Multiphase Turbulent Flow

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Overview

- **Multiphase Flow Basics**
  - General Features and Challenges
  - Characteristics and definitions
- **Conservation Equations and Modeling Approaches**
  - Fully Resolved
  - Eulerian-Lagrangian
  - Eulerian-Eulerian
    - Averaging & closure
  - When to use what approach?
- **Preferential concentration**
- **Examples**
    - Modified instability of a Shear Layer
    - Sediment suspension in a turbulent channel flow
    - Numerical simulation example: Mesh-free methods in multiphase flow

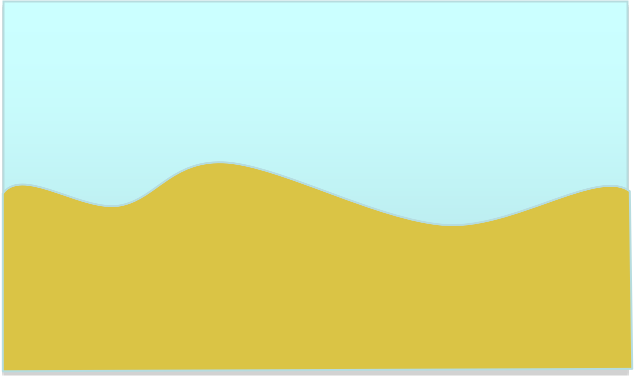
What is a multiphase flow?

- **In the broadest sense, it is a flow in which two or more phases of matter are dynamically interacting**
  - Distinguish multiphase and/or multicomponent
    - Liquid/Gas or Gas/Liquid
    - Gas/Solid
    - Liquid/Liquid
      - *Technically, two immiscible liquids are "multi-fluid", but are often referred to as a "multiphase" flow due to their similarity in behavior*

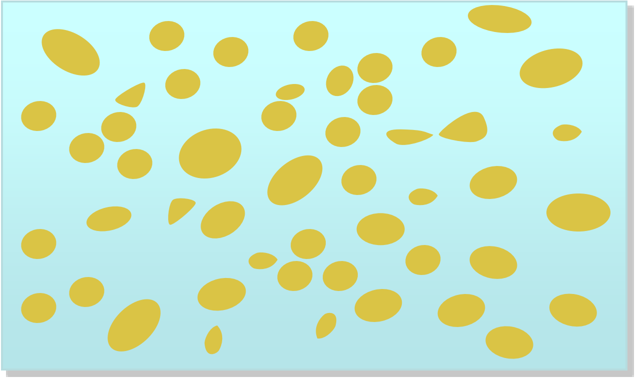
	Single component	Multi-component
Single phase	Water Pure nitrogen	Air H_20+oil emulsions
Multi-phase	Steam bubble in H_20 Ice slurry	Coal particles in air Sand particle in H_20

Dispersed/Interfacial

- **Flows are also generally categorized by distribution of the components**
  - "separated" or "interfacial"
    - both fluids are more or less contiguous throughout the domain



  - "dispersed"
    - One of the fluids is dispersed as non-contiguous isolated regions within the other (continuous) phase.
    - The former is the "dispersed" phase, while the latter is the "carrier" phase.
    - One can now describe/classify the geometry of the dispersion:
      - *Size & geometry*
      - *Volume fraction*



Gas-Liquid Flow

Bubbly Pipe Flow – heat exchangers in power plants, A/C units

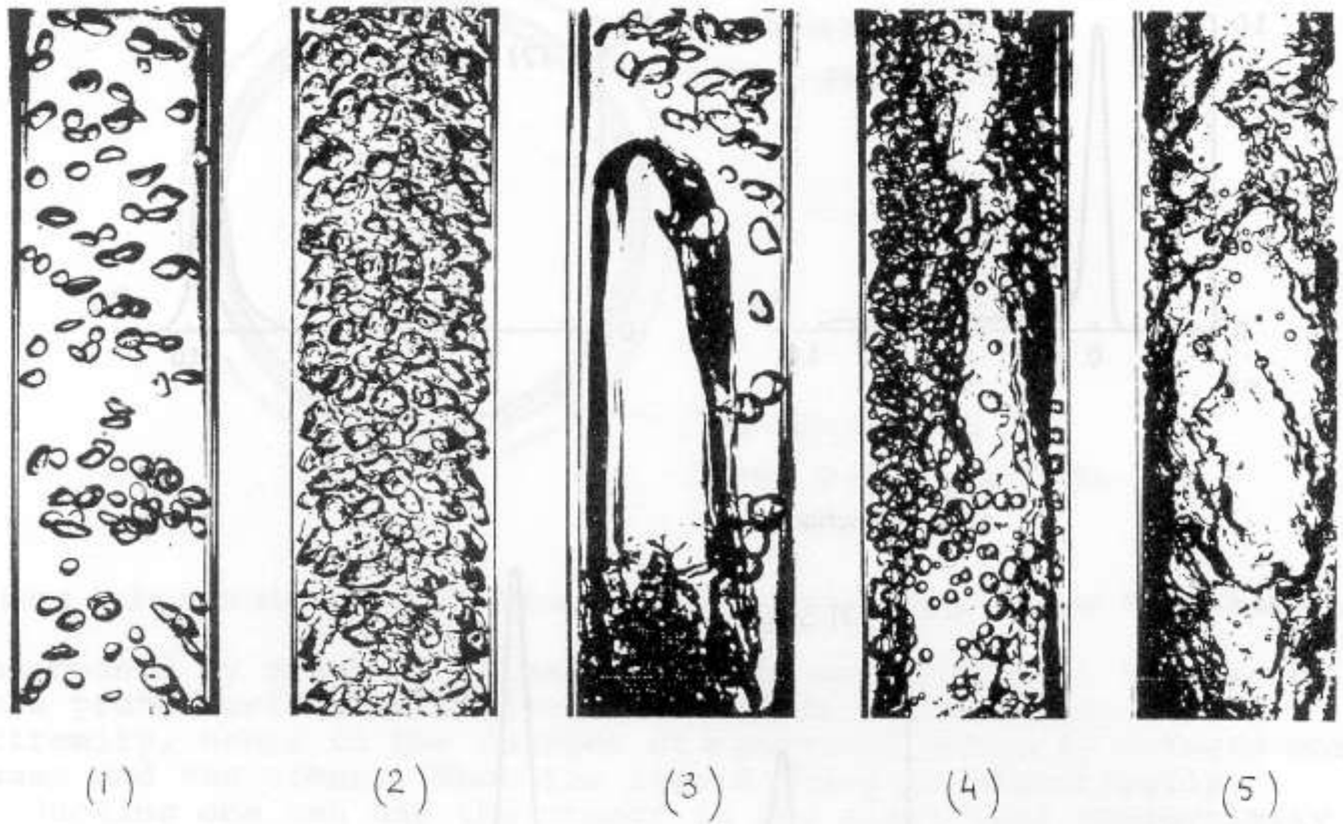
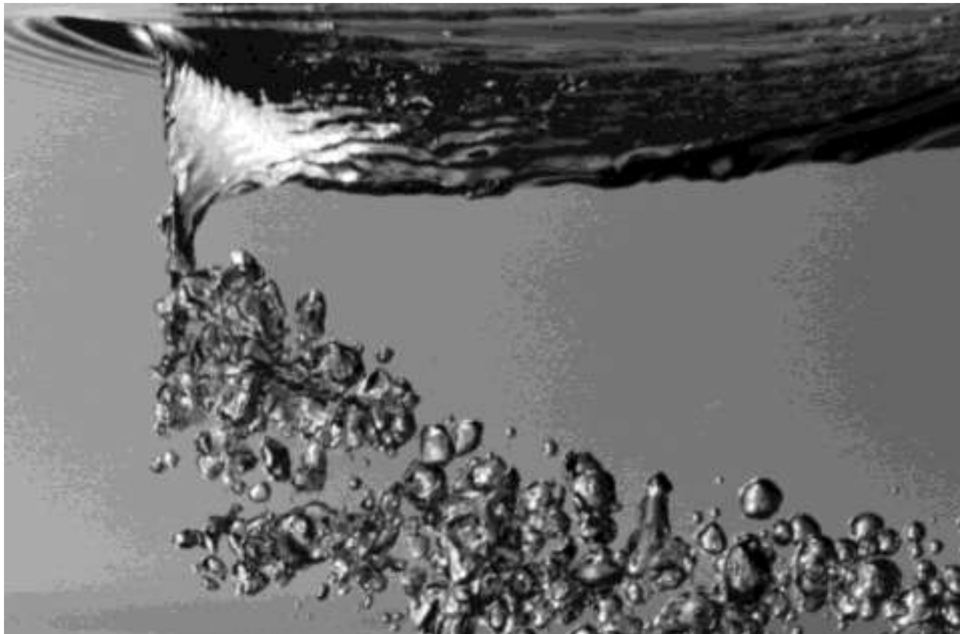


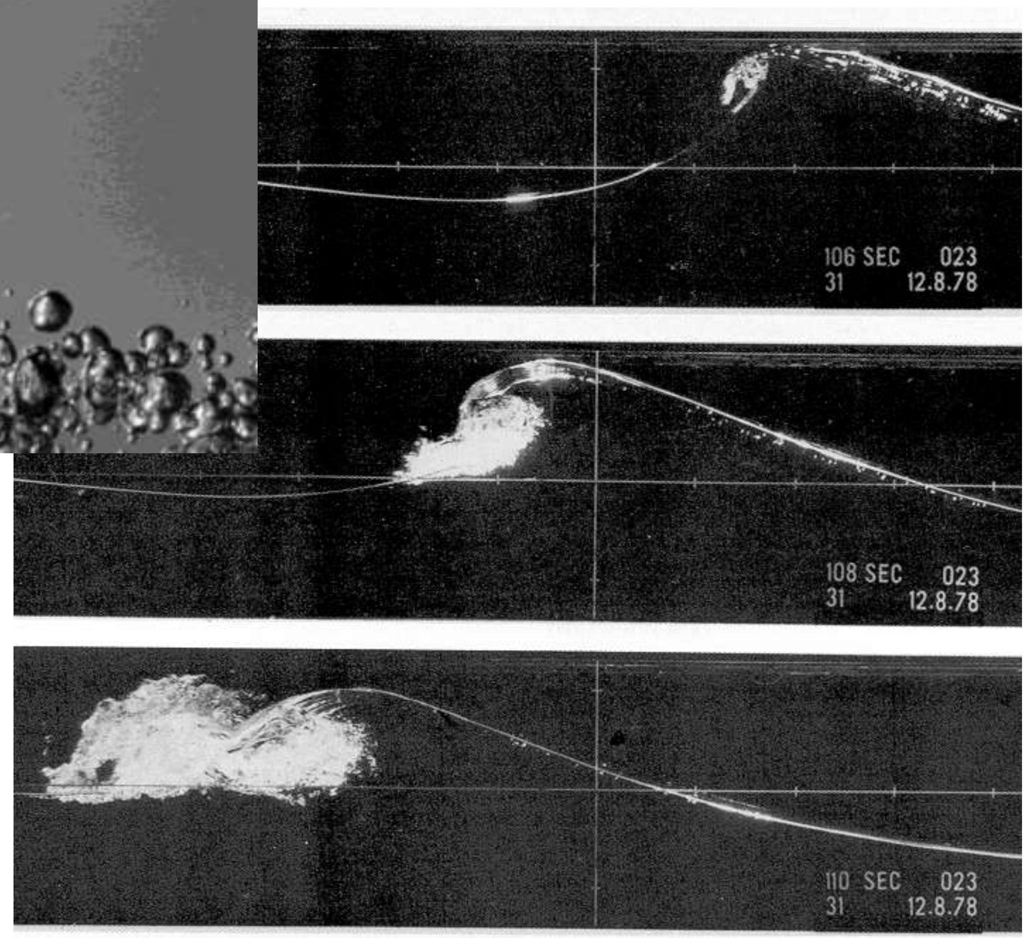
Figure 1.6: Upward Cocurrent Flow in a Vertical Pipe Air-water Flow Patterns (Roumy, 1969) (1) Independent bubbles, (2) Packed bubbles, (3) Slug flow, (4) Churn flow, (5) Annular flow. Pipe diameter : 32 mm

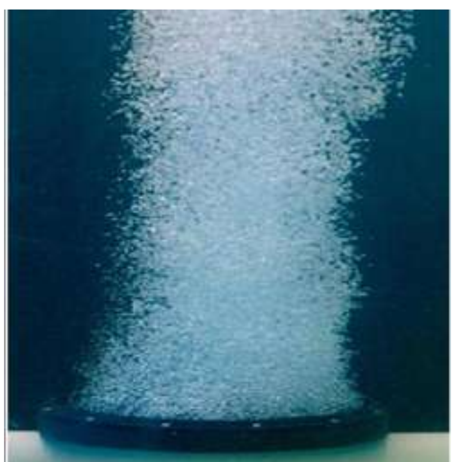
Gas-Liquid Flow (cont)



Aeration:

-produced by wave action
- used as reactor in chemical processing
- enhanced gas-liquid mass transfer



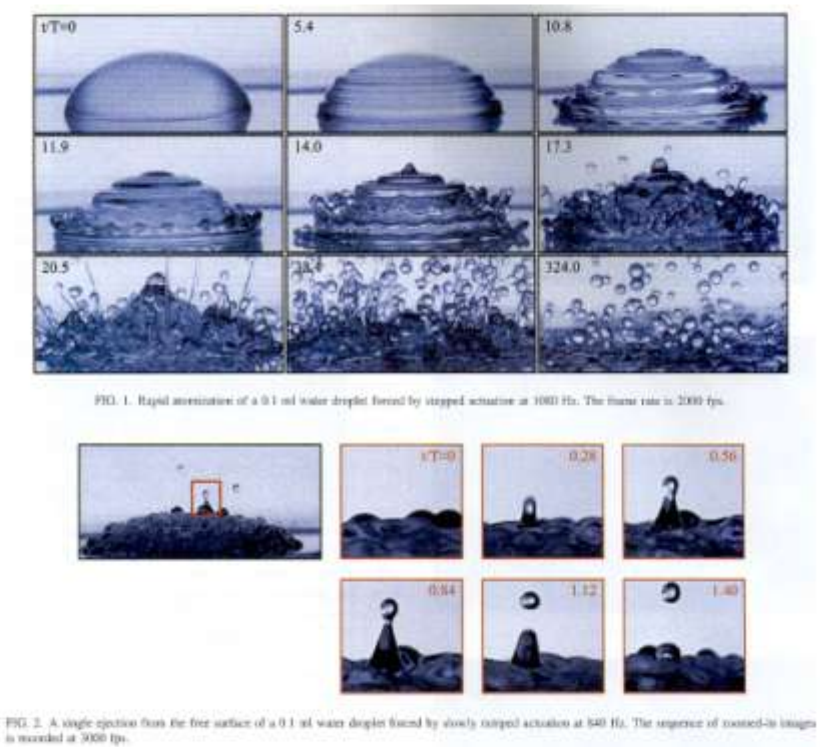


Gas-Liquid Flow (cont)

Ship wakes – detectability
Cavitation – noise, erosion of structures

Liquid-Gas Flow

Weather – cloud formation
Biomedical – inhalant drug delivery



Vukasinovic, Glezer, Smith (2000)

http://www.mywindpowersystem.com/2009/07/wind-power-when-nature-gets-angry-the-worst-wind-disasters-of-the-world/

Gas-Liquid Flow

Energy production – liquid fuel combustion
Biomedical – inhalant drug delivery

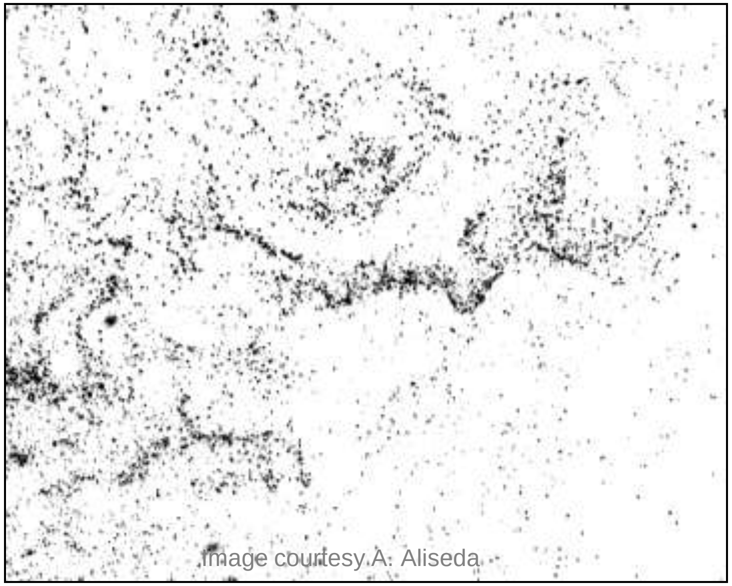
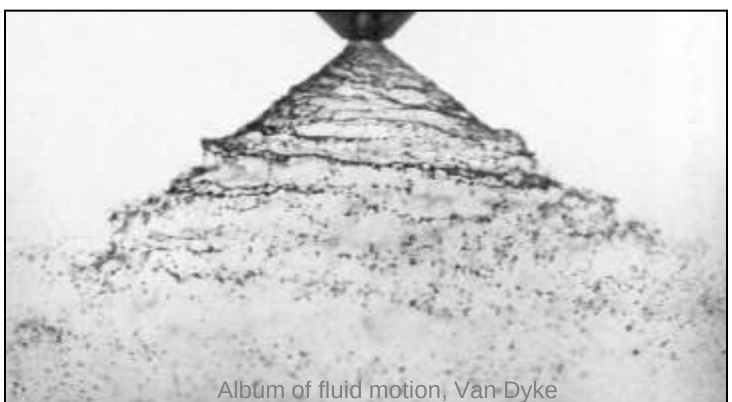
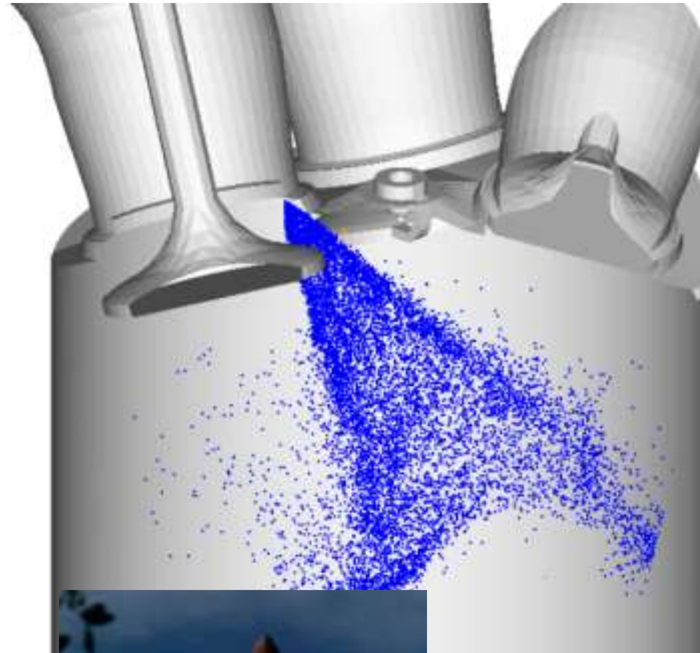


Image courtesy A. Aliseda


Album of fluid motion, Van Dyke

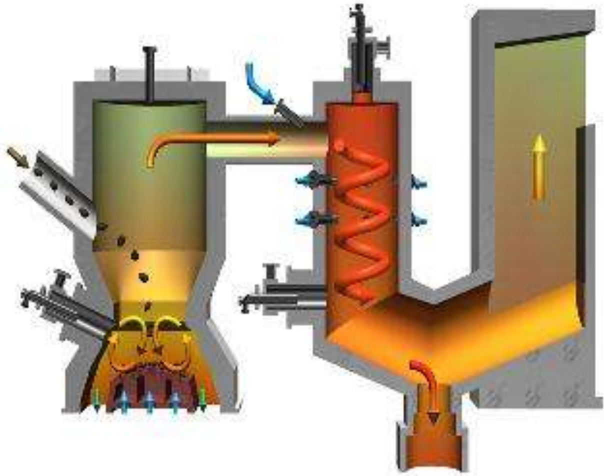

http://convergecfd.com/applications/engine/sparkignited/

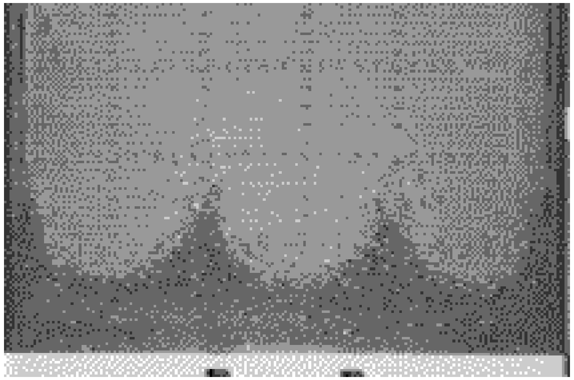
Gas-Solid Flow

Environmental – avalanche, pyroclastic flow, ash plume, turbidity currents

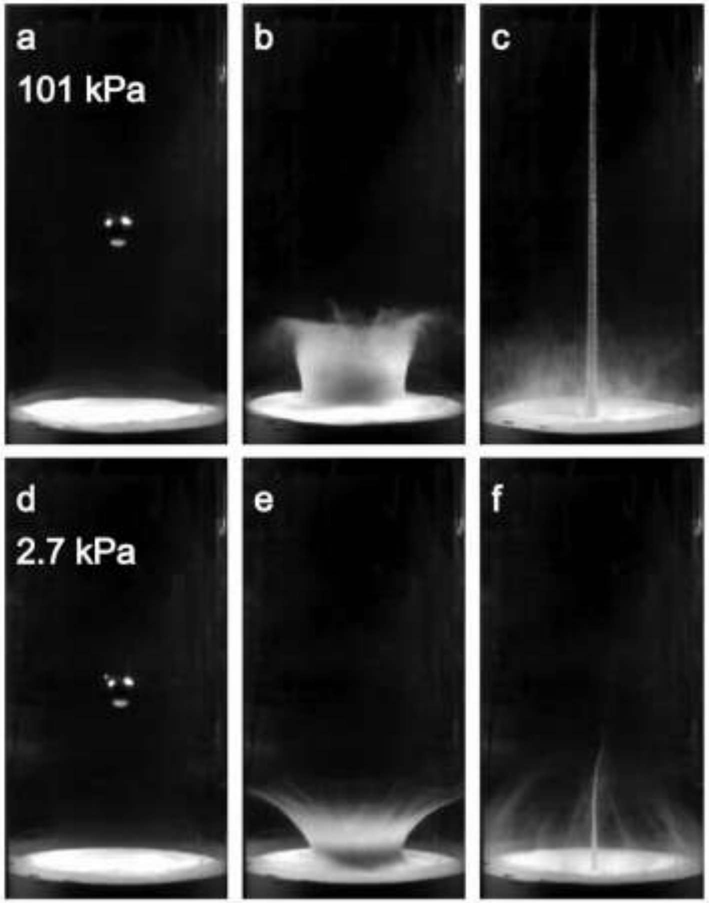
Gas-Solid (dense)

Granular Flow – collision dominated dynamics; chemical processing





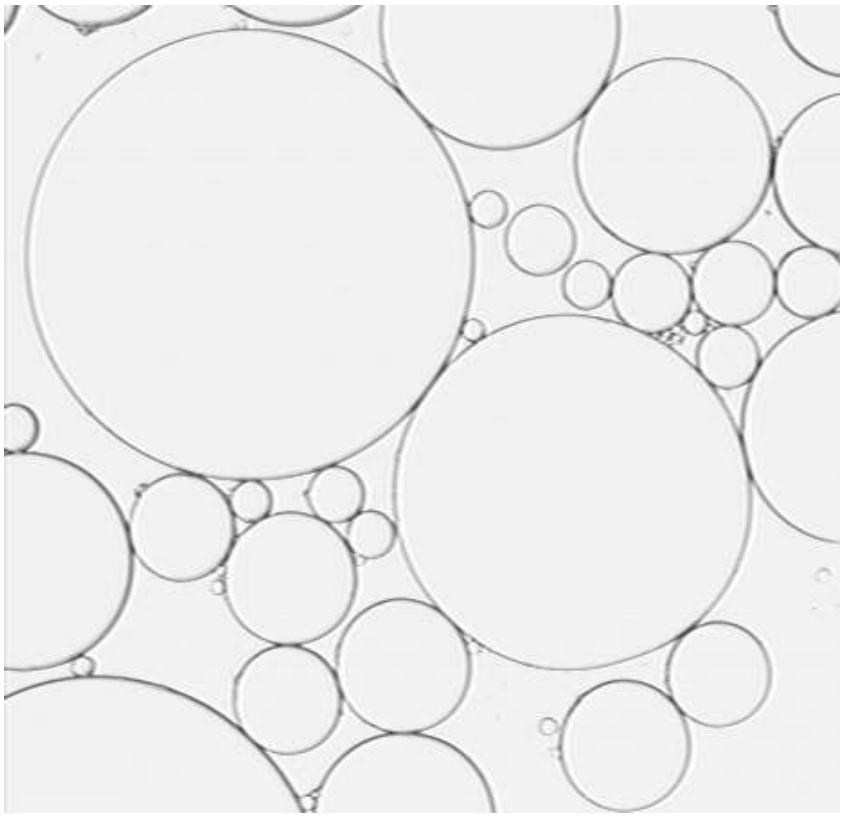
http://www.its.caltech.edu/~granflow/homepage.html



http://jfi.uchicago.edu/~jaeger/group/

Liquid-Liquid

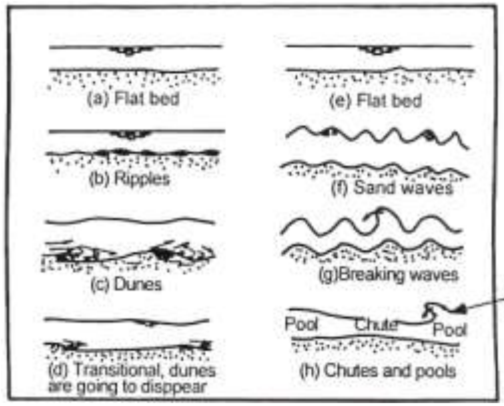
Chemical production – mixing and reaction of immiscible liquids

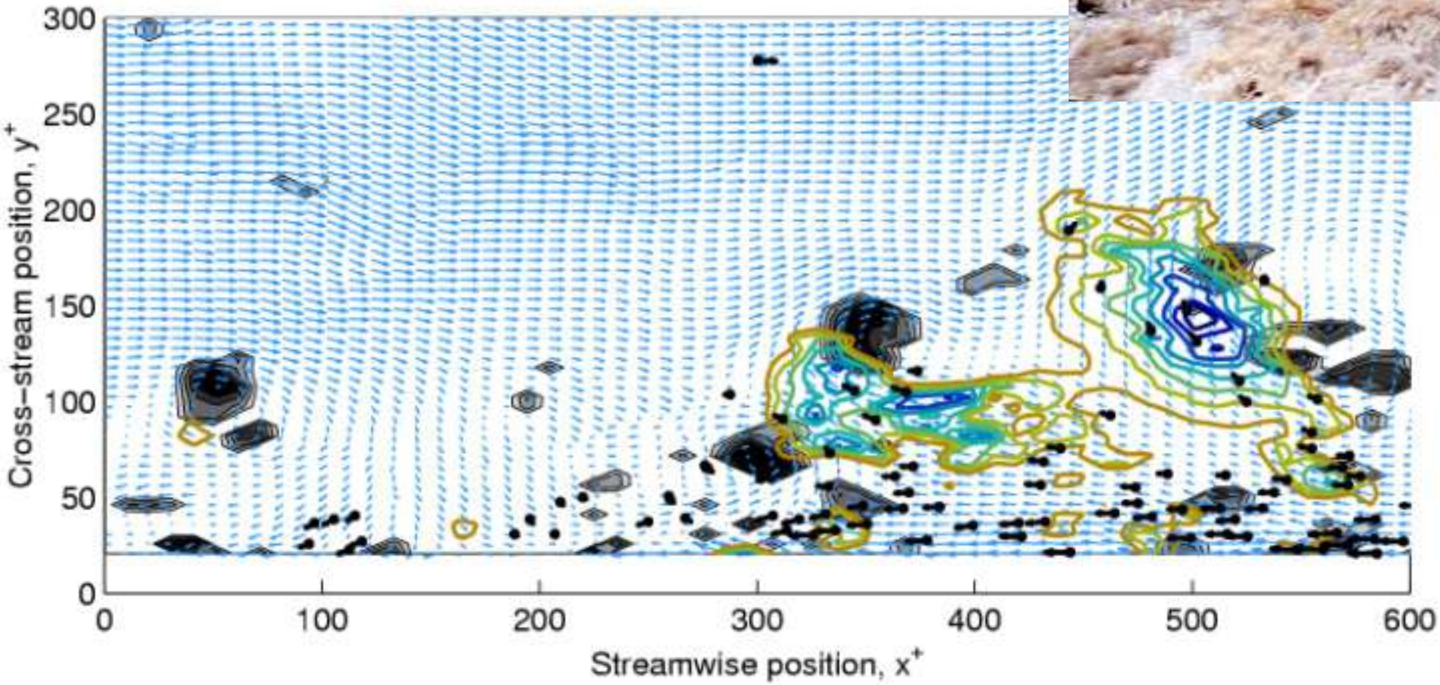


http://www.physics.emory.edu/students/kdesmond/2DEmulsion.html

Solid-Liquid

Sediment Transport – pollution, erosion of beaches, drainage and flood control





Solid-Liquid

Settling/sedimentation, turbidity currents

http://www.physics.utoronto.ca/~nonlin/turbidity/turbidity.html

FIG. 1. Visualization of fluid vorticity (red) and solid particles (white) of an initially spherical suspension falling due to gravity. Case A: (a)–(c); Case B: (d)–(f).

Solid-Gas

Material processing – generation of particles & composite materials
Energy production – coal combustion

Solid-Gas

Aerosol formation – generation of particles & environmental safety

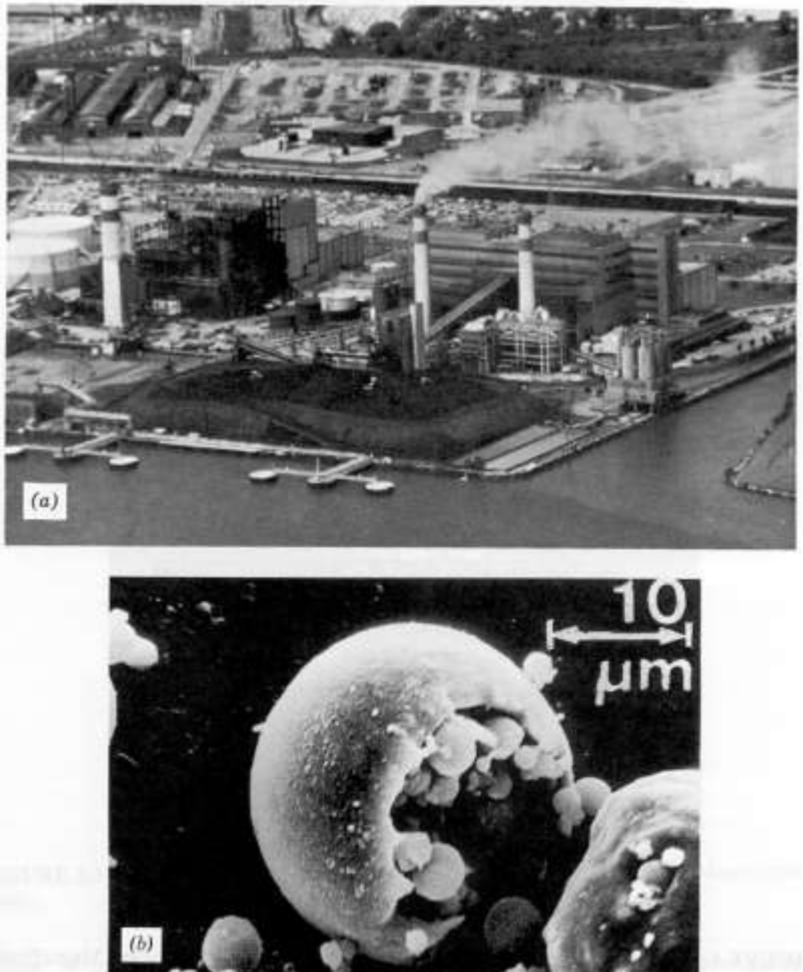


FIGURE 1.1 (*a*) Coal-burning power plant. (*b*) Scanning electron microscope (SEM) photograph of coal fly ash particles.

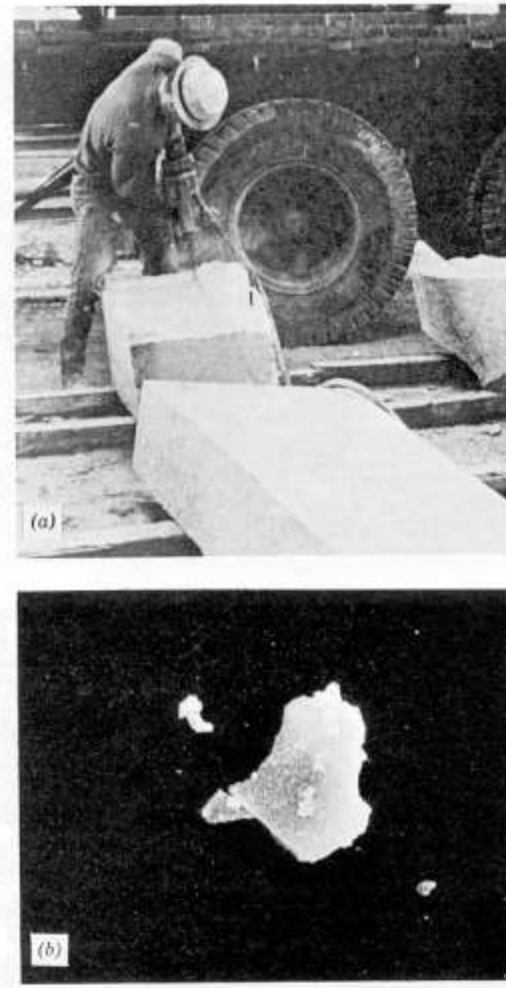
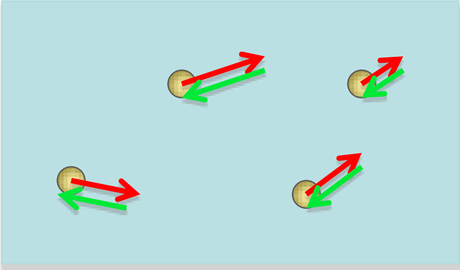


FIGURE 1.2 (*a*) Granite cutting. (*b*) SEM photograph of quartz particles. Magnification 2650×.

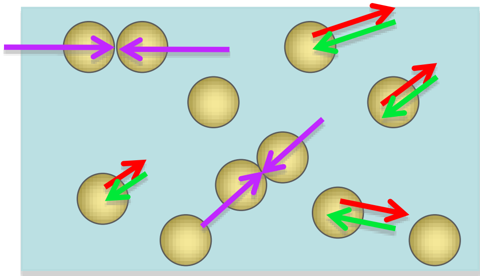
Classification by regime

- **Features/challenges**
  - Dissimilar materials (density, viscosity, etc)
  - Mobile and possibly stochastic interface boundary
  - Typically turbulent conditions for bulk motion
- **Coupling**
  - **One-way coupling:** Sufficiently dilute such that fluid feels no effect from presence of particles. Particles move in dynamic response to fluid motion.

  - **Two-way coupling:** Enough particles are present such that momentum exchange between dispersed and carrier phase interfaces alters dynamics of the carrier phase.



  - **Four-way coupling:** Flow is dense enough that dispersed phase collisions are significant momentum exchange mechanism
    - *Depends on particle size, relative velocity, volume fraction*



Viscous response time

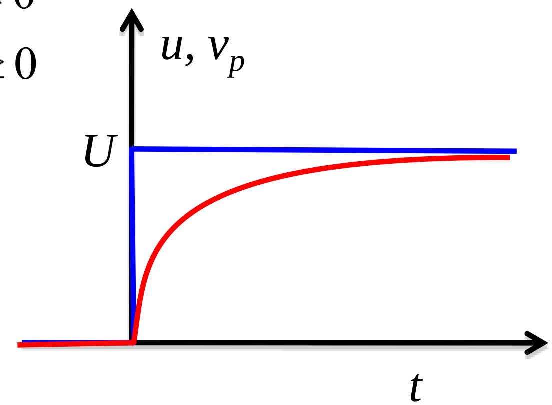
- **To first order, viscous drag is usually the dominant force on the dispersed phase**

$$m_p \frac{d\vec{v}_p}{dt} = 3\pi\mu D\left(\vec{u} - \vec{v}_p\right)$$

$$u = \begin{cases} 0 \text{ for } t < 0 \\ U \text{ for } t \geq 0 \end{cases}$$

$$\frac{dv_p}{dt} = \frac{18\mu}{\rho_p D^2}\left(U - v_p\right) = \frac{1}{\tau_p}\left(U - v_p\right)$$

$$v_p = U\left[1 - \exp\left(-t/\tau_p\right)\right]$$



  - This defines the typical particle “viscous response time”

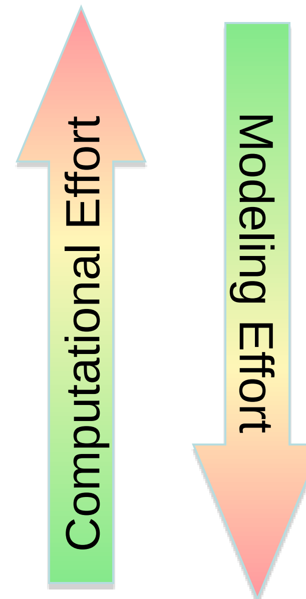
$$\tau_p = \frac{\rho_p D^2}{18\mu}$$

    - Can be altered for finite Re drag effects, added mass, etc. as appropriate

- **Stokes number:**
  - ratio of particle response time to fluid time scale: $St = \dfrac{\tau_p}{\tau_f}$

Modeling approach?

- **How to treat such a wide range of behavior?**
  - A single approach has not proved viable
    - Fully Resolved : complete physics
    - Eulerian-Lagrangian : idealized isolated particle
    - Eulerian-Eulerian : two co-existing fluids



Fully Resolved Approach

- **Solve conservation laws in coupled domains**
  1. separate fluids
     - Each contiguous domain uses appropriate transport coefficients
     - Apply boundary jump conditions at interface
     - Boundary is moving and may be deformable
  2. single fluid with discontinuous properties
     - Boundary becomes a source term

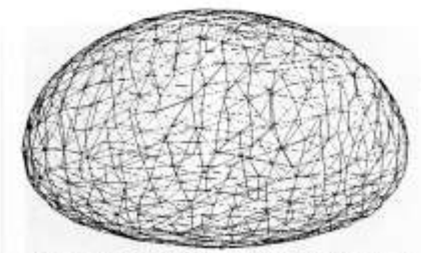
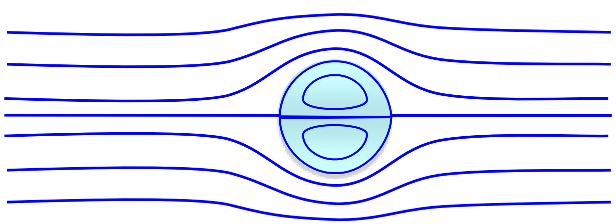
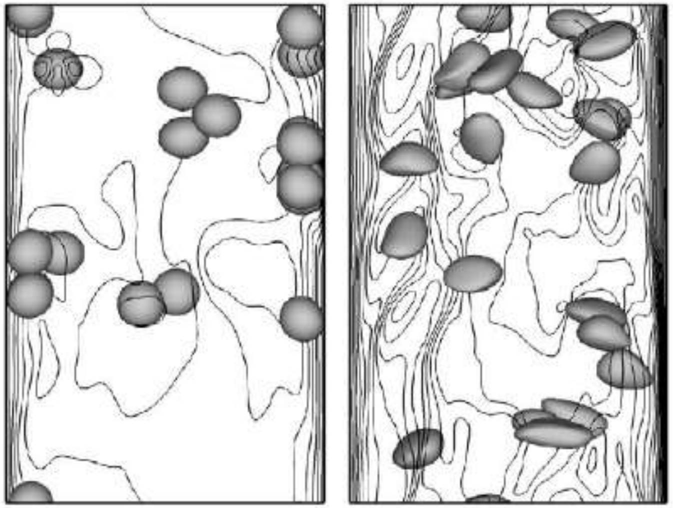


FIG. 11. The interface grid for a rising bubble corresponding to the last time in Fig. 10a.

- **Examples**
  - Stokes flow of single liquid drop
    - Simple analytical solution



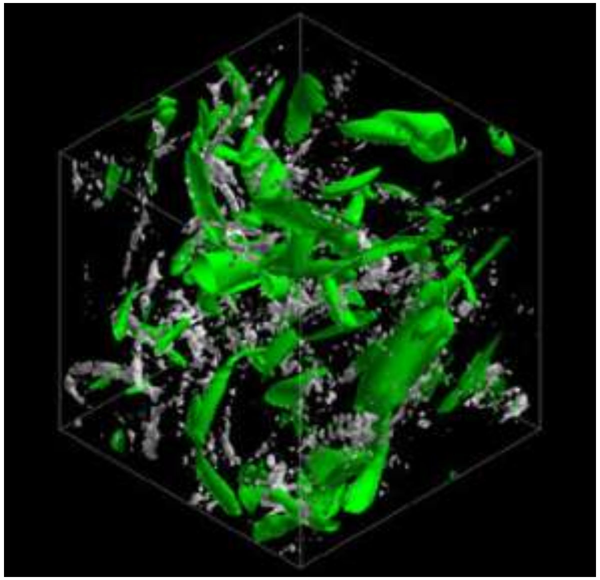
  - Small numbers of bubbles/drop
    - Quiescent or weakly turbulent flow



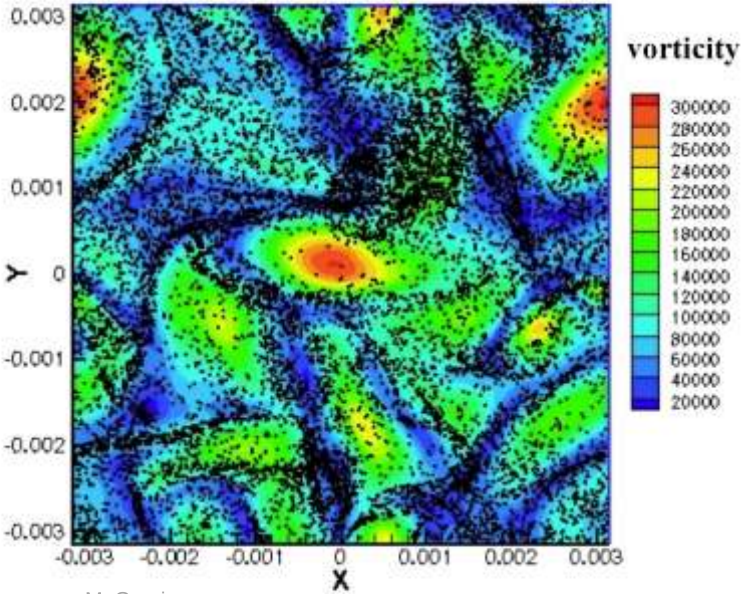
G Tryggvason, S Thomas, J Lu, B Aboulhasanzadeh (2010)

Eulerian-Lagrangian

- **Dispersed phase tracked via individual particles**
  - Averaging must be performed to give field properties
    - (concentration, average and r.m.s. velocity, etc.)
- **Carrier phase is represented as an Eulerian single fluid**
  - Two-way coupling must be implemented as distributed source term



Collins & Keswani (2004)



M. Garcia, http://www.cerfacs.fr/cfd/FIGURES/IMAGES/vort_stokes2-MG.jpg

Particle Motion: tracer particle

- **Equation of motion for spherical particle at small Re_p:**

$$m_p \frac{d\vec{v}_p}{dt} = 3\pi\mu D\left(\vec{u} - \vec{v}_p\right) + \frac{1}{2} m_f \left[\frac{D\vec{u}}{Dt} - \frac{d\vec{v}_p}{dt}\right] + m_f \frac{D\vec{u}}{Dt} - m_p\left(1 - \frac{\rho_g}{\rho_p}\right)\vec{g}$$

Inertia **Viscous drag** **Added mass** **Pressure gradient** **buoyancy**

– Where

$m_p = \rho_p \frac{\pi D^3}{6}$, the particle mass

$m_f = \rho_f \frac{\pi D^3}{6}$, fluid mass of same volume as particle

$D =$ particle diameter

$\mu =$ fluid viscosity

$\vec{u} =$ fluid velocity

$\vec{v}_p =$ particle velocity

$\rho_g =$ fluid density

$\rho_p =$ particle material density

– Possible alterations:

- Finite Re_p drag corrections
- Influence of local velocity gradients (Faxen Corrections)
- Lift force (near solid boundary, finite Re_p)

Two-Fluid Equations

- **Apply averaging operator to mass and momentum equations**
  - Drew (1983), Simonin (1991)
    - Phase indicator function

$$\frac{\partial \chi_k}{\partial t} + u_{I,j}\frac{\partial \chi_k}{\partial x_j} = 0$$

    - Averaging operator
  - Assume no inter-phase mass flux, incompressible carrier phase
    - Mass

$$\frac{\partial}{\partial t}\left(\alpha_k \rho_k\right) + \frac{\partial}{\partial x_j}\left(\alpha_k \rho_k U_{k,j}\right) = 0$$

    - Momentum

$$\alpha_k \rho_k \left(\frac{\partial U_{k,i}}{\partial t} + U_{k,j}\frac{\partial U_{k,i}}{\partial x_j} \right) = -\alpha_k \frac{\partial P_1}{\partial x_i} + \alpha_k \rho_k g_i + \frac{\partial}{\partial x_j}\left(\alpha_k \tau_{k,ij}\right) - \frac{\partial}{\partial x_j}\left(\alpha_k \rho_k \left\langle u_i' u_j' \right\rangle_k\right) + I_{k,i}$$

Two-Fluid Equations (cont)

- **Interphase momentum transport**
  - For large particle/fluid density ratios, quasi-steady viscous drag is by far the dominant term
  - For small density ratios, additional force terms can be relevant
    - Added mass
    - Pressure term
    - Bassett history term
  - For sediment, $\rho_2/\rho_1 \sim 2.5 > 1$ (k =1 for fluid, k =2 for dispersed phase)
    - Drag still first order effect, but other terms will likely also contribute

$$\mathbf{I}_2 = -\mathbf{I}_1 = \left\langle X_k \rho_1 \frac{3 C_D}{4d} |\mathbf{v}_r| \mathbf{v}_r \right\rangle_2$$

$$\mathbf{v}_r = \mathbf{u}_2 - \mathbf{u}_1 \qquad C_D = \frac{24\left[1 + 0.15\,\mathrm{Re}_p^{0.687}\right]}{\mathrm{Re}_p} \qquad \mathrm{Re}_p = \frac{\rho_1 |\mathbf{v}_r| d}{\mu_1}$$

Closure requirements

- **Closure**
  - Closure is needed for:
    - Particle fluctuations
    - Particle/fluid cross-correlations
    - Fluid fluctuations
  - Historically, the earliest models used a gradient transport model
    - Shown to be inconsistent for many applications
  - Alternative: Provide separate evolution equation for each set of terms
    - Particle kinetic stress equation
    - Particle/fluid covariance equation
    - Fluid kinetic stress equation
      - *Also required for single-phase RANS models*
    - Also will require third-moment correlations models to complete the closure

Simpler Two-Fluid Models

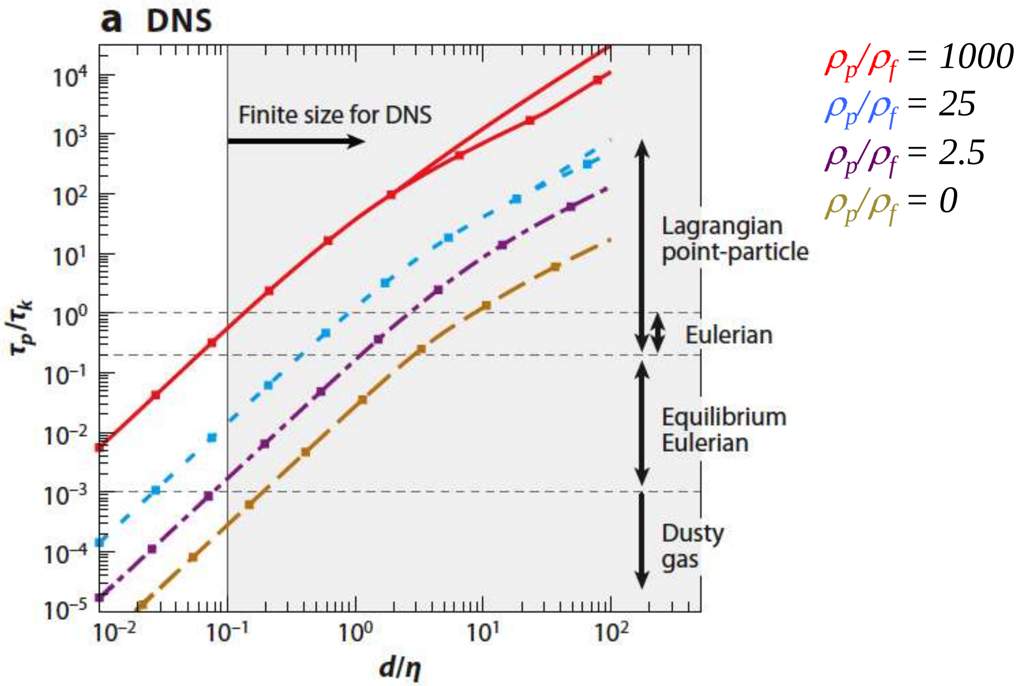
- **For St < 1, the particles tend to follow the fluid motion with greater fidelity**
  - Asymptotic expansions on the equation of motion lead to a closed expression for the particle field velocity, in terms of the local fluid velocity and spatial derivatives (Ferry & Balachandran, 2001)

$$\mathbf{v} = \mathbf{u} + \begin{cases} -St(1-\beta)\dfrac{D\mathbf{u}}{Dt} & \text{for} \quad |\mathbf{w}| \ll St \\ \mathbf{w} - St(1-\beta)\dfrac{D\mathbf{u}}{Dt} & \text{for} \quad |\mathbf{w}| \sim O(St) \\ \mathbf{w} - St\left[(1-\beta)\dfrac{D\mathbf{u}}{Dt} + \mathbf{w}\cdot\nabla\mathbf{u}\right] & \text{for} \quad |\mathbf{w}| \sim O(1). \end{cases}$$

  - Where $St = \dfrac{\tau_p}{\tau_f}$ $\quad \mathbf{w} = \tau_p \mathbf{g}$ $\quad \beta = \dfrac{3}{2\rho + 1}$
  - This is referred to as the "Eulerian Equilibrium" regime (Balachandar 2009).
    - Also, similar to "dusty gas" formulation by Marble (1970)
- **For larger St, the dispersed phase velocity at a point can be multivalued!**

- **When is a given approach best?**
  - What approach is best, depends on:
    - D/η: Particle size and fluid length scales (typically Kolmogorov)
    - τ_p/τ_f: Particle response time and fluid time scales
    - Total number of particles: Scale of system
    - α, $\Phi = \rho_p\alpha$: Loading of the dispersed phase (volume or mass fraction)

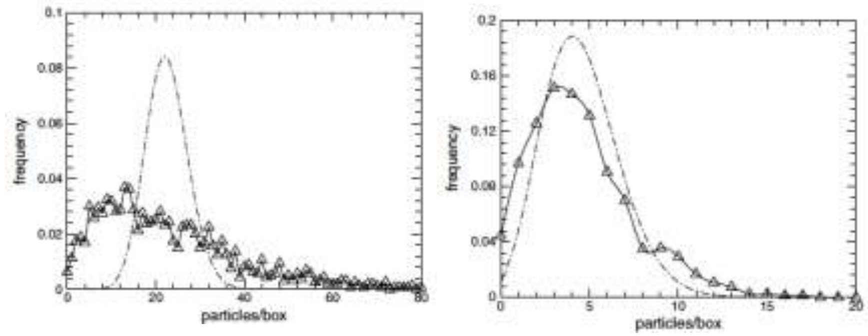
$$\frac{\tau_p}{\tau_f} = \frac{2\rho+1}{36}\frac{1}{\phi(\mathrm{Re})}\left(\frac{d}{\eta}\right)^2$$



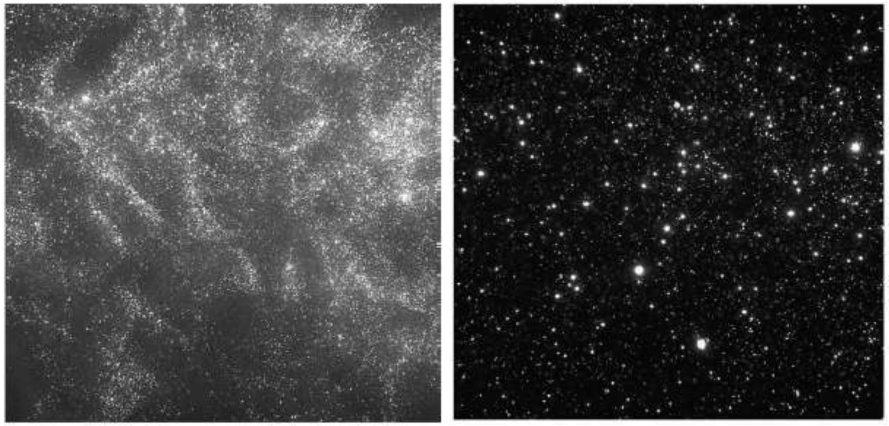
Balachandar (2009)

Preferential Concentration

- **From early studies, it was observed that inertial particles can be segregated in turbulent flows**
  - Heavy particles are ejected from regions of strong vorticity
  - Light particles are attracted to vortex cores



St = 1.33　　　　St = 8.1



- **Small *St* approx. shows trend**
  - Taking divergence of velocity…

$$\nabla \cdot \mathbf{u}_p = -St(1-\beta)\left[\|S_{ij}\|^2 - \|\Omega\|^2\right]$$

Wood, Hwang & Eaton (2005)

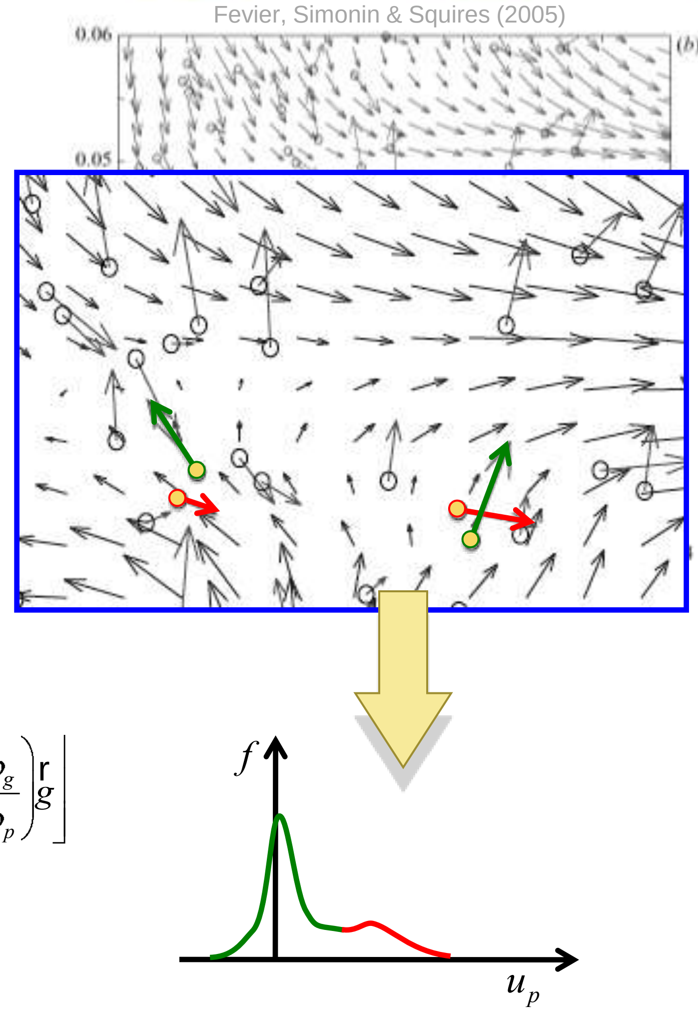
PDF formulation

- **Consequences of inertia**
  - Implies history of particle matters
  - Particles can have non-unique velocity
    - How can models account for this?
- **Probability distribution function**
  - $f(\mathbf{x},\mathbf{u},t)$ = phase space pdf

$$\frac{\partial f}{\partial t} = -\nabla_x \cdot \left(\mathbf{u}_p f\right) - \nabla_{u_p} \cdot \left(\mathbf{a}_p f\right)$$

$$\mathbf{u}_p = \frac{d\mathbf{x}_p}{dt} \qquad \nabla_x \cdot (\;) = \partial / \partial x_i \qquad \nabla_{u_p} \cdot (\;) = \partial / \partial u_{p,i}$$

$$\mathbf{a}_p = \frac{d\mathbf{u}_p}{dt} = \frac{1}{m_p}\left[3\pi\mu D\left(\vec{u} - \vec{v}_p\right) + \frac{1}{2} m_f \left[\frac{D\vec{u}}{Dt} - \frac{d\vec{v}_p}{dt}\right] + m_f \frac{D\vec{u}}{Dt} - m_p\left(1 - \frac{\rho_g}{\rho_p}\right)\vec{g}\right]$$



  - Instantaneous point quantities come as moments of the pdf over velocity phase space

$$n(\mathbf{x},t) = \int_{-\infty}^{+\infty} f\left(\mathbf{x}_\mathbf{p},\mathbf{u}_\mathbf{p},t\right) d\mathbf{u}_\mathbf{p} \qquad \hat{u}_p(\mathbf{x},t) = \int_{-\infty}^{+\infty} u_p f\left(\mathbf{x}_\mathbf{p},\mathbf{u}_\mathbf{p},t\right) d\mathbf{u}_\mathbf{p}$$

Examples

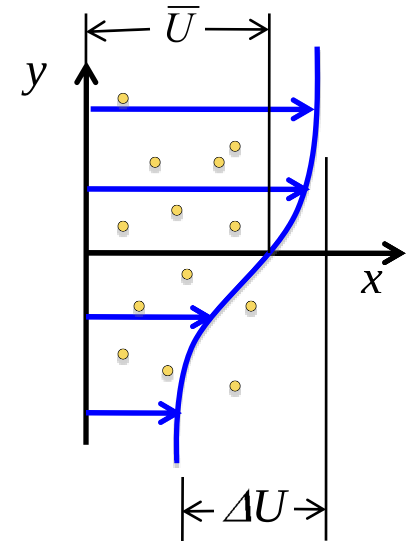
- **Effect of particles on shear layer instability**

- **Particle-Fluid Coupling in sediment transport**

- **Case studies in interface tracking methods**

Effect of particles on KH Instabilty

- **How does the presence of a dynamics dispersed phase influence the instability growth of a mixing layer?**



$$\frac{\partial u_{f_j}}{\partial x_j}=0,$$

$$\frac{\partial u_{f_i}}{\partial t}+u_{f_j}\frac{\partial u_{f_i}}{\partial x_j}=-\frac{\partial p}{\partial x_i}-\frac{\lambda}{2}\frac{\tau_f}{\tau_p}\gamma\alpha_p(u_{f_i}-u_{p_i}),$$

$$\frac{\partial \alpha_p}{\partial t}+\frac{\partial}{\partial x_j}(\alpha_p u_{p_j})=0,$$

$$\frac{\partial u_{p_i}}{\partial t}+u_{p_j}\frac{\partial u_{p_i}}{\partial x_j}=\frac{\lambda}{2}\frac{\tau_f}{\tau_p}(u_{f_i}-u_{p_i}),$$

$$\psi(x,y,t)=\phi(y)\exp[i(kx-\omega t)],$$

$$a_p'(x,y,t)=a_p(y)\exp[i(kx-\omega t)],$$

$$u_p'(x,y,t)=u_p(y)\exp[i(kx-\omega t)],$$

$$v_p'(x,y,t)=v_p(y)\exp[i(kx-\omega t)],$$

$$(kU-iC_f)\left(\frac{d^2\phi}{dy^2}-k^2\phi\right)-k\frac{d^2U}{dy^2}\phi+iC_f\left(\frac{du_p}{dy}-ikv_p\right)=\omega\left(\frac{d^2\phi}{dy^2}-k^2\phi\right),$$

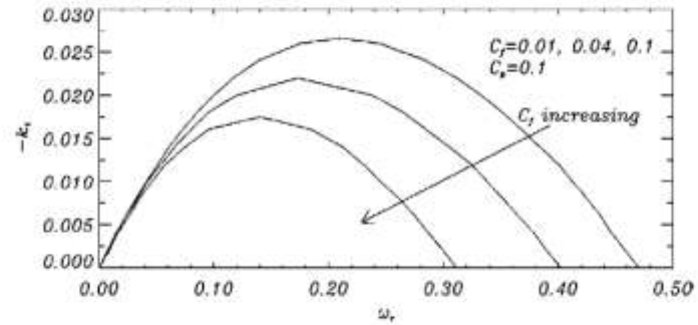
$$iC_p\frac{d\phi}{dy}+(kU-iC_p)u_p-i\frac{dU}{dy}v_p=\omega u_p,$$

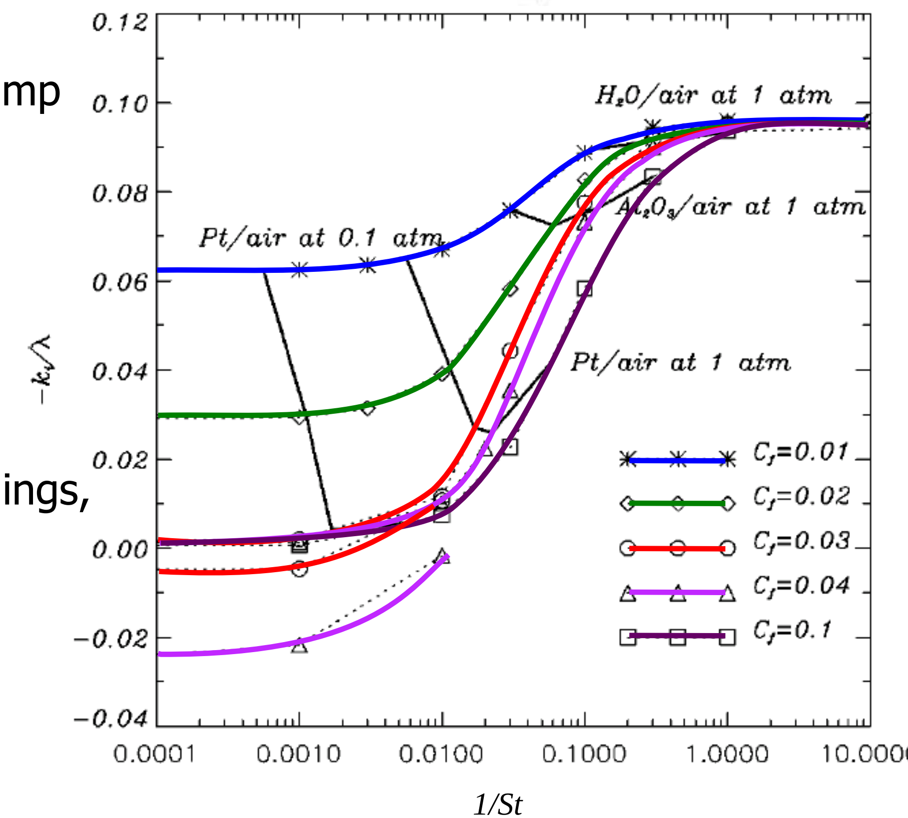
$$kC_p\phi+(kU-iC_p)v_p=\omega v_p,$$

$$a_p=\frac{kAu_p-iA\frac{dv_p}{dy}}{\omega-kU}.$$

Results

- **Effect of particles**
  - At small St, particles follow flow exactly, and there is no dynamic response. Flow is simply a heavier fluid.
  - As St is increased, dynamic slip becomes prevalent, and helps damp the instability
  - At large St, particles have no response to perturbation and are static
  - Effect is stronger, for higher loadings, but shear layer remains weakly unstable





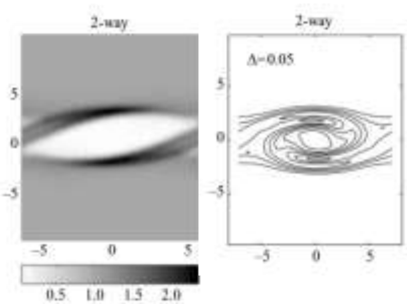
Mechanism

- **Particles damp instability**
  - Particles act as a mechanism to redistribute vorticity from the core back to the braid, in opposition to the K-H instability

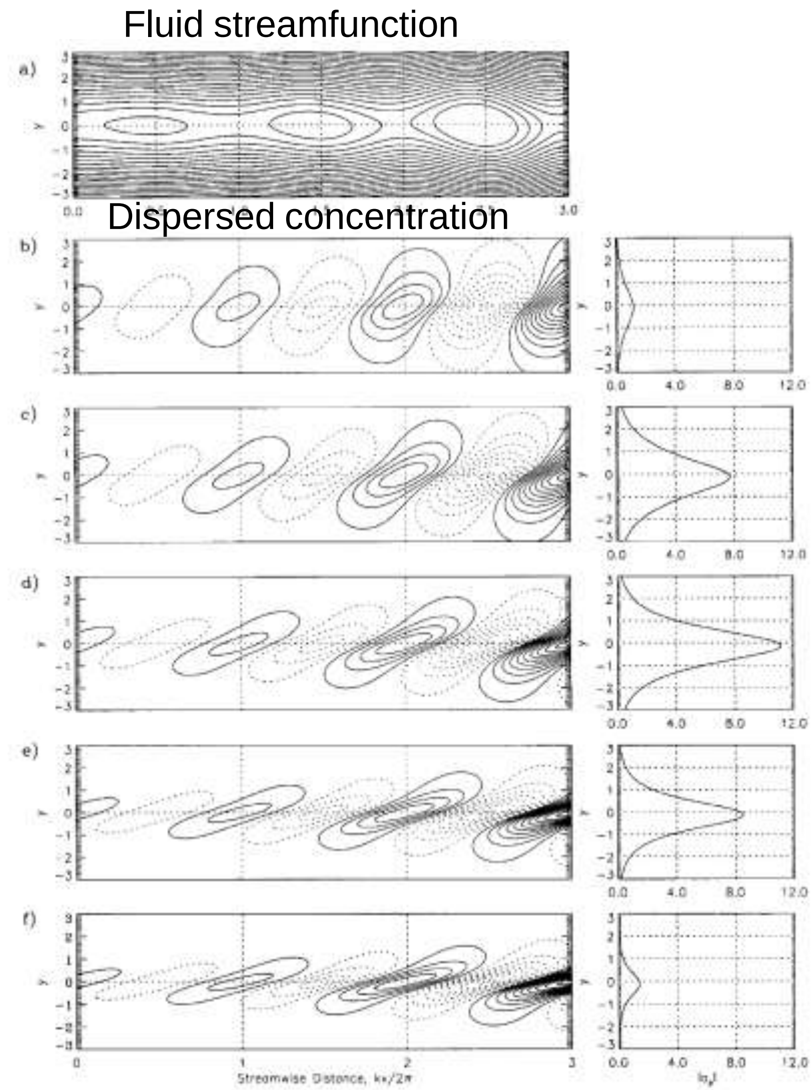
$$\nabla\cdot\mathbf{u}_p$$

$$\Omega|_{y=0} = -\frac{C_f\left(iku_p + \dfrac{dv_p}{dy}\right)}{(\omega - kU)(\omega - kU + i(C_f + C_p))}\frac{dU}{dy}$$

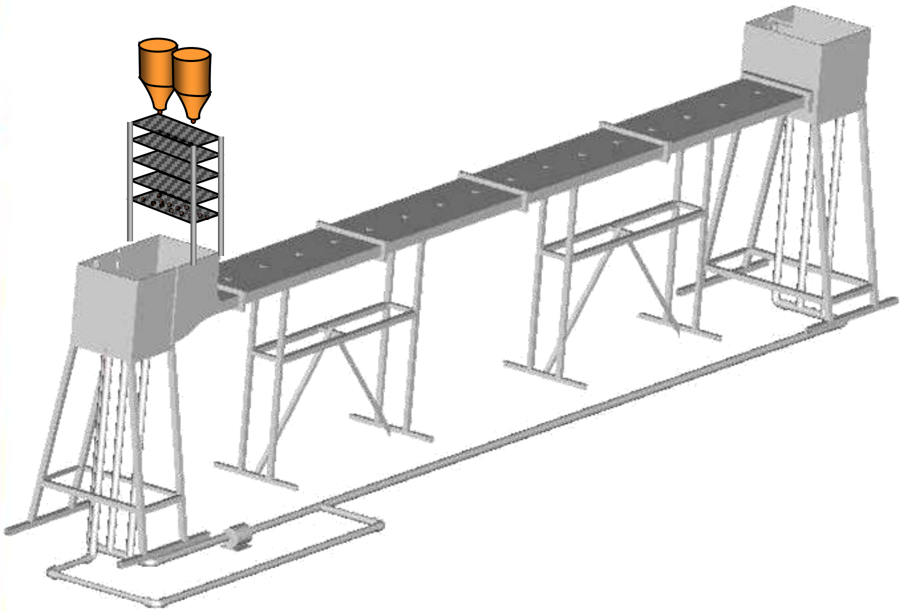
- **Limitations**
  - Results at large St do not capture effects of multi-value velocity

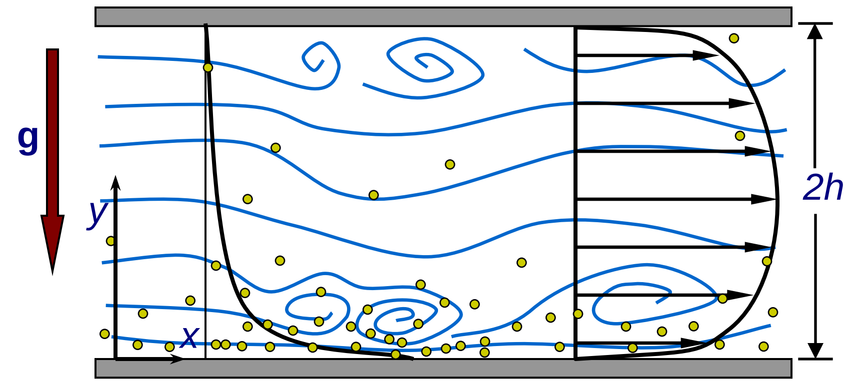


Meiburg et al. (2000)



Sediment Transport in Channel Flow





- **Planar Horizontal Water Channel**
  - $4 \times 36 \times 488$ cm, recirculating flow
  - Pressure gradient measurements show fully-developed by $x = 250$ cm
  - Particles introduce to settling chamber outlet across span

Experimental Conditions

- **Both single-phase and two-phase experiments conducted**
- **Carrier Fluid Conditions**
  - Water, $Q = 7.6$ l/s
  - $U_c = 59$ cm/s, $u_\tau = 2.8$ cm/s, $Re_\tau = 570$
  - Flowrate kept the same for two-phase experiments
  - Tracer particles: 10 μm silver-coated, hollow glass spheres, $SG = 1.4$

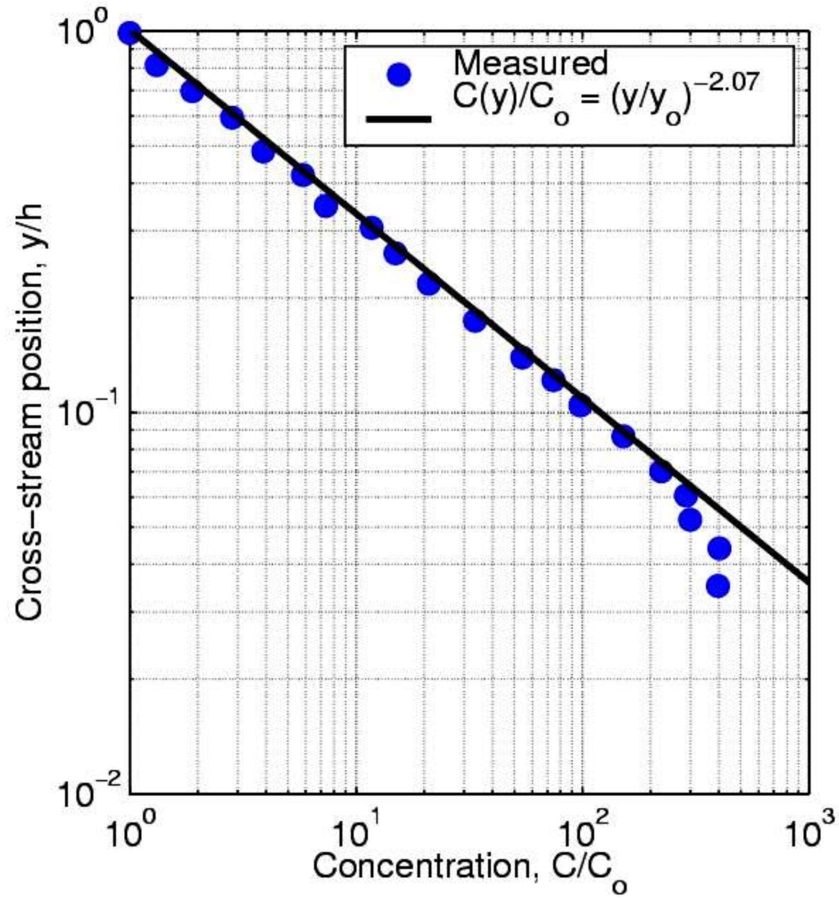
- **Dispersed Phase Conditions**
  - Glass beads: (specific gravity, $SG = 2.5$)
  - Standard sieve size range: $180 < D < 212$ μm
  - Settling velocity, $v_s = 2.2$ to 2.6 cm/s
  - Corrected Particle Response Time, $\tau_p = 4.5$ ms
  - $St^+ = \tau_p/\tau^+ \sim 4$
  - Bulk Mass Loading: $dM/dt = 4$ gm/s, $M_p/M_f \sim 5 \times 10^{-4}$
  - Bulk Volume Fraction, $\alpha = 2 \times 10^{-4}$

Mean Concentration Profile

– Concentration follows a power law
  - Equivalent to Rouse distribution for infinite depth

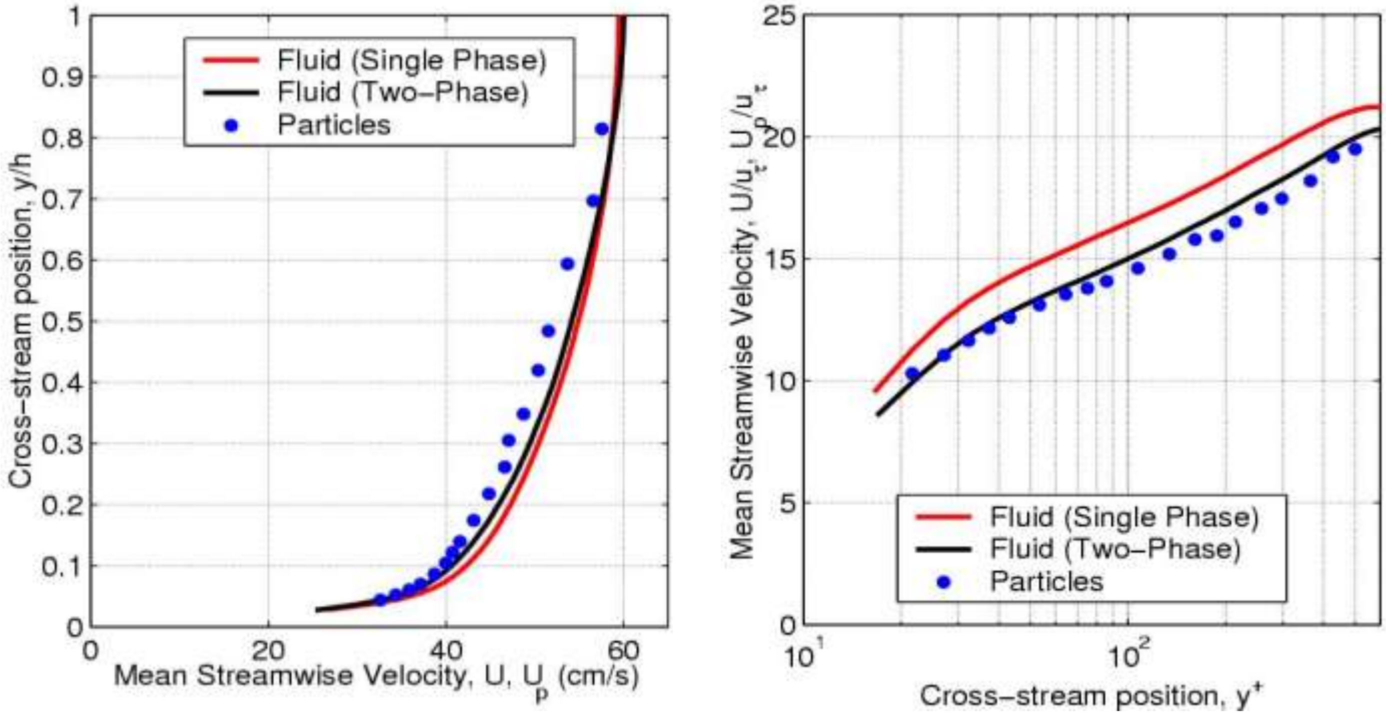
$$\frac{C(y)}{C(y_o)} = \left(\frac{h-y}{y}\,\frac{y_o}{h-y_o}\right)^a \approx \left(\frac{y_o}{y}\right)^a$$

$$a = \frac{V_s}{\kappa u_*} = \frac{2.4\ [illegible]}{(0.4)(\ldots)} = [illegible]$$



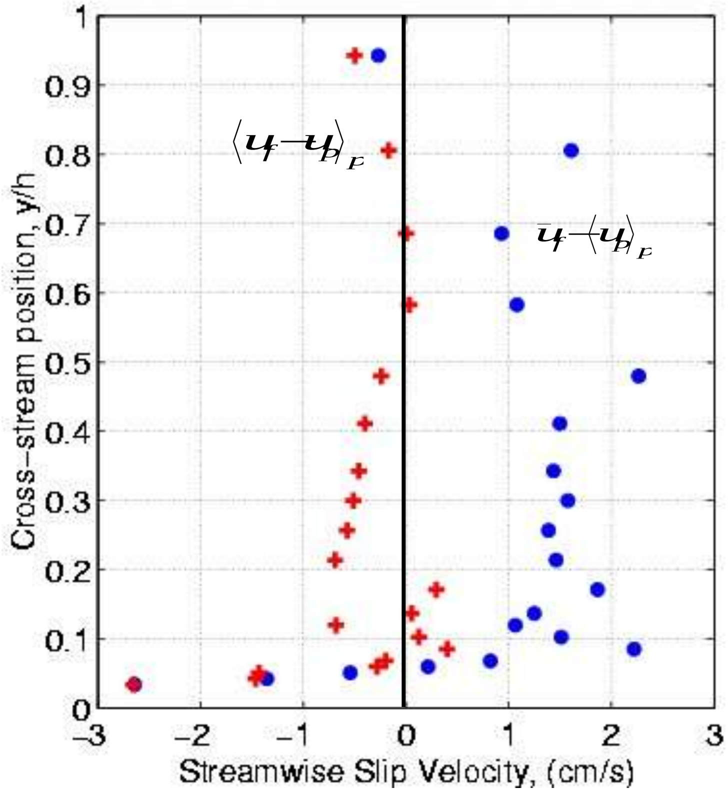
  - Based on mixing length theory, but still gives good agreement

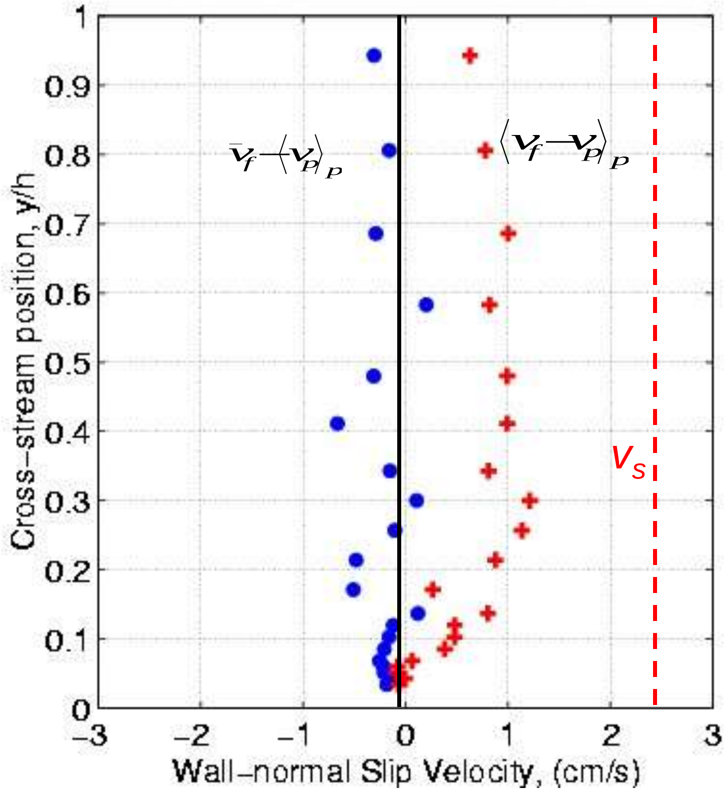
Mean Velocity



– Particles alter mean fluid profile
  - Skin friction increased by 7%; qualitatively similar to effect of fixed roughness
– Particles lag fluid over most of flow
  - Observed in gas/solid flow (much large Stokes number… likely not same reasons)
  - Particles on average reside in slower moving fluid regions?
    - *Reported by Kaftori et al, 1995 for $\rho_p/\rho_f = 1.05$ (current is heavier ~ 2.5)*
    - *Organization of particles to low speed side of structures – a la Wang & Maxey (1993)?*
– Particles begin to lead fluid near inner region – transport lag across strong gradient

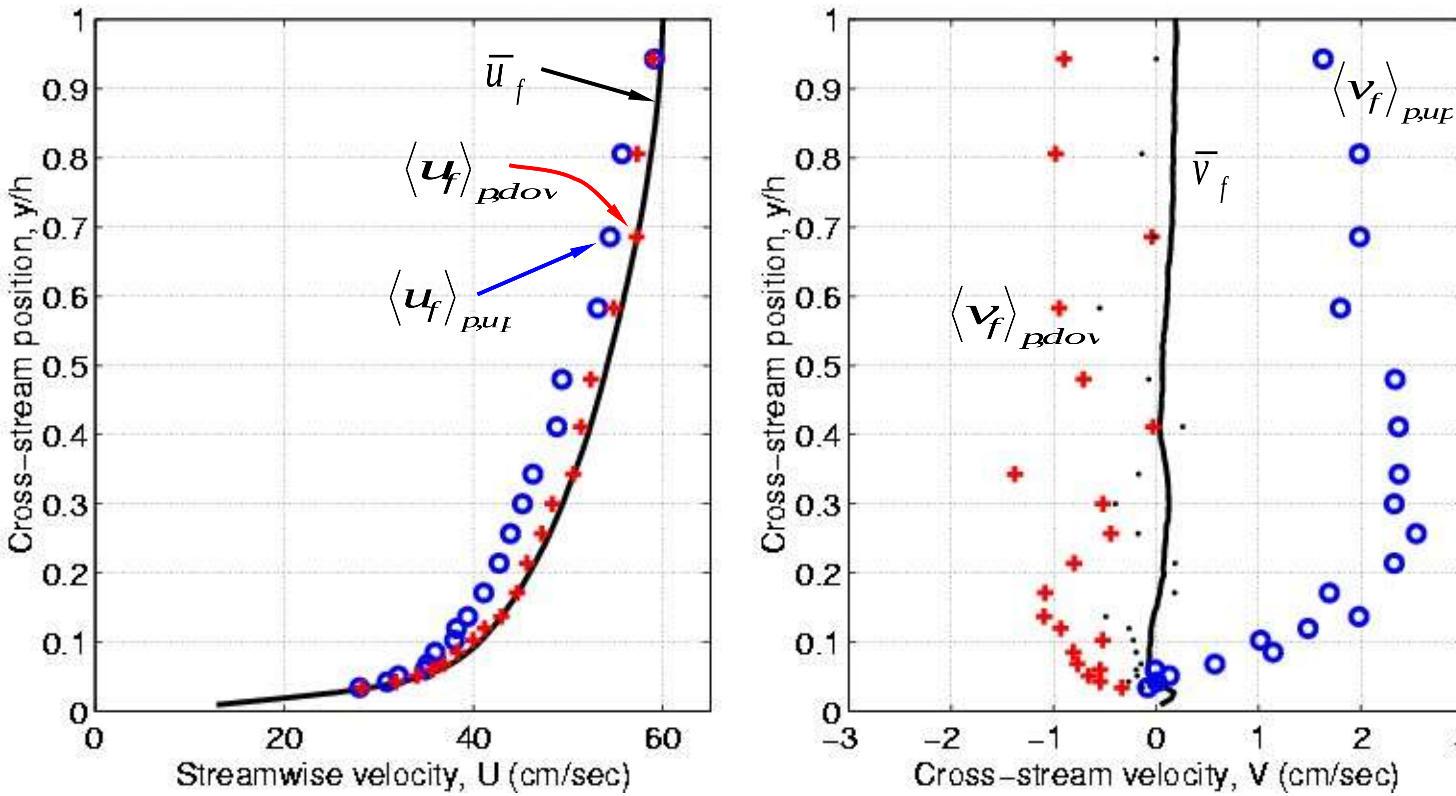
Particle Slip Velocity, $\bar{u}_f - \langle u_p \rangle_p$ $\langle u_f - u_p \rangle_p$





- Streamwise direction
  - Particle-conditioned slip (✚) is generally small in outer flow
  - Mean slip (•) and particle conditioned slip are similar in near wall region
- Wall-normal direction
  - Mean slip (•) is negligible
  - Particle-conditioned slip (✚) approximately 40% of steady-state settling velocity (2.4 cm/s)

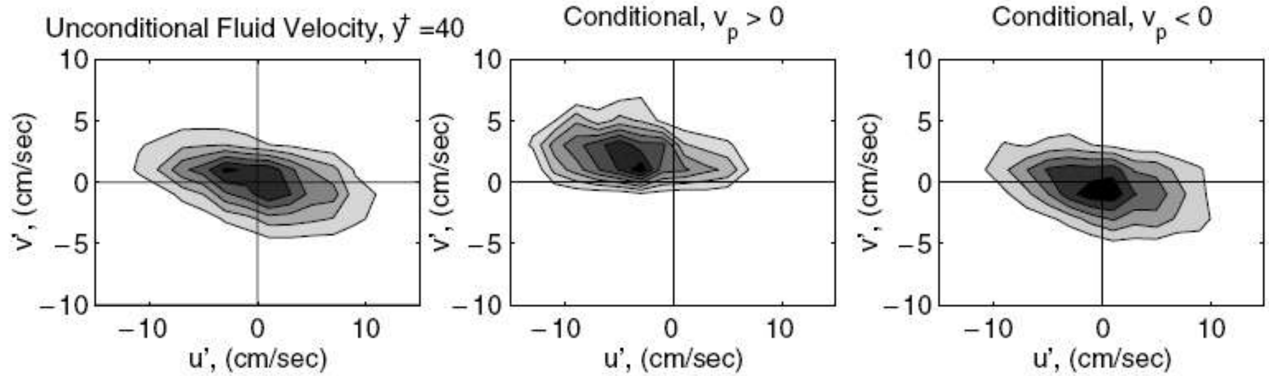
Particle Conditioned Fluid Velocity



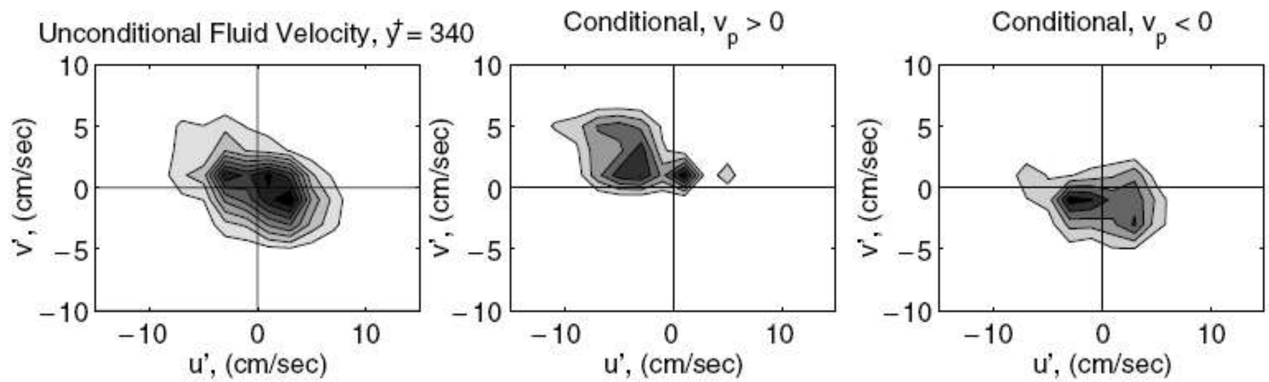
– Average fluid motion at particle locations:
  - Upward moving particles are in fluid regions moving slower than mean fluid
  - Downward moving particles are in fluid regions which on average are the same as the fluid
  - Indicates preferential structure interaction of particle suspension

Suspension Quadrant Analysis

- Conditionally sampled fluid velocity fluctuations
  - Upward moving particles primarily in quadrant II
  - Downward moving particles are almost equally split in quadrant III and IV

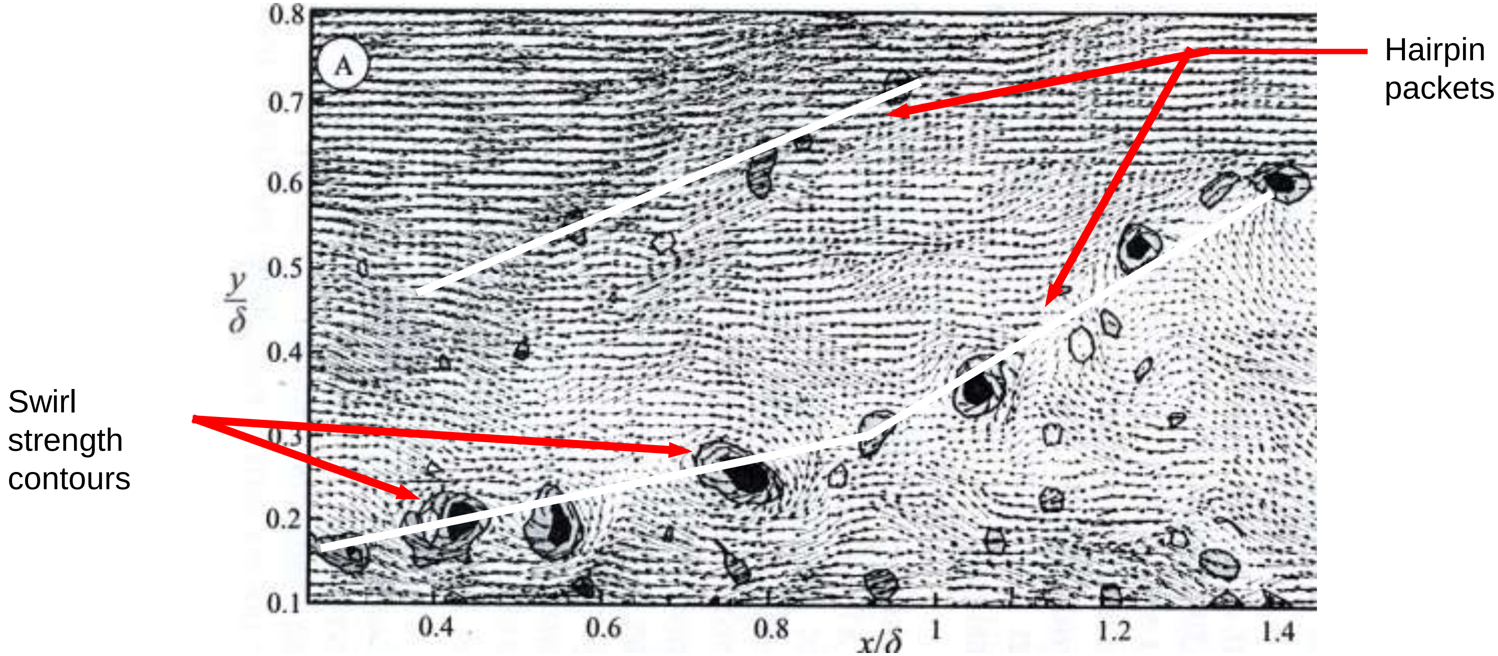


- Persistent behavior
  - Similar quadrant behavior in far outer region
  - Distribution tends towards axisymmetric case in outer region



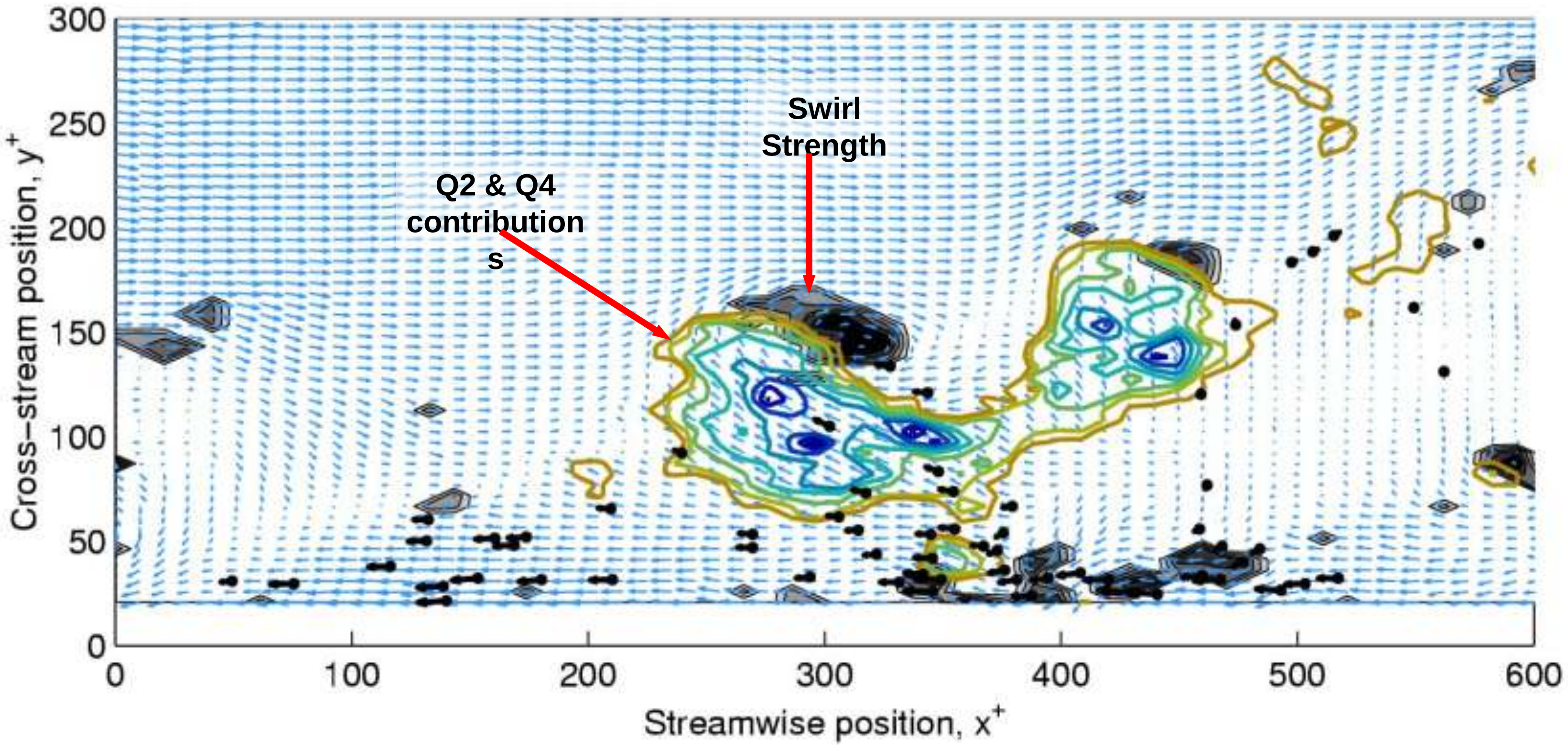
Expected Structure: Hairpin "packets"

- **Visualization of PIV data in single-phase boundary layer**
  - Adrian, Meinhart & Tomkins (2000), *JFM*
  - Use "swirl strength" to find head of hairpin structures
    - Eigenvalues of 2-D deformation rate tensor, swirl strength is indicated by magnitude of complex component

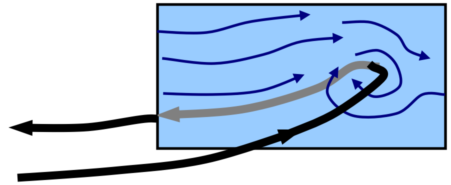


  - Spacing ~ 200 wall units
  - Packet growth angle can increase or decrease, +10° on average
  - Packets were observed in 80% of images (Re_θ = 7705)

Event structures: Quadrant II hairpin



- **Similar structures found**
  - Appropriate spacing
  - Not as frequent
    - Re effects? (Re_θ = 1183)
    - Smaller field of view?
- **Evidence suggests packets contribute to particle suspension**



Particle Kinetic Stress

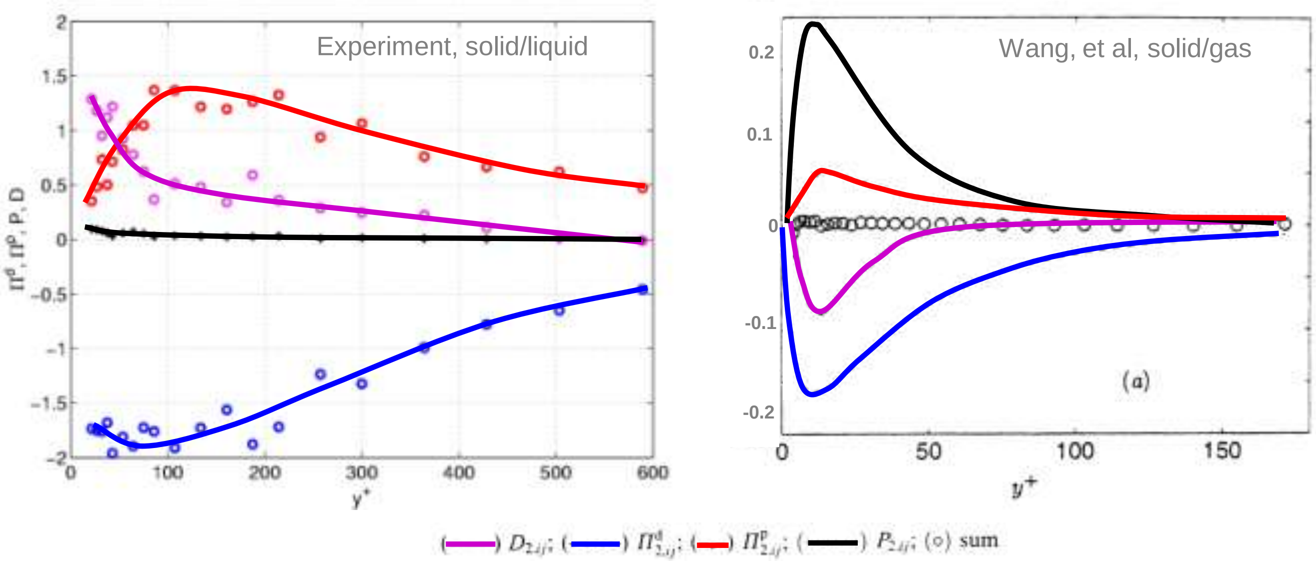
- **Turbulence budget for particle stresses**
  - (Wang, Squires, Simonin,1998)

$$\cancelto{0}{\left[\frac{\partial}{\partial t}+U_{2,m}\frac{\partial}{\partial x_m}\right]}\left\langle u'_{2,i}u'_{2,j}\right\rangle_2 = P_{2,ij}+D_{2,ij}+\Pi^d_{2,ij}+\Pi^p_{2,ij}$$

  - Production by mean shear $$P_{2,ij}=-\left\langle u'_{2,i}u'_{2,m}\right\rangle\frac{\partial U_{2,j}}{\partial x_m}-\left\langle u'_{2,j}u'_{2,m}\right\rangle\frac{\partial U_{2,i}}{\partial x_m}$$
  - Transport by fluctuations $$D_{2,ij}=-\frac{1}{\alpha_2}\frac{\partial}{\partial x_m}\left[\alpha_2\left\langle u'_{2,i}u'_{2,j}u'_{2,m}\right\rangle_2\right]$$
  - Momentum coupling to fluid $$\Pi^d_{2,ij}=-\left\langle\frac{\rho_1}{\rho_2}\frac{3}{2}\frac{C_d}{d}\left|\mathbf{v}_r\right|u'_{2,i}u'_{2,j}\right\rangle_2$$
    - *(destruction)*
  - Momentum coupling to fluid $$\Pi^p_{2,ij}=\left\langle\frac{\rho_1}{\rho_2}\frac{3}{4}\frac{C_d}{d}\left|\mathbf{v}_r\right|\left[u'_{1,i}u'_{2,j}+u'_{1,j}u'_{2,i}\right]\right\rangle_2$$
    - *(production)*

Particle Kinetic Stress Budget

- **Streamwise Particle/Fluid Coupling: $\Pi^{d}_{2,11}$, $\Pi^{p}_{2,11}$**
  - Compare results to Wang, Squires, & Simonin (1998)
    - Gas/solid flow (ρ_2/ρ_1=2118), Re_τ = 180, No gravity, St^+~700
    - Computations, all 4 terms are computed; Experiments, all but $D_{2,ij}$computed



  - Interphase terms are qualitatively similar
    Similar general shapes, $\Pi^d_{11} > \Pi^p_{11}$
  - Quantitative difference
    - Magnitudes different: Π^d_{11} / Π^p_{11}~1.3 vs 3, overall magnitudes are 10 to 20 times greater
      - *Interphase terms are expected to increase with decreased St^+*
    - Dominant interphase transfer (Π) greatly diminishes importance of mean shear (P)
    - Turbulent transport (D) has opposite sign because of small shear production (P)