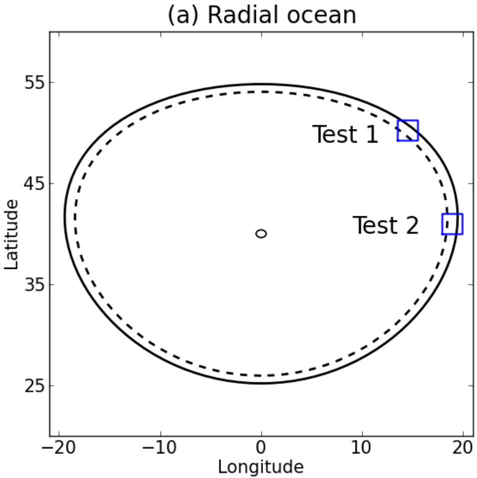
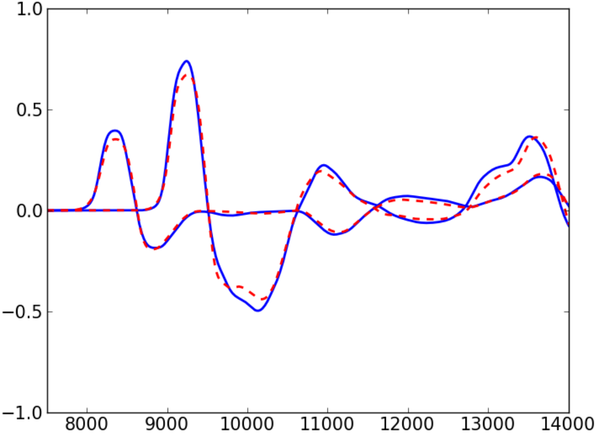


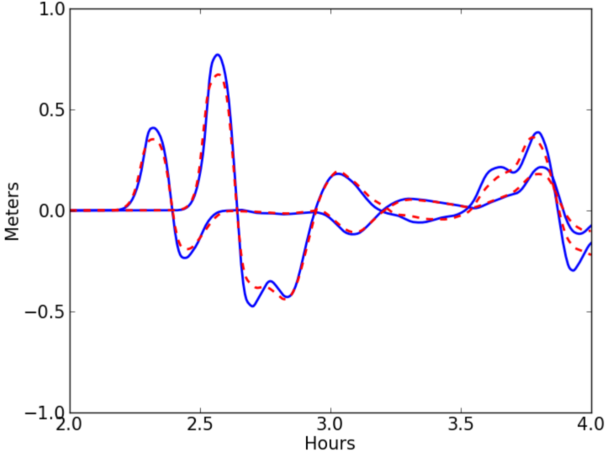
Radial ocean verification study

Comparison of Gauges 1 and 2 from Test 1 and 2:

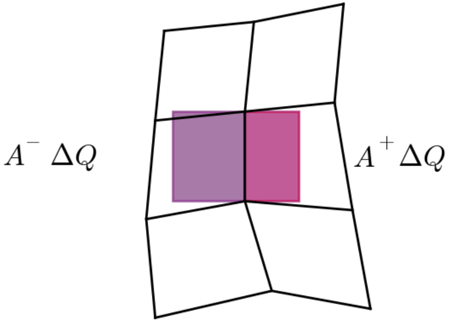


Radial ocean verification study

Comparison of Gauges 1 and 2 with more refined grids (Test 1):



Wave propagation algorithm on a quadrilateral grid



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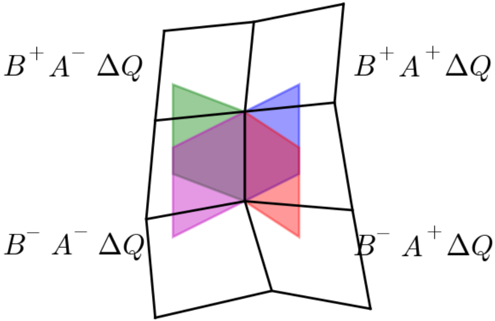
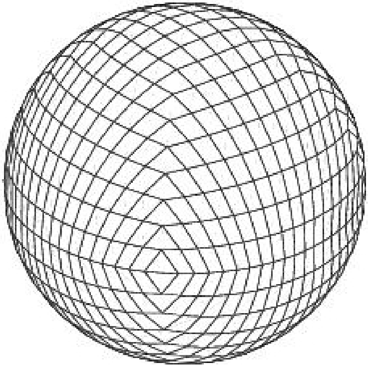
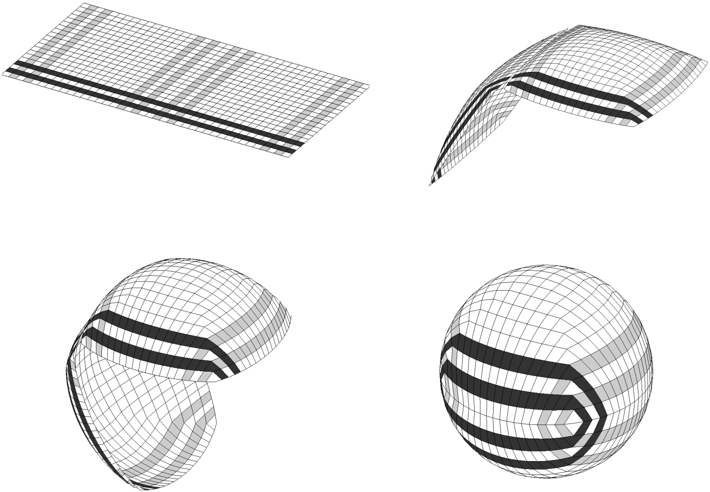


Wave propagation algorithm on a quadrilateral grid

This approach works very well, even in highly distorted cells.



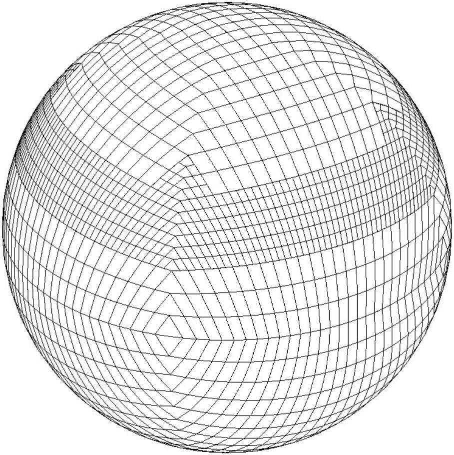
Mapping from rectangle to sphere



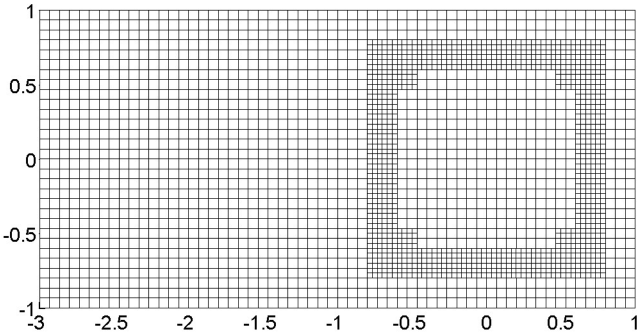
Shallow water on rotating sphere

Calhoun, Helzel, RJL, SIAM Review 2008 [link]

Berger, Calhoun, Helzel, RJL, Phil. Trans. R. Soc. A 2009 [link]

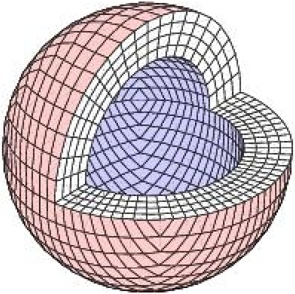


AMR on “rectangular” patches



Our approach for shells

Above approach can be used on sphere and then extended radially:



Useful for atmosphere, mantle convection,
volcanic ash plumes, etc.

Some references

- Clawpack: **www.clawpack.org**
- GeoClaw: **www.clawpack.org/geoclaw**
- Recent paper with references and codes:

  The GeoClaw software for depth-averaged flows with adaptive refinement,
  by M. J. Berger, D. L. George, RJL, and K. M. Mandli,

  www.clawpack.org/links/awr10/
  or... **arXiv:1008.0455v1**
- Paper for *Acta Numerica* in preparation,
  to appear.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