Turbulence Summer School
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Large Eddy Simulation, Dynamic Model, and Applications

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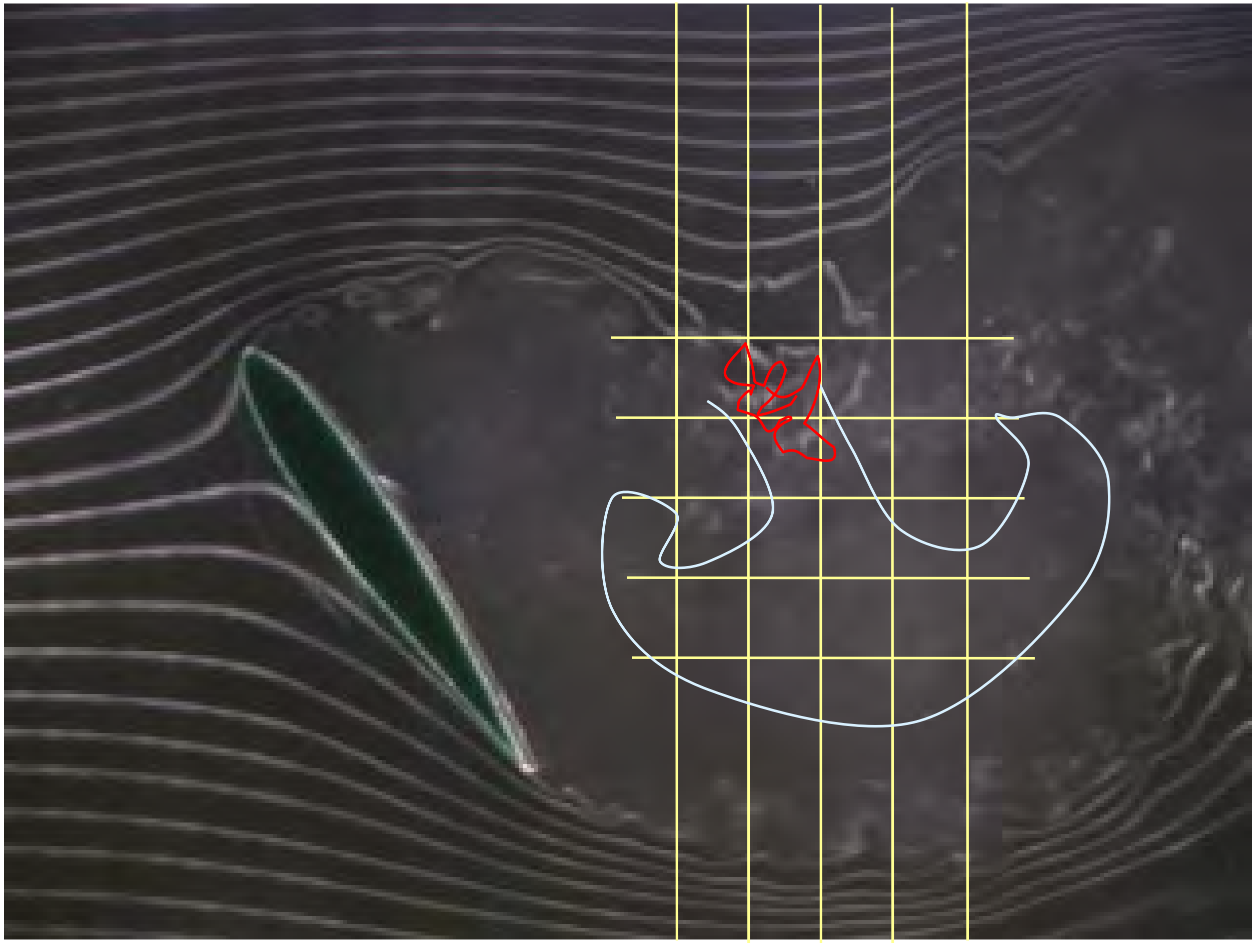
Turbulence modeling:

Large-eddy-simulation (LES)



Large-eddy-simulation (LES) and filtering:

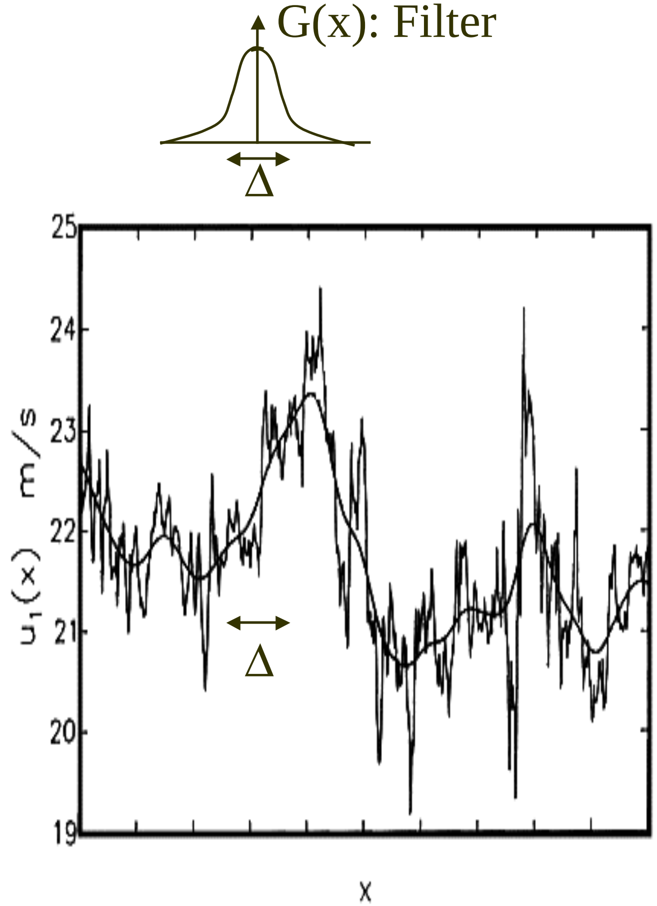
N-S equations:

$$\frac{\partial u_j}{\partial t} + \frac{\partial u_k u_j}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + \nu \nabla^2 u_j \qquad \frac{\partial u_j}{\partial x_j} = 0$$

Filtered N-S equations:

$$\frac{\partial \tilde{u}_j}{\partial t} + \frac{\partial \widetilde{u_k u_j}}{\partial x_k} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_j} + \nu \nabla^2 \tilde{u}_j$$

$$\frac{\partial \tilde{u}_j}{\partial t} + \tilde{u}_k \frac{\partial \tilde{u}_j}{\partial x_k} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_j} + \nu \nabla^2 \tilde{u}_j - \frac{\partial}{\partial x_k}\tau_{jk}$$



where SGS stress tensor is:

$$\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$$

Effects of τ_{ij} upon resolved motions: Energetics (kinetic energy):

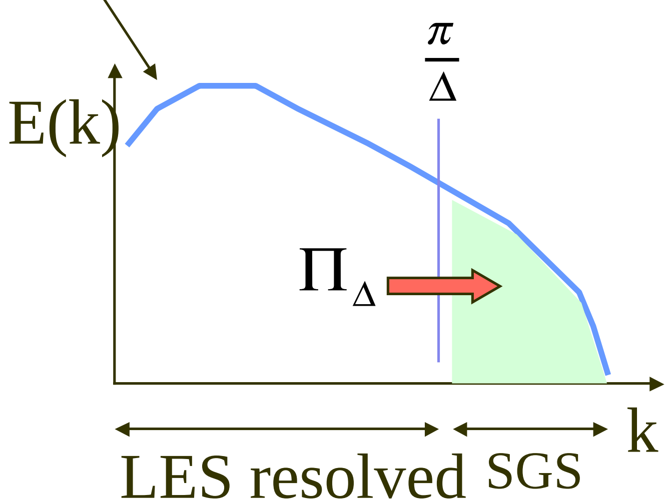
$$\frac{\partial \frac{1}{2}\tilde{u}_j\tilde{u}_j}{\partial t} + \tilde{u}_k \frac{\partial \frac{1}{2}\tilde{u}_j\tilde{u}_j}{\partial x_k} = -\frac{\partial}{\partial x_j}(....) - 2\nu \tilde{S}_{jk}\tilde{S}_{jk} - \left(-\tau_{jk}\tilde{S}_{jk}\right)$$

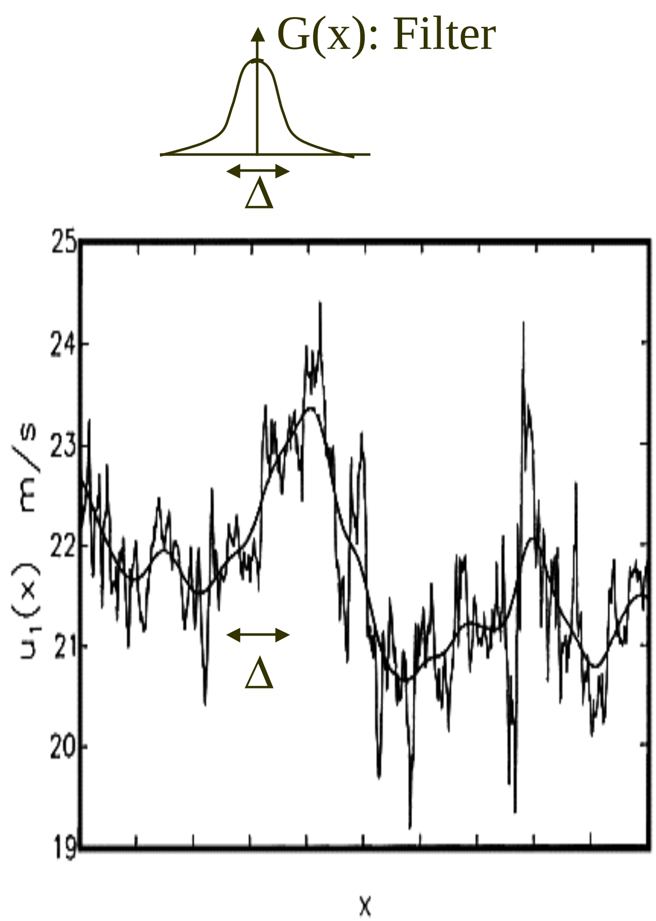
$$\tilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)$$

$$\Pi_\Delta = -\left\langle \tau_{jk}\tilde{S}_{jk} \right\rangle$$

Inertial-range flux

$$\varepsilon = \frac{u'^3}{L}$$





“SGS energy dissipation”:

$$\Pi_\Delta = -\left\langle \tau_{jk} \tilde{S}_{jk} \right\rangle$$

If we wish to “control” dissipation of energy we can set τ_{ij} proportional to $-S_{ij}$

E.g. Smagorinsky-Lilly model:

$$\tau_{ij}^{d} = -\nu_{sgs}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) = -2\nu_{sgs}\tilde{S}_{ij}$$

$$\nu_{sgs} = ?? = (velocity - scale) \times (length - scale)$$

$$\tau_{ij}^{d} = -\nu_{sgs}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right) = -2\nu_{sgs}\tilde{S}_{ij}$$

Length-scale: ~ Δ (instead of *L*),

Velocity-scale ~ Δ |*S*|

$$\nu_{sgs} \sim \Delta^2 \,|\tilde{S}\,|$$

$$\nu_{sgs} = \left(c_s\Delta\right)^2 |\tilde{S}\,|$$

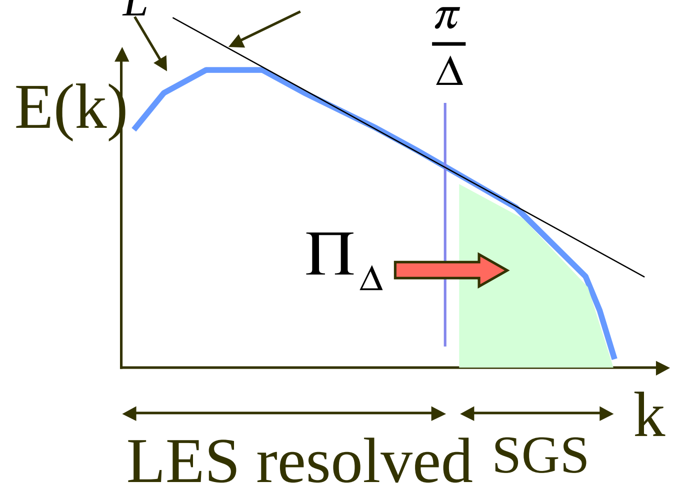
c_s: *"Smagorinsky constant"*

Theoretical calibration of c_s (D.K. Lilly, 1967):

$$\Pi_\Delta = \varepsilon = -\left\langle \tau_{ij}\tilde{S}_{ij} \right\rangle \qquad \tau_{ij} = -2(c_s\Delta)^2 \,|\tilde{S}|\, \tilde{S}_{ij}$$

$$\varepsilon = c_s^2 \Delta^2 2 \left\langle |\tilde{S}|\, \tilde{S}_{ij}\tilde{S}_{ij} \right\rangle$$

$$\varepsilon \approx c_s^2 \Delta^2 2^{3/2} \left\langle \tilde{S}_{ij}\tilde{S}_{ij} \right\rangle^{3/2}$$



$$\varepsilon = \frac{u'^3}{L} \qquad E(k) = c_K \varepsilon^{2/3} k^{-5/3}$$

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$$\varepsilon = c_s^2\Delta^2\,2\left\langle |\tilde{S}|\,\tilde{S}_{ij}\tilde{S}_{ij}\right\rangle$$

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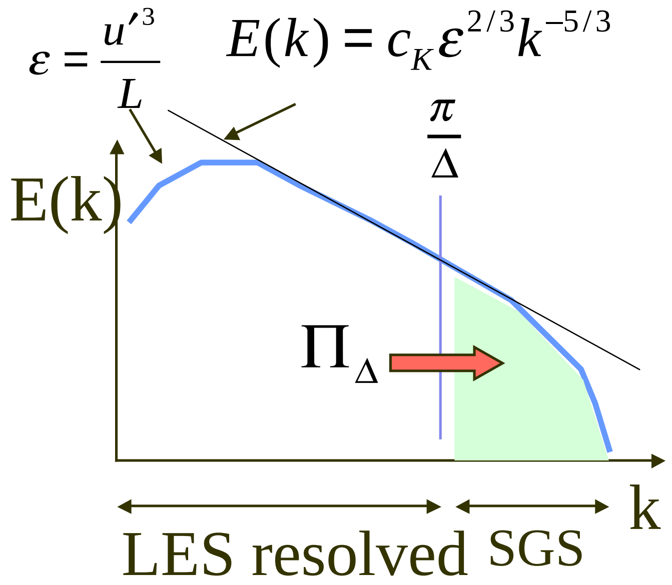
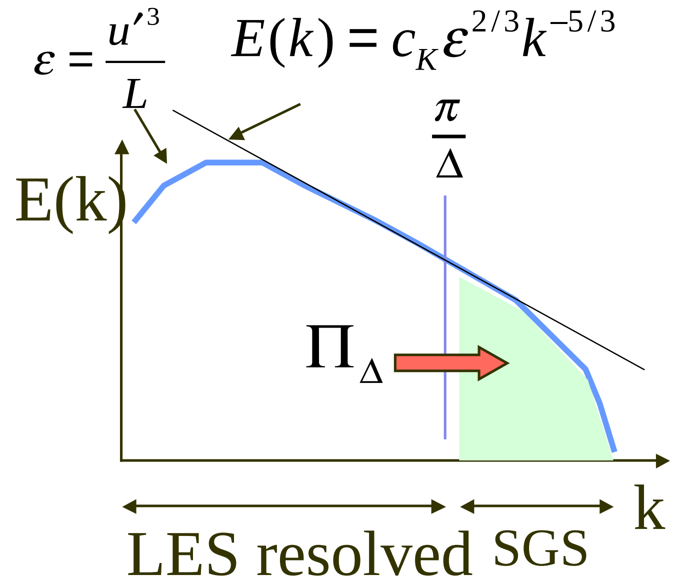


$$\left\langle \tilde{S}_{ij}\tilde{S}_{ij}\right\rangle = \frac{1}{2}\left\langle \frac{\partial \tilde{u}_i}{\partial x_j}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)\right\rangle =$$

$$= \frac{1}{2}\iiint_{|\mathbf{k}|<\pi/\Delta} [k_j{}^2\Theta_{ii}(\mathbf{k}) + k_i k_j \Theta_{ij}(\mathbf{k})]d^3\mathbf{k} = \frac{1}{2}\iiint_{|\mathbf{k}|<\pi/\Delta} [k^2(\frac{E(k)}{4\pi k^2}(\delta_{ii} - \frac{k^2}{k^2})) + 0]d^3\mathbf{k}$$

$$= c_K\varepsilon^{2/3}\frac{1}{2}\int_0^{\pi/\Delta} k^{-5/3+2}\frac{3-1}{4\pi k^2}4\pi k^2 dk = c_K\varepsilon^{2/3}\int_0^{\pi/\Delta} k^{1/3}dk = c_K\varepsilon^{2/3}\frac{3}{4}\left(\frac{\pi}{\Delta}\right)^{4/3}$$

Theoretical calibration of c_s (D.K. Lilly, 1967):



$$\Pi_\Delta = \varepsilon = -\left\langle \tau_{ij} \tilde{S}_{ij} \right\rangle \qquad \tau_{ij} = -2(c_s \Delta)^2 \, |\tilde{S}| \, \tilde{S}_{ij}$$

$$\varepsilon = c_s^2 \Delta^2 2 \left\langle |\tilde{S}| \, \tilde{S}_{ij} \tilde{S}_{ij} \right\rangle$$

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$$\left\langle \tilde{S}_{ij} \tilde{S}_{ij} \right\rangle = \frac{1}{2} \left\langle \frac{\partial \tilde{u}_i}{\partial x_j} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) \right\rangle =$$

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$$\varepsilon \approx c_s^2 \Delta^2 2^{3/2} \left(c_K \varepsilon^{2/3} \frac{3}{4} \left(\frac{\pi}{\Delta} \right)^{4/3} \right)^{3/2}$$

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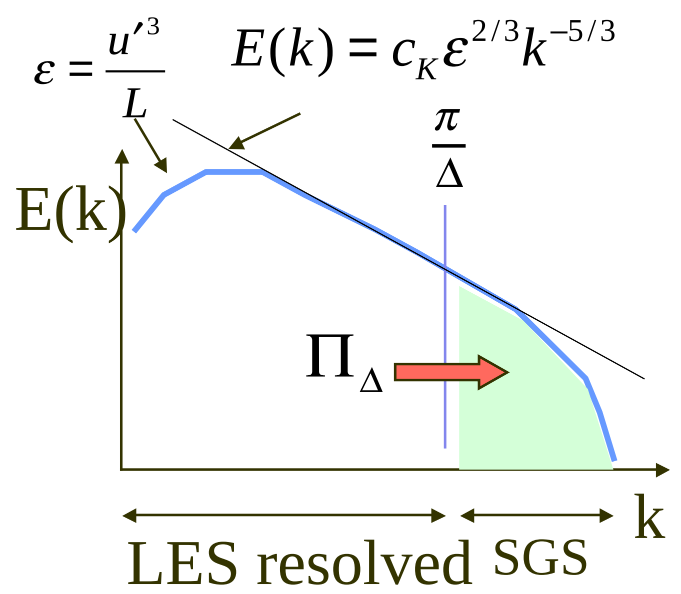

$$\left\langle \tilde{S}_{ij}\tilde{S}_{ij} \right\rangle = \frac{1}{2}\left\langle \frac{\partial \tilde{u}_i}{\partial x_j}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) \right\rangle =$$

$$= \frac{1}{2} \iiint_{|\mathbf{k}|<\pi/\Delta} [k_j{}^2 \Theta_{ii}(\mathbf{k}) + k_i k_j \Theta_{ij}(\mathbf{k})] d^3\mathbf{k} = \frac{1}{2} \iiint_{|\mathbf{k}|<\pi/\Delta} [k^2 (\frac{E(k)}{4\pi k^2}(\delta_{ii} - \frac{k^2}{k^2})) + 0] d^3\mathbf{k}$$

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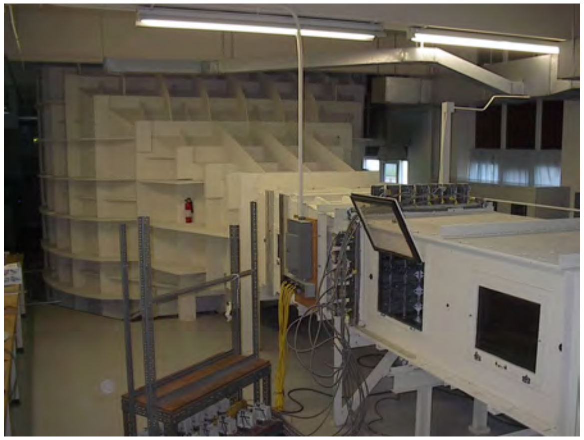
$$\varepsilon \approx c_s^2 \Delta^2 2^{3/2} \left(c_K \varepsilon^{2/3} \frac{3}{4}\left(\frac{\pi}{\Delta}\right)^{4/3} \right)^{3/2}$$

$$\Rightarrow 1 \approx c_s^2 \pi^2 \left(\frac{3c_K}{2}\right)^{3/2} \Rightarrow c_s = \left(\frac{3c_K}{2}\right)^{-3/4} \pi^{-1}$$

$$c_K = 1.6 \Rightarrow c_s \approx 0.16$$

c_s=0.16 works well for isotropic, high Reynolds number turbulence

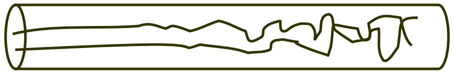


But in practice (complex flows)

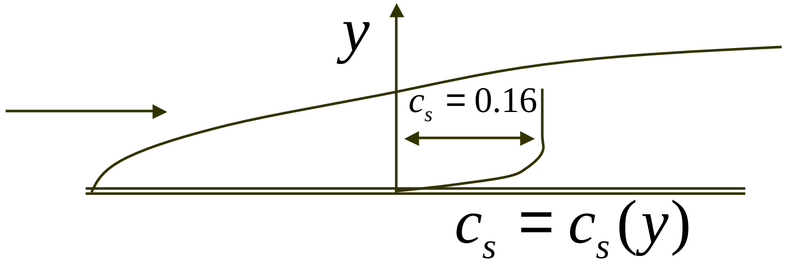
$$c_s = c_s(\mathbf{x}, t)$$

Ad-hoc tuning?

Examples: Transitional pipe flow: from 0 to 0.16



Near wall damping for wall boundary layers (Piomelli et al 1989)



How does c_s vary under realistic conditions? Interrogate data:

Measure: $$\Pi_\Delta = -\left\langle \tau_{jk} \tilde{S}_{jk} \right\rangle$$

Measure: $$\frac{\Pi_\Delta^{Smag}}{c_s^2} = 2\Delta^2 \left\langle |\tilde{S}| \tilde{S}_{ij} \tilde{S}_{ij} \right\rangle$$

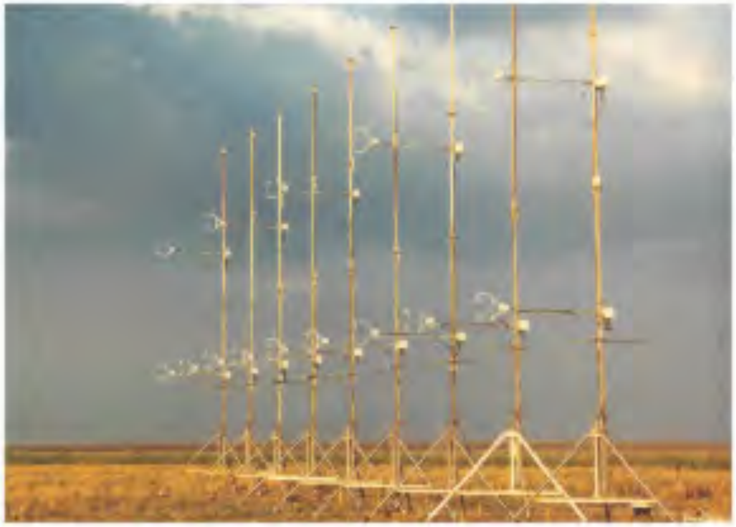
Obtain"empirical" Smagorinsky coefficient *= f(x,conditions…):*

$$c_s = \left(\frac{-\left\langle \tau_{jk} \tilde{S}_{jk} \right\rangle}{2\Delta^2 \left\langle |\tilde{S}| \tilde{S}_{ij} \tilde{S}_{ij} \right\rangle} \right)^{1/2}$$

An example result from atmospheric turbulence…:

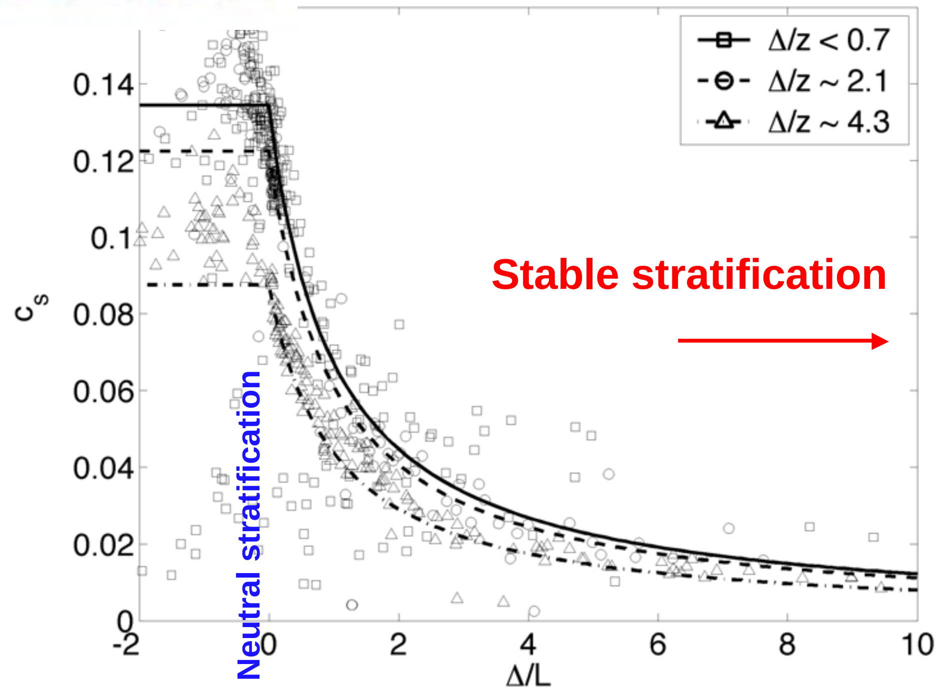
Measure "empirical" Smagorinsky coefficient for atmospheric surface layer as function of height and stability (thermal forcing or damping):

HATS - 2000
(with NCAR researchers: Horst, Sullivan)
Kettleman City (Central Valley, CA)



$$c_s = \left(\frac{-\langle \tau_{jk} \tilde{S}_{jk} \rangle}{2\Delta^2 \langle |\tilde{S}| \tilde{S}_{ij} \tilde{S}_{ij} \rangle} \right)^{1/2}$$

Example result: effect of atmospheric stability on coefficient from sonic anemometer measurements in atmospheric surface layer (Kleissl et al., J. Atmos. Sci. 2003)



$$c_s = c_s(\mathbf{x}, t)$$

How to avoid “tuning” and case-by-case adjustments of model coefficient in LES?

The Dynamic Model

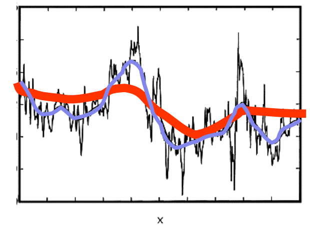
(Germano et al. Physics of Fluids, 1991)

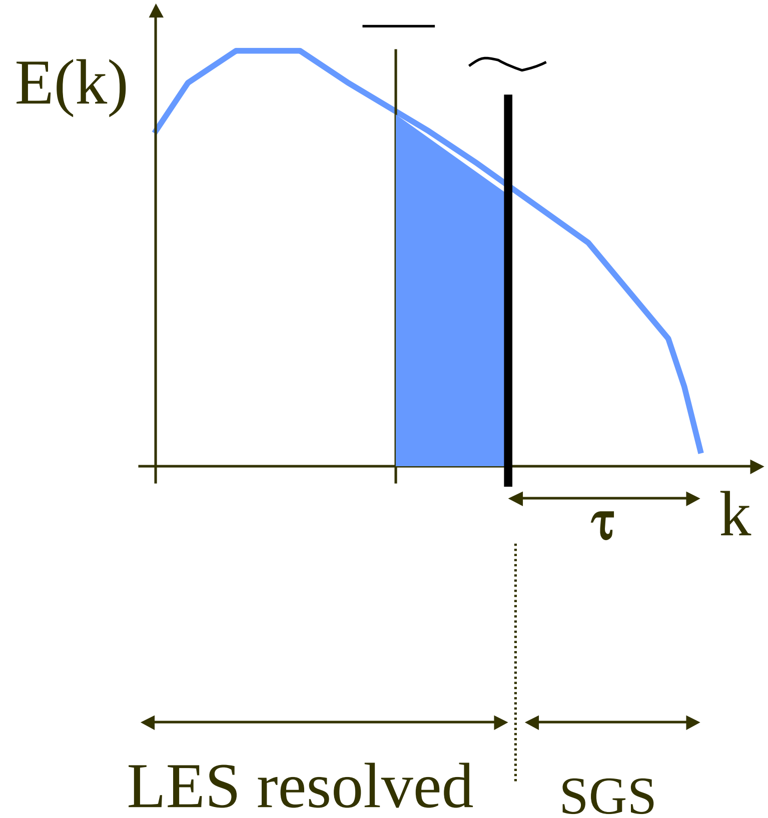
Germano identity and dynamic model

(Germano et al. 1991):

Exact ("rare" in turbulence):

$$\overline{\widetilde{u_i u_j}} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j = \overline{\widetilde{u_i u_j}} \qquad\qquad - \bar{\tilde{u}}_i \bar{\tilde{u}}_j$$



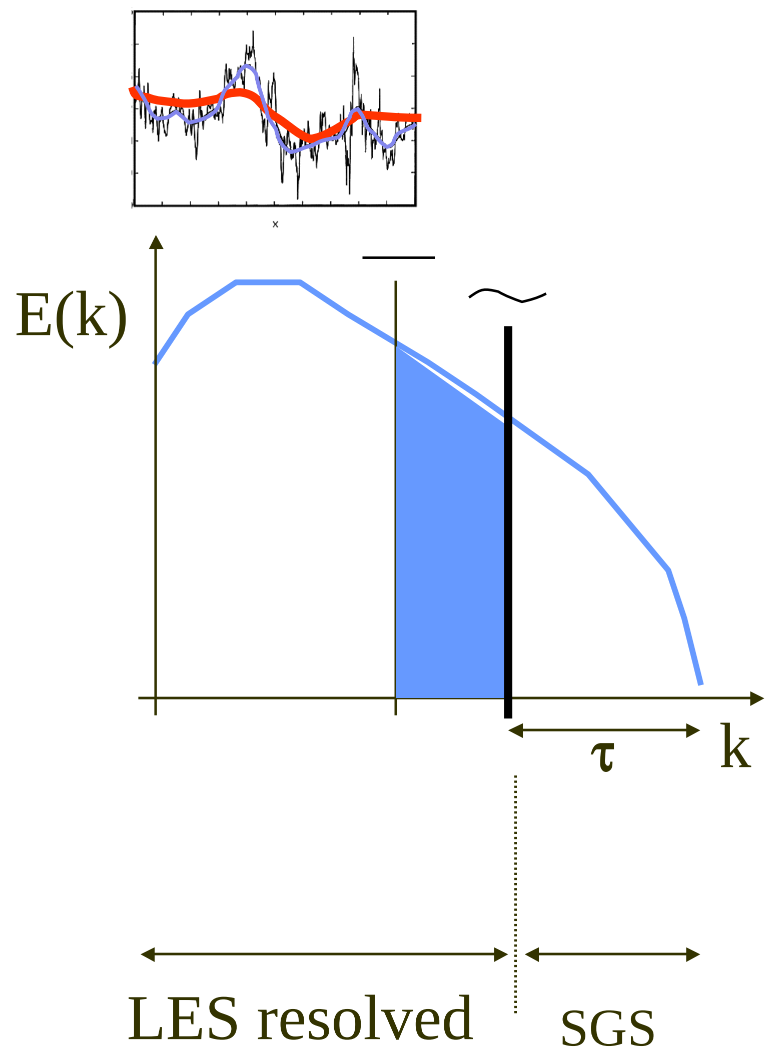


Germano identity and dynamic model

(Germano et al. 1991):

Exact ("rare" in turbulence):

$$\overline{\widetilde{u_i u_j}} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j = \overline{\widetilde{u_i u_j}} \quad - \overline{\tilde{u}_i \tilde{u}_j} \; + \overline{\tilde{u}_i \tilde{u}_j} \quad - \bar{\tilde{u}}_i \bar{\tilde{u}}_j$$



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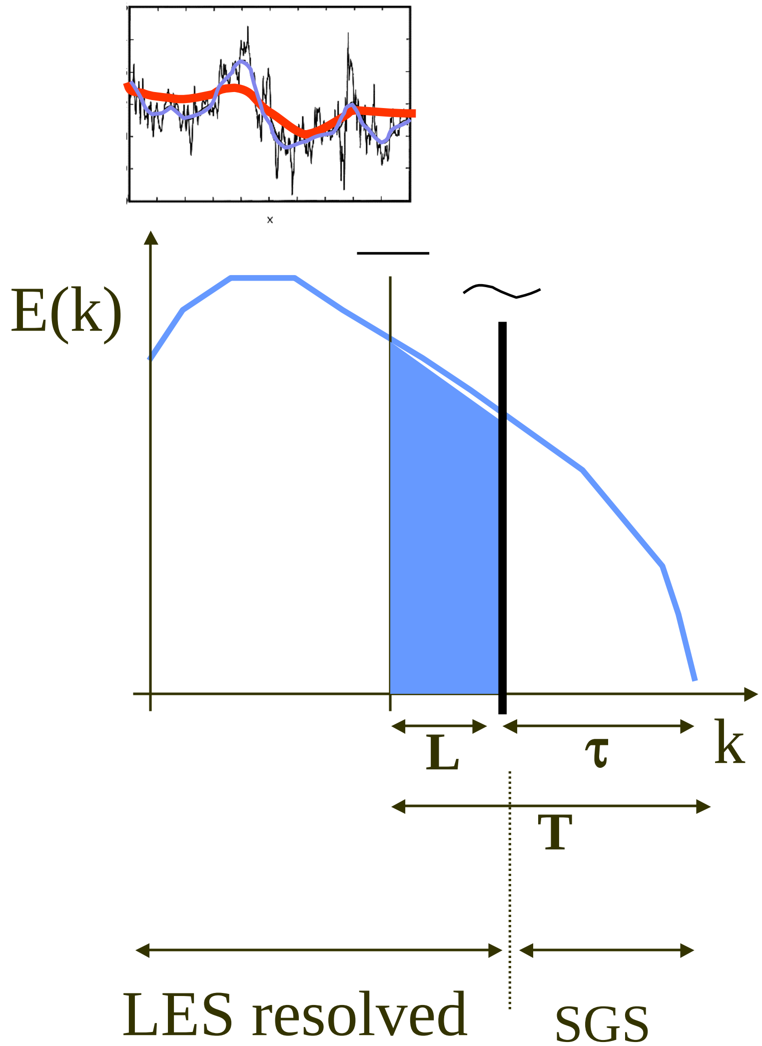
$$\underbrace{\overline{\widetilde{u_i u_j}} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j}_{T_{ij}} = \underbrace{\overline{\widetilde{u_i u_j}} - \overline{\tilde{u}_i \tilde{u}_j}}_{\bar{\tau}_{ij}} + \underbrace{\overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j}_{L_{ij}}$$

$$T_{ij} = \bar{\tau}_{ij} + L_{ij}$$

$$L_{ij} - (T_{ij} - \bar{\tau}_{ij}) = 0$$



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$$T_{ij} = \bar{\tau}_{ij} + L_{ij}$$

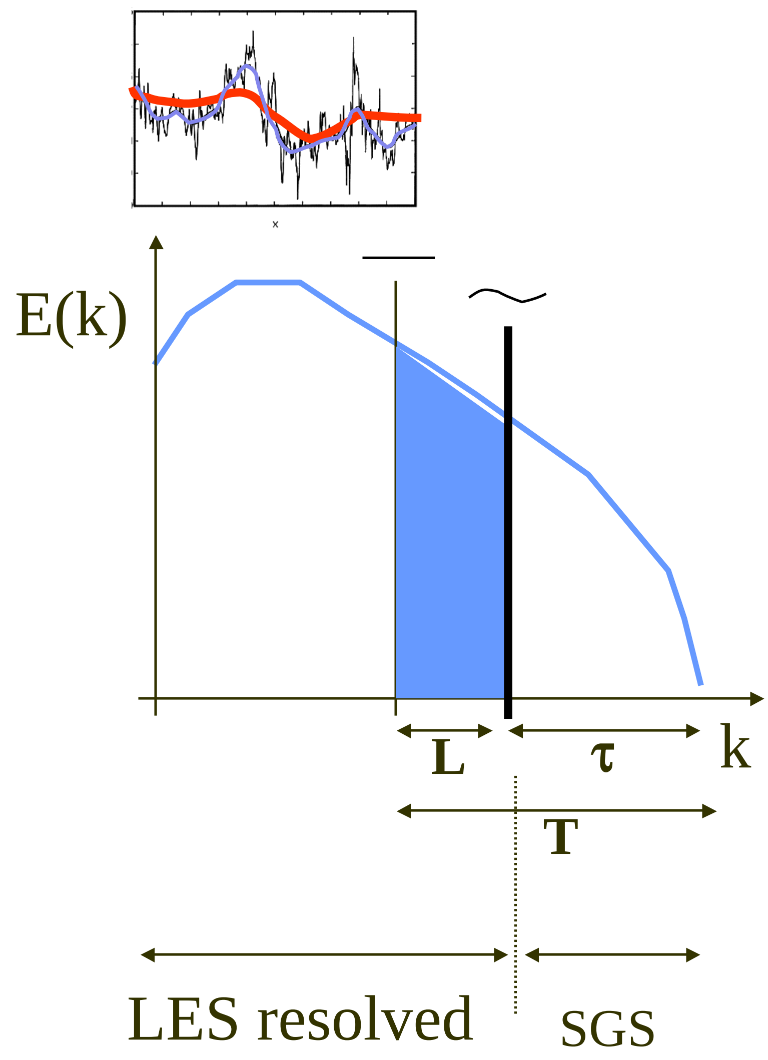
$$L_{ij} - (T_{ij} - \bar{\tau}_{ij}) = 0$$

$$-2(c_s 2\Delta)^2 \, |\bar{\tilde{S}}| \, \bar{\tilde{S}}_{ij} \qquad -2(c_s \Delta)^2 \, \overline{|\tilde{S}| \, \tilde{S}_{ij}}$$

Assumes scale-invariance:

$$L_{ij} - c_s^2 M_{ij} = 0$$

where $M_{ij} = 2\Delta^2 \left(\overline{|\tilde{S}| \, \tilde{S}_{ij}} - 4 \, |\bar{\tilde{S}}| \, \bar{\tilde{S}}_{ij} \right)$



Germano identity and dynamic model

(Germano et al. 1991):

$$L_{ij} - c_s^2 M_{ij} = 0$$

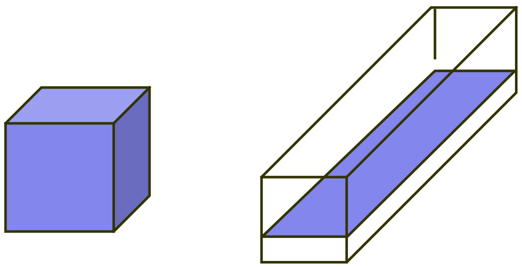
Over-determined system:
solve in "some average sense"
(minimize error, Lilly 1992):

$$\mathrm{E} = \left\langle \left(L_{ij} - c_s^2 M_{ij} \right)^2 \right\rangle$$

Minimized when:

$$c_s^2 = \frac{\left\langle L_{ij} M_{ij} \right\rangle}{\left\langle M_{ij} M_{ij} \right\rangle}$$

Averaging over regions of
statistical homogeneity
or fluid trajectories



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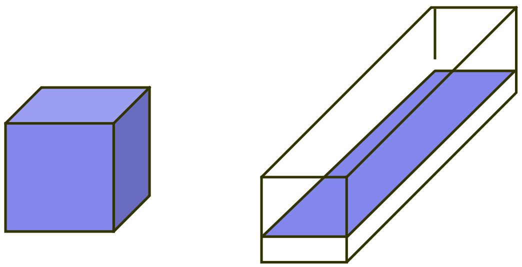


Exercise:

(a) Prove the above equation, and

(b) repeat entire formulation for the SGS heat flux vector

modeled using an SGS diffusivity (find C_{scalar})

$$q_i = \widetilde{u_i T} - \tilde{u}_i \tilde{T}$$

$$q_i = C_{scalar} \Delta^2 \, |\tilde{S}| \, \frac{\partial \tilde{T}}{\partial x_i}$$

Similarity, tensor eddy-viscosity, and mixed models

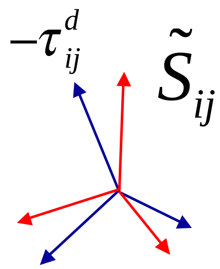
$$\tau_{ij}^{mnl} = C_{nl}\Delta^2 \frac{\partial \tilde{u}_i}{\partial x_k}\frac{\partial \tilde{u}_j}{\partial x_k} - 2(C_S\Delta)^2 |\tilde{S}|\tilde{S}_{ij}$$

Two-parameter dynamic mixed model

$$L_{ij} \equiv T_{ij} - \bar{\tau}_{ij} = \overline{\tilde{u}_i\tilde{u}_j} - \bar{\tilde{u}}_i\bar{\tilde{u}}_j$$

$$T_{ij} = -2(C_S 2\Delta)^2 |\bar{\tilde{S}}|\bar{\tilde{S}}_{ij} + C_{nl}(2\Delta)^2 \frac{\partial \bar{\tilde{u}}_i}{\partial x_k}\frac{\partial \bar{\tilde{u}}_j}{\partial x_k}$$

- Mixed tensor Eddy Viscosity Model:
  - *Taylor-series expansion of similarity (Bardina 1980) model*
    (Clark 1980, Liu, Katz & Meneveau (1994), …)
  - *Deconvolution:*
    (Leonard 1997, Geurts et al, Stolz & Adams, Winckelmans etc..)
  - Significant d*irect empirical evidence, experiments:*
    - Liu et al. (JFM 1999, 2-D PIV)
    - Tao, Katz & CM (J. Fluid Mech. 2002):
      tensor alignments from 3-D HPIV data
    - Higgins, Parlange & CM (Bound Layer Met. 2003):
      tensor alignments from ABL data
    - From DNS: Horiuti 2002, Vreman et al (LES), etc…



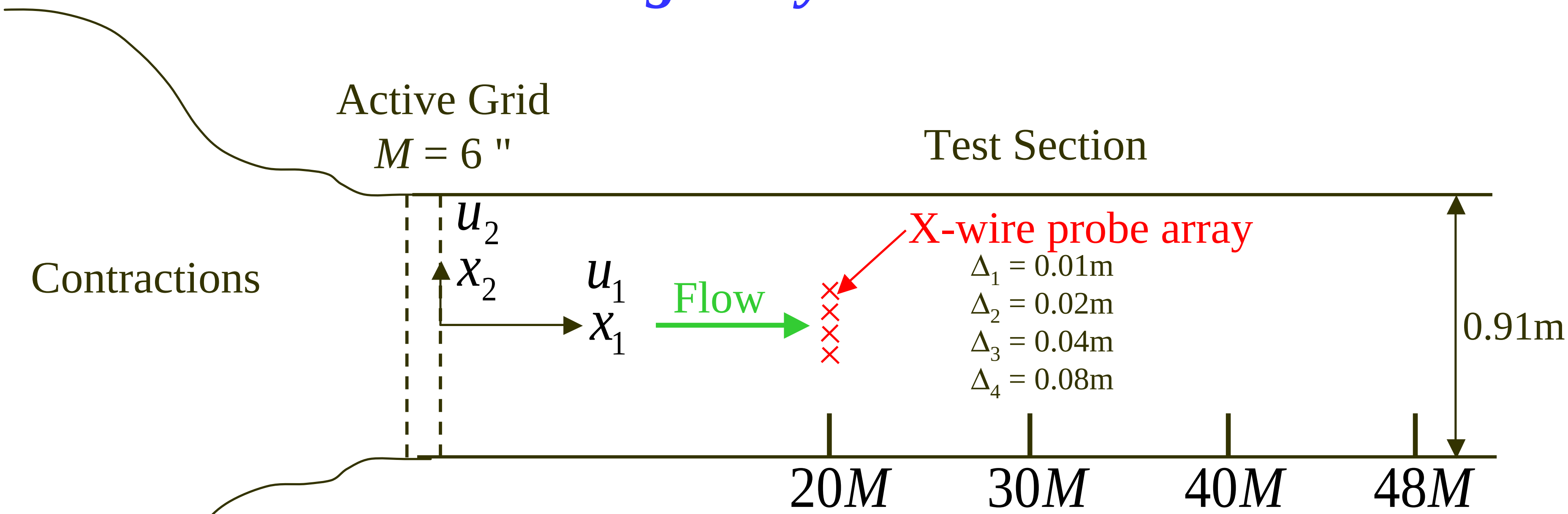
Reality check:

Do simulations with these closures produce realistic statistics of $\tilde{u}_i(x,t)$?

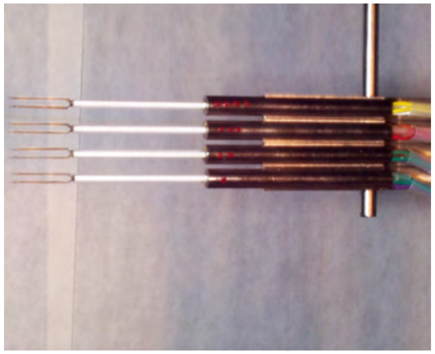
- *Need good data*
- *Need good simulations*

  - *Next: Summary of results from Kang et al. (JFM 2003)*
    - *Smagorinsky model,*
    - *Dynamic Smagorinsky model,*
    - *Dynamic 2-parameter mixed model*

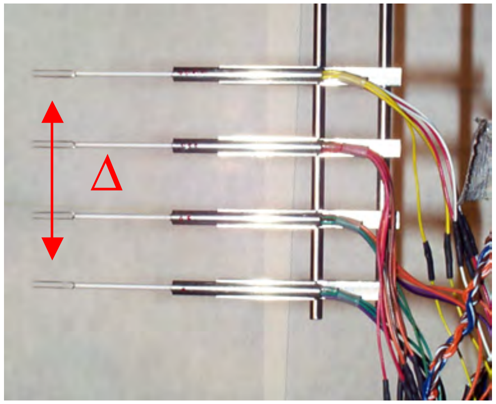
Remake of Comte-Bellot & Corrsin (1967) decaying isotropic turbulence experiment at high Reynolds number



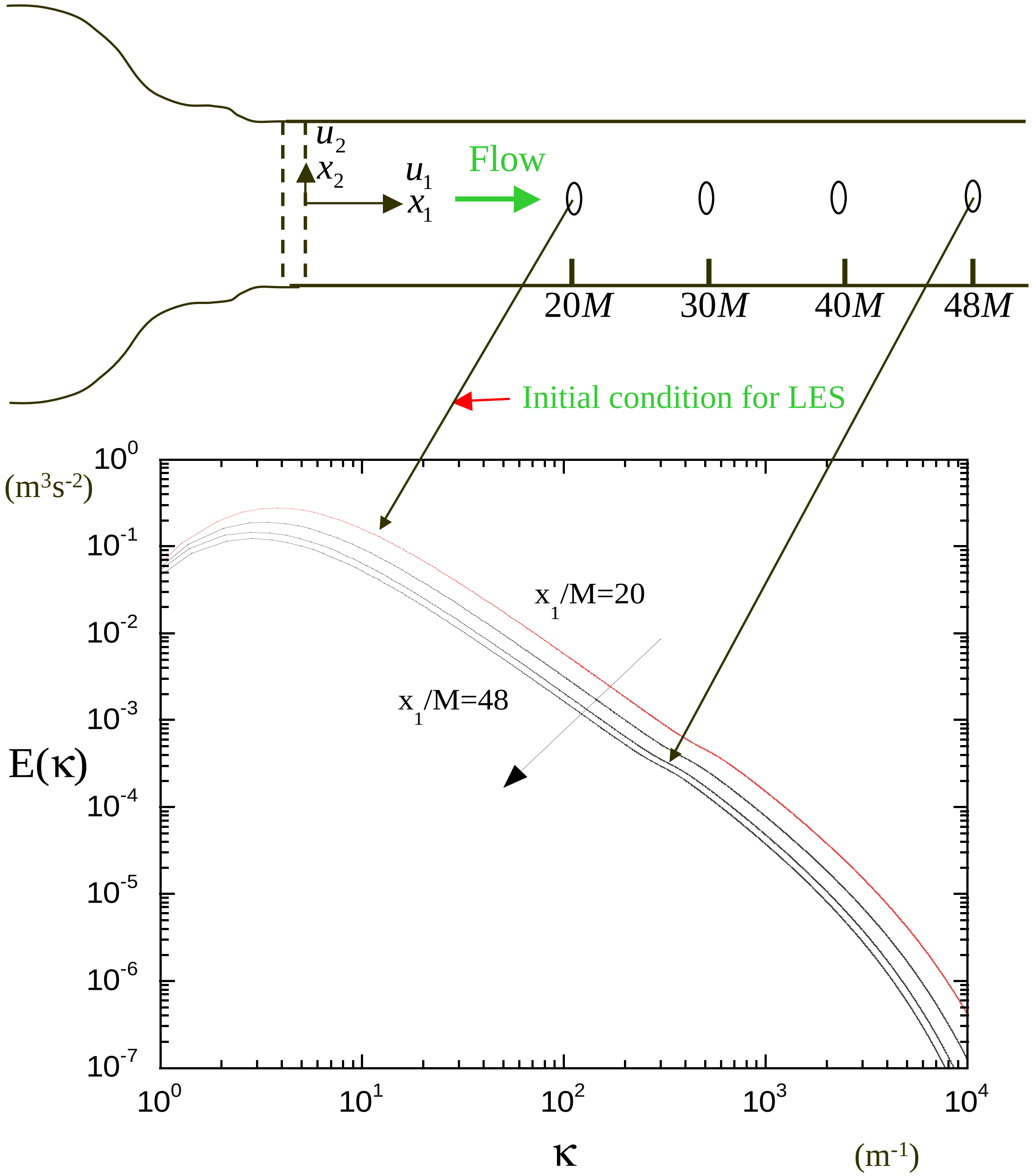
h_1 = 0.005m
Δ_1 = 0.01m



h_3 = 0.02m
Δ_3 = 0.04m



Results



u2
x2
u1
x1
Flow
20M
30M
40M
48M
Initial condition for LES
(m3s-2)
E(κ)
x1/M=20
x1/M=48
κ
(m-1)
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^1
10^2
10^3
10^4

LES of Temporally Decaying Turbulence

Experiment	Taylor Hypothesis	LES
Spatially Decaying Grid Turbulence 2-D Box Filter	→ $x_1 = \langle u_1 \rangle t$	Temporally Decaying Turbulence 3-D Filter

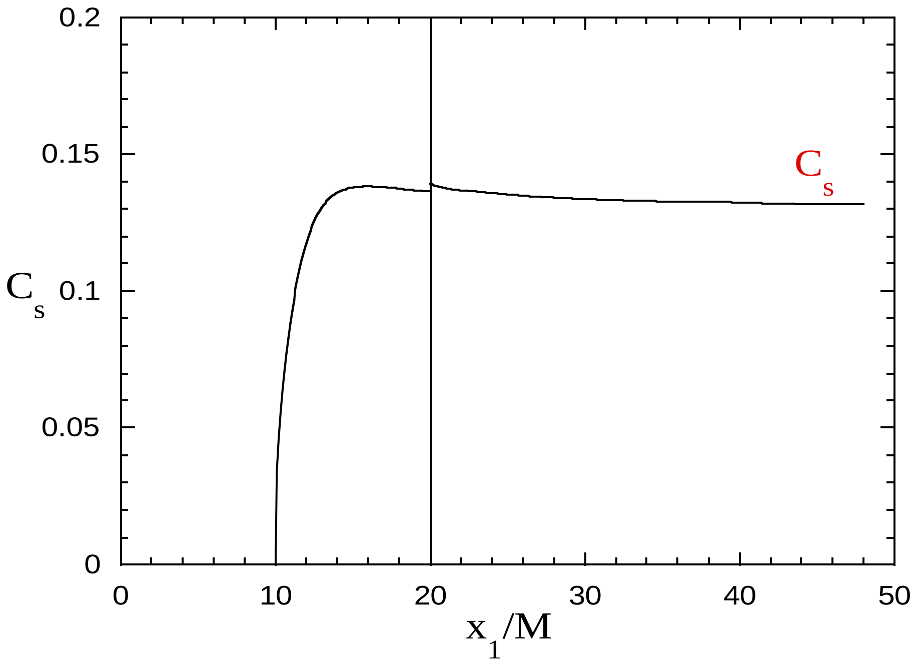
- Pseudo-spectral code: 128^3 nodes, carefully dealiased (3/2N)
- All parameters are equivalent to those of experiments.
- Initial energy distribution: 3-D energy spectrum at $x_1/M = 20$

- LES Models: standard Smagorinsky-Lilly model,
  dynamic Smagorinsky and
  dynamic mixed tensor eddy-visc. model

Results: Dynamic Model Coefficients

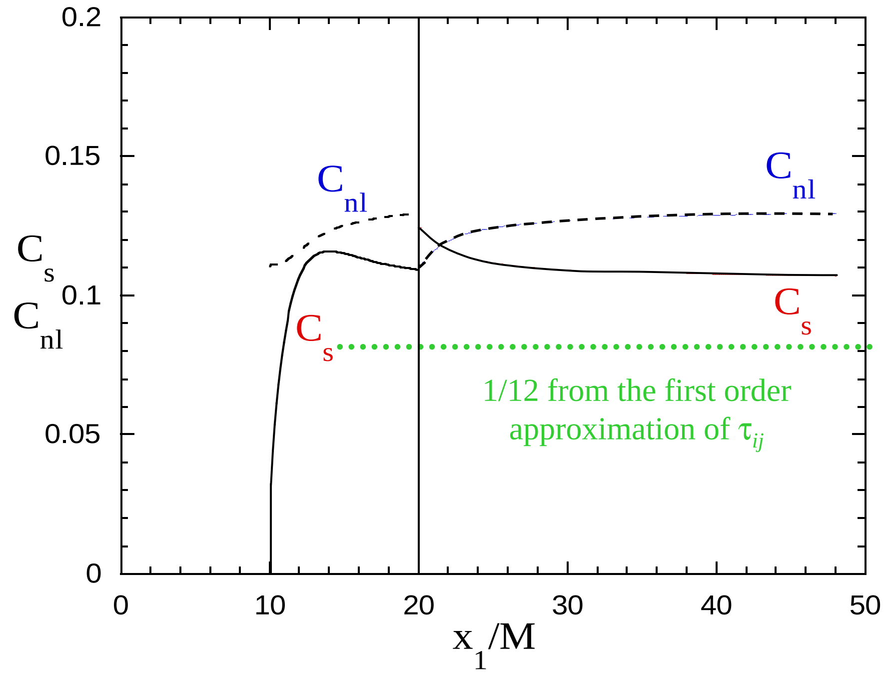
- **Dynamic Smagorinsky**

$$\tau_{ij}^{dyn-Smag} = -2\frac{\langle L_{ij}M_{ij}\rangle}{\langle M_{ij}M_{ij}\rangle}\Delta^2|\tilde{S}|\tilde{S}_{ij}$$



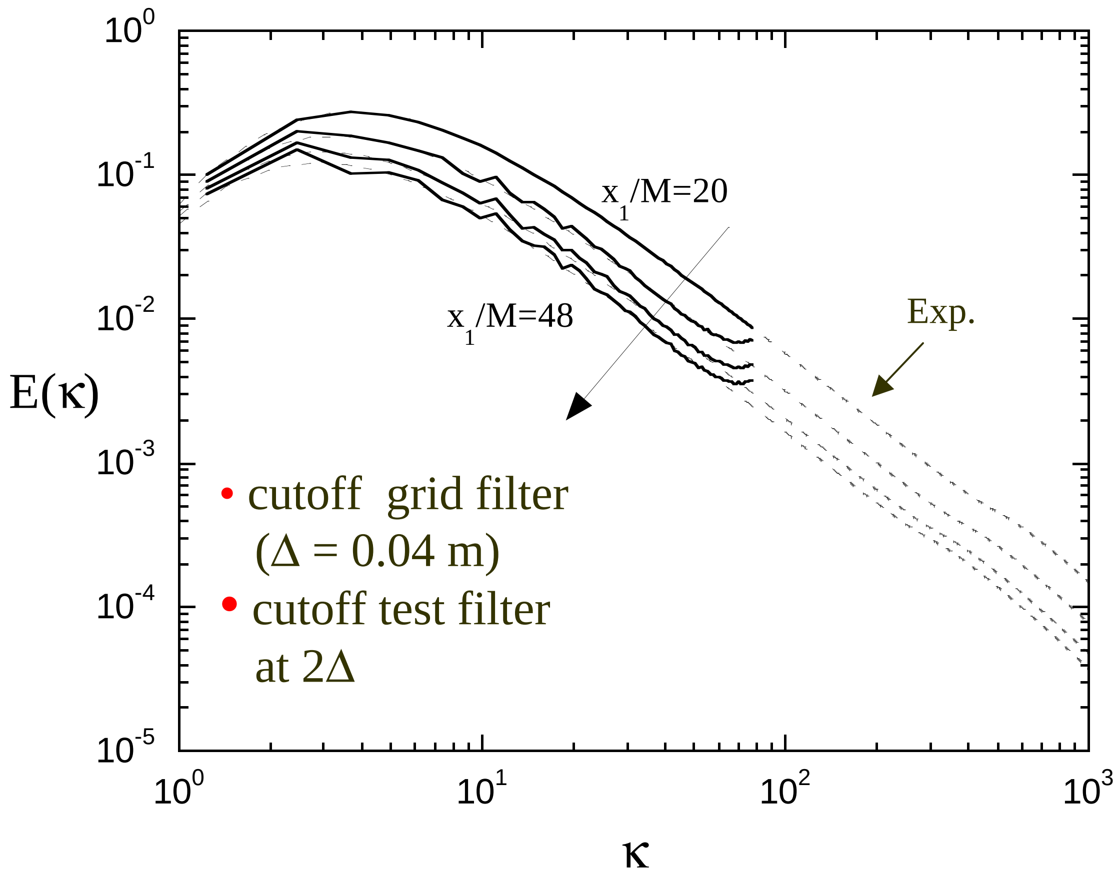
- **Dynamic Mixed tensor eddy visc:**

$$\tau_{ij}^{mnnl} = C_{nl}\Delta^2\frac{\partial\tilde{u}_i}{\partial x_k}\frac{\partial\tilde{u}_j}{\partial x_k} - 2(C_S\Delta)^2|\tilde{S}|\tilde{S}_{ij}$$



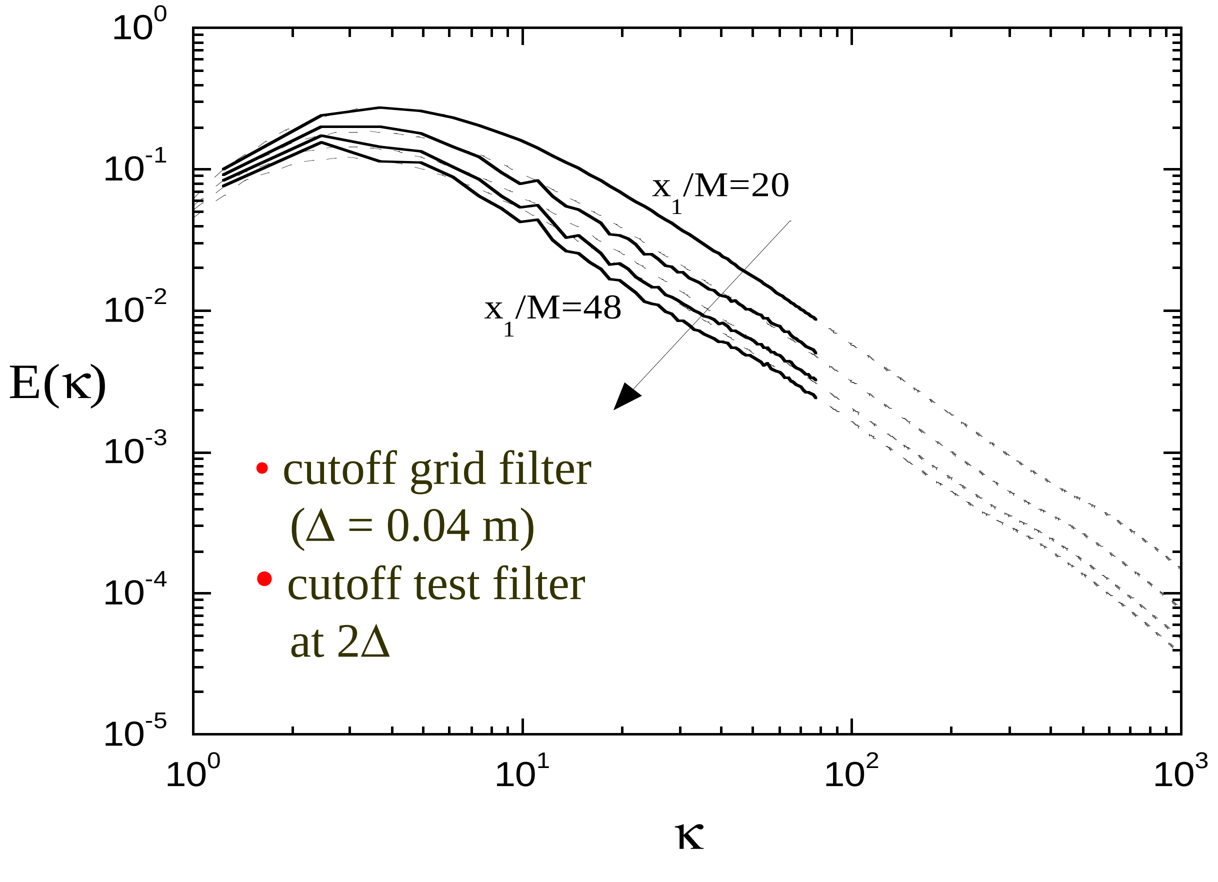
3-D Energy Spectra (LES vs experiment)

- Dynamic Smagorinsky



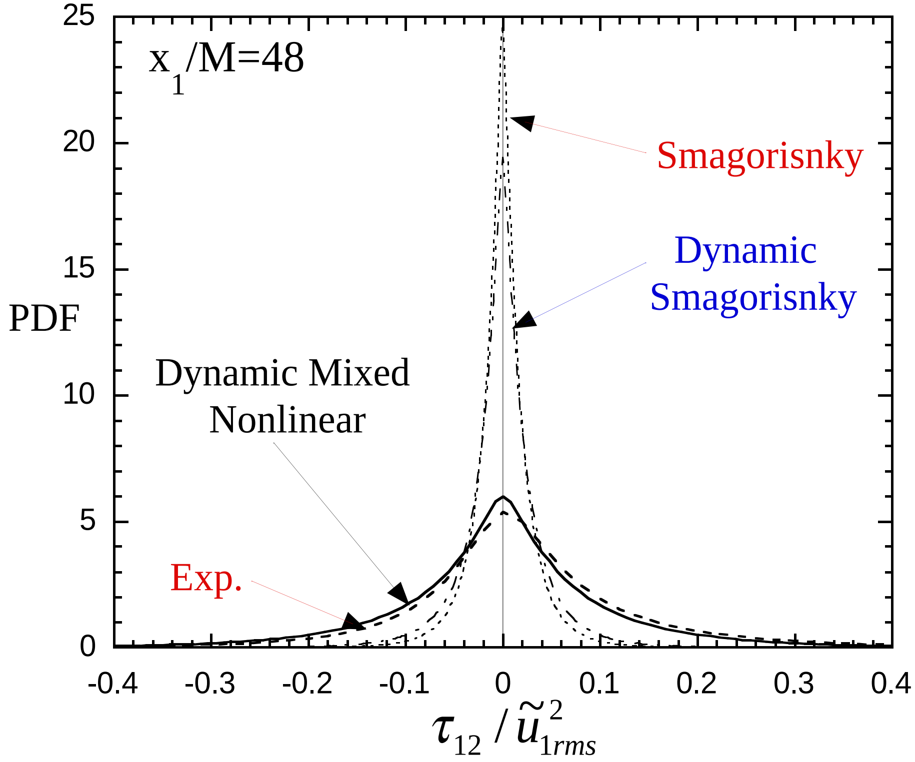
3-D Energy Spectra (LES vs experiment)

➢ Dynamic Mixed tensor eddy-visc. model



PDF of SGS Stress (LES vs experiment)

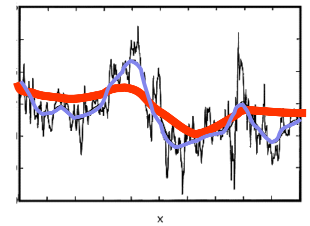
- SGS Stress $\tau_{12} \equiv \widetilde{u_1 u_2} - \tilde{u}_1 \tilde{u}_2$



Dynamic mixed tensor eddy-visc model predicts PDF of the SGS stress accurately.

Germano identity and dynamic model

(Germano et al. 1991):



Exact ("rare" in turbulence):

$$\overline{\widetilde{u_i u_j}} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j = \overline{\widetilde{u_i u_j}} \quad - \overline{\tilde{u}_i \tilde{u}_j} \; + \overline{\tilde{u}_i \tilde{u}_j} \quad - \bar{\tilde{u}}_i \bar{\tilde{u}}_j$$

$$T_{ij} = \qquad \bar{\tau}_{ij} \quad + \quad L_{ij}$$

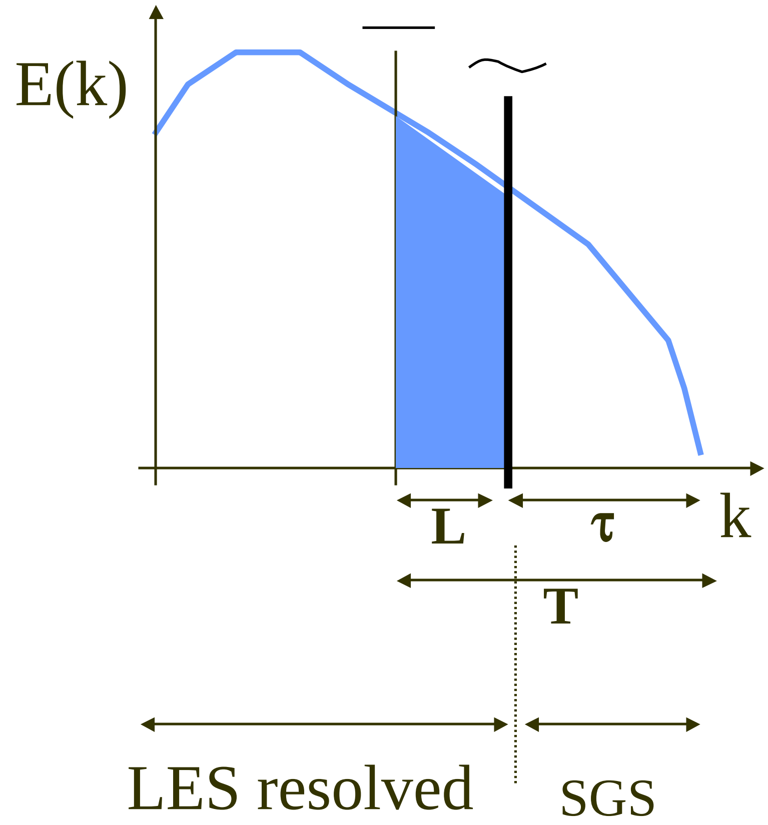
$$L_{ij} - (T_{ij} - \bar{\tau}_{ij}) = 0$$

$$-2(c_s 2\Delta)^2 \, |\bar{\tilde{S}}| \bar{\tilde{S}}_{ij} \qquad -2(c_s \Delta)^2 \, \overline{|\tilde{S}| \tilde{S}_{ij}}$$

Assumes scale-invariance:

$$L_{ij} - c_s^2 M_{ij} = 0$$

where $M_{ij} = 2\Delta^2 \left(\overline{|\tilde{S}| \tilde{S}_{ij}} - 4 |\bar{\tilde{S}}| \bar{\tilde{S}}_{ij} \right)$



Germano identity and dynamic model

(Germano et al. 1991):

$$L_{ij} - c_s^2 M_{ij} = 0$$

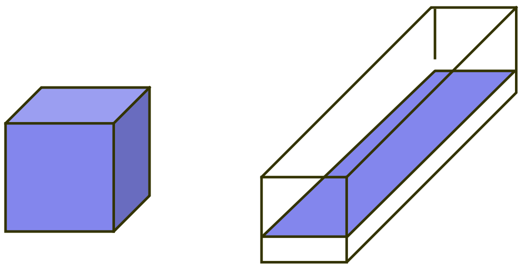
Over-determined system:
solve in "some average sense"
(minimize error, Lilly 1992):

$$\mathrm{E} = \left\langle \left(L_{ij} - c_s^2 M_{ij} \right)^2 \right\rangle$$

Minimized when:

$$c_s^2 = \frac{\langle L_{ij} M_{ij} \rangle}{\langle M_{ij} M_{ij} \rangle}$$

Averaging over regions of
statistical homogeneity
or fluid trajectories



Problem: what to do for non-homogeneous flows without directions over which to average ("learn", or "assimilate" larger-scale statistics?)

Lagrangian dynamic model (M, Lund & Cabot, JFM 1996):

Average in time, following fluid particles for Galilean invariance:

$$\langle E \rangle = \int_{-\infty}^{t} \left(L_{ij} - C_s^2 M_{ij} \right)^2 \frac{1}{T} e^{-\frac{(t-t')}{T}} dt'$$

Lagrangian dynamic model (M, Lund & Cabot, JFM 1996):

Average in time, following fluid particles for Galilean invariance:

$$\langle E \rangle = \int_{-\infty}^{t} \left(L_{ij} - C_s^2 M_{ij} \right)^2 \frac{1}{T} e^{-\frac{(t-t')}{T}} dt'$$

$$\delta \langle E \rangle = 0 \;\Rightarrow\; C_s^2 = \frac{\int_{-\infty}^{t} L_{ij} M_{ij} \frac{1}{T} e^{-\frac{(t-t')}{T}} dt'}{\int_{-\infty}^{t} M_{ij} M_{ij} \frac{1}{T} e^{-\frac{(t-t')}{T}} dt'}$$

$$\Im_{LM} = \int_{-\infty}^{t} L_{ij} M_{ij} \frac{1}{T} e^{-\frac{(t-t')}{T}} dt'$$

$$\Im_{MM} = \int_{-\infty}^{t} M_{ij} M_{ij} \frac{1}{T} e^{-\frac{(t-t')}{T}} dt'$$

Lagrangian dynamic model (M, Lund & Cabot, JFM 1996):

Average in time, following fluid particles for Galilean invariance:

$$\langle E \rangle = \int_{-\infty}^{t} \left(L_{ij} - C_s^2 M_{ij} \right)^2 \; \frac{1}{T} e^{-\frac{(t-t')}{T}} \, dt'$$

$$\delta \langle E \rangle = 0 \;\Rightarrow\; C_s^2 = \frac{\int_{-\infty}^{t} L_{ij} M_{ij} \; \frac{1}{T} e^{-\frac{(t-t')}{T}} \, dt'}{\int_{-\infty}^{t} M_{ij} M_{ij} \; \frac{1}{T} e^{-\frac{(t-t')}{T}} \, dt'}$$

$$\Im_{LM} = \int_{-\infty}^{t} L_{ij} M_{ij} \; \frac{1}{T} e^{-\frac{(t-t')}{T}} \, dt'$$

$$\Im_{MM} = \int_{-\infty}^{t} M_{ij} M_{ij} \; \frac{1}{T} e^{-\frac{(t-t')}{T}} \, dt'$$

With exponential weight-function, equivalent to relaxation forward equations:

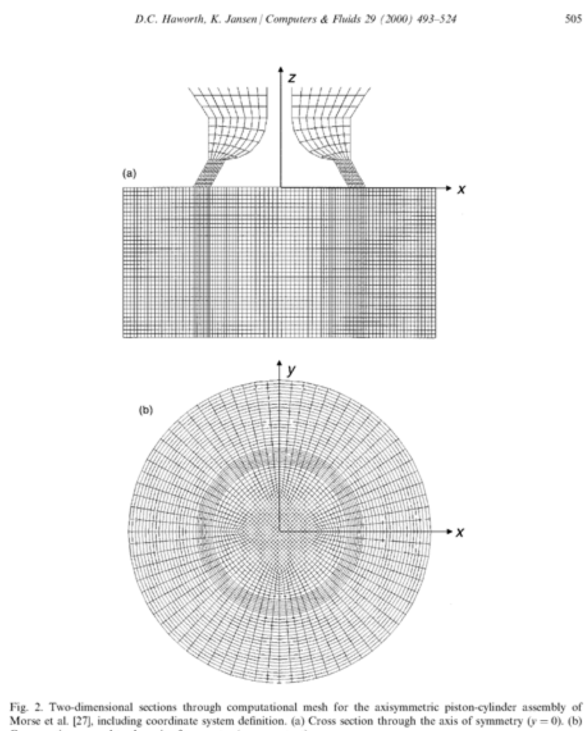
$$\frac{\partial \Im_{LM}}{\partial t} + \tilde{u}_k \frac{\partial \Im_{LM}}{\partial x_k} = \frac{1}{T} (L_{ij} M_{ij} - \Im_{LM})$$

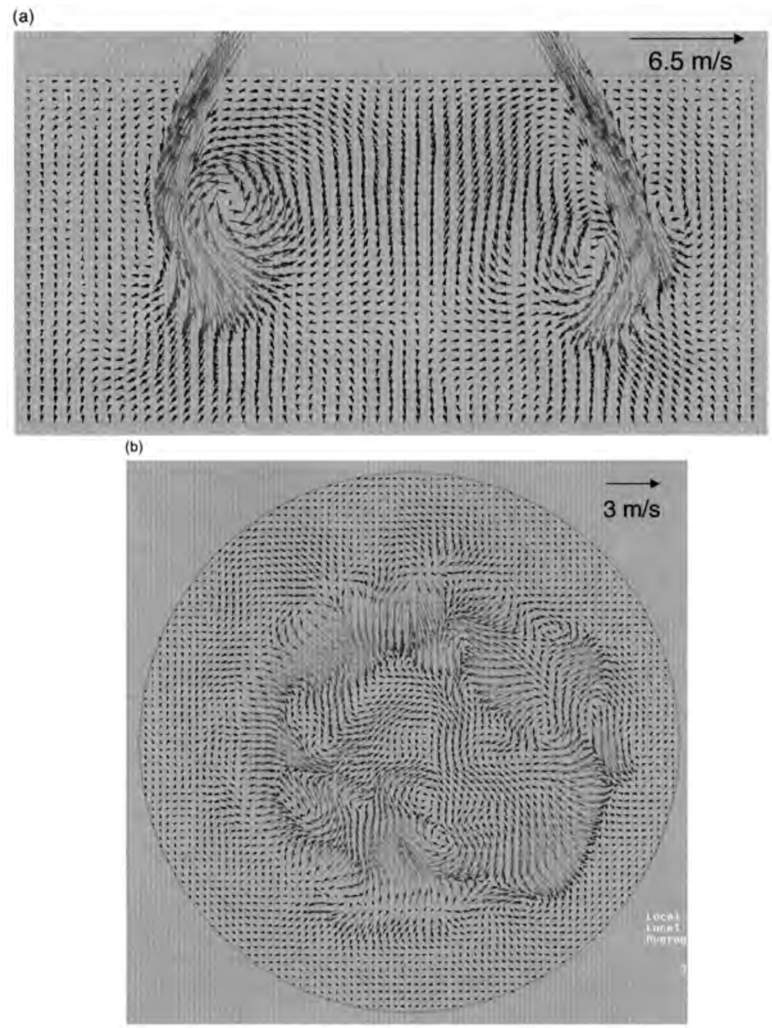
$$\frac{\partial \Im_{MM}}{\partial t} + \tilde{u}_k \frac{\partial \Im_{MM}}{\partial x_k} = \frac{1}{T} (M_{ij} M_{ij} - \Im_{MM})$$

$$c_s^2 = \frac{\Im_{LM}(\mathbf{x}, t)}{\Im_{MM}(\mathbf{x}, t)}$$

Lagrangian dynamic model has allowed applying the Germano-identity to a number of complex-geometry engineering problems

LES of flows in internal combustion engines:
Haworth & Jansen (2000)
Computers & Fluids **29**.





Examples:

LES of flow over wavy walls
Armenio & Piomelli (2000)
Flow, Turb. & Combustion.

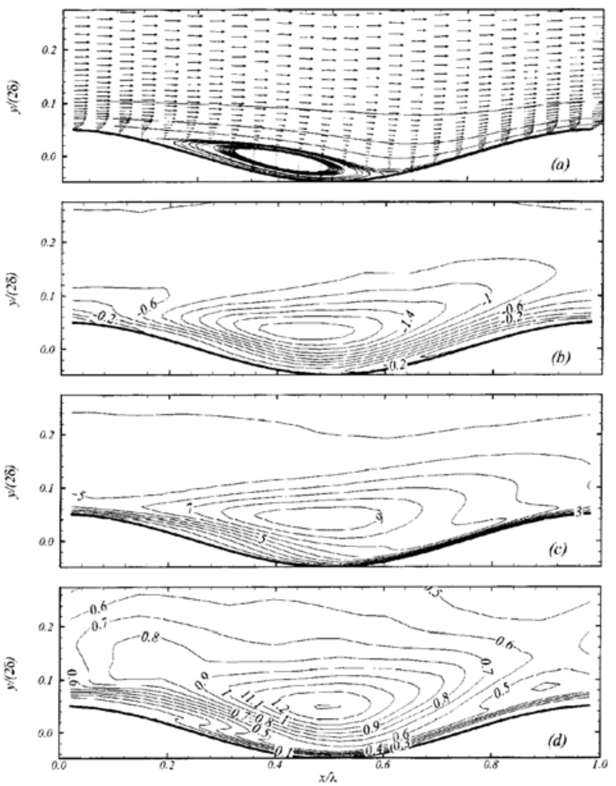
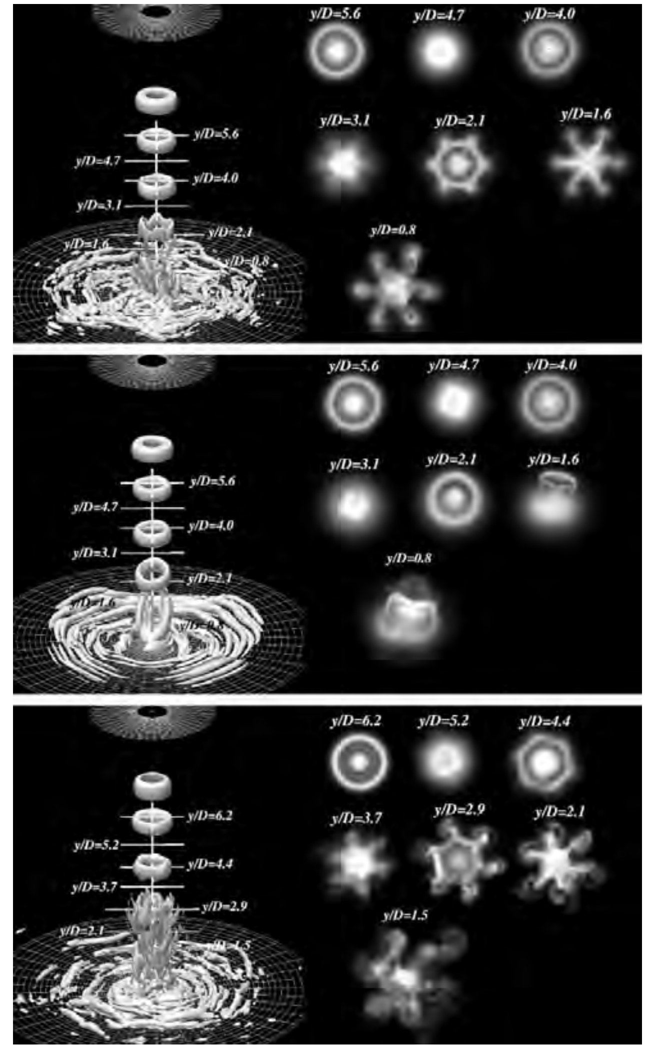


Figure 12. Wavy channel, $2a/\lambda = 0.1$; C1W simulation. (a) Mean velocity vectors and streamlines; (b) Reynolds stress $\langle u'v' \rangle$; (c) q^2; (d) normalized eddy viscosity.

4.2. Large-amplitude wave

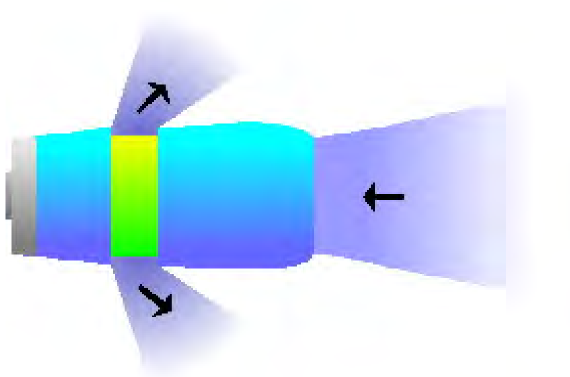
The grid and the flow parameters used for the simulation of the flow over a large-amplitude wavy wall were reported in Table III. As previously pointed out, the parameters have been chosen to fit the experiments of B93 and the LES of HS99.

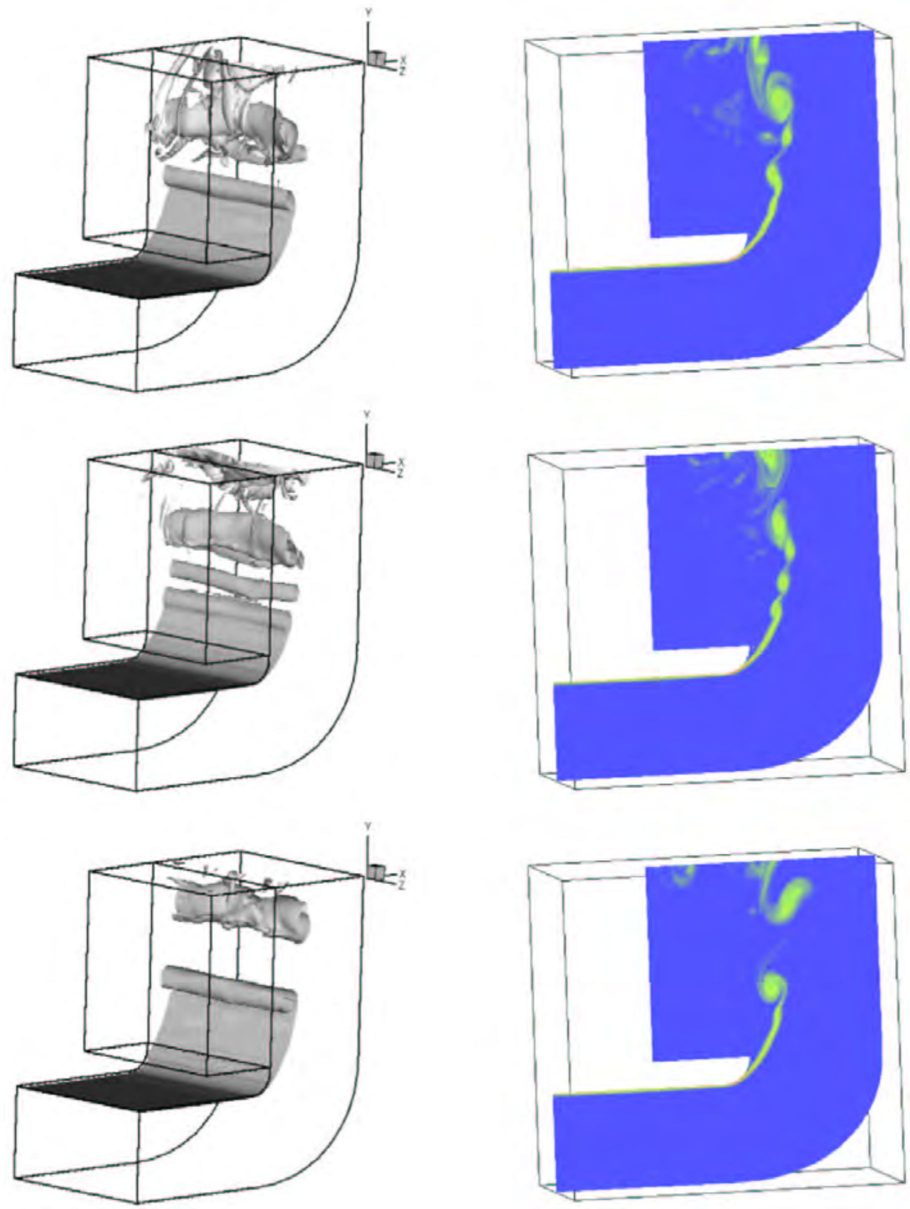
LES of structure of impinging jets:
Tsubokura et al. (2003)
Int Heat Fluid Flow **24**.



Examples:

LES of flow in thrust-reversers
Blin, Hadjadi & Vervisch (2002)
J. of Turbulence.

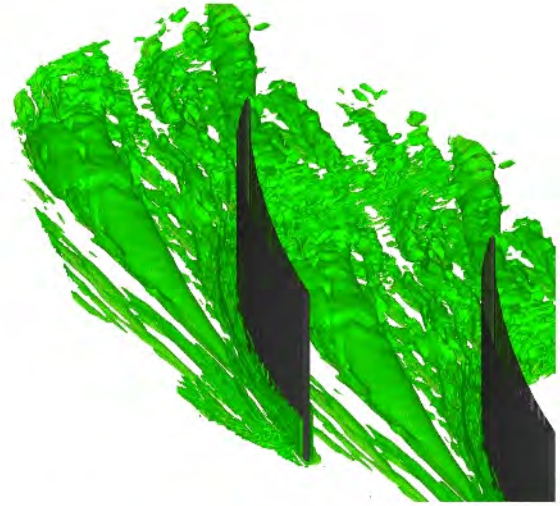


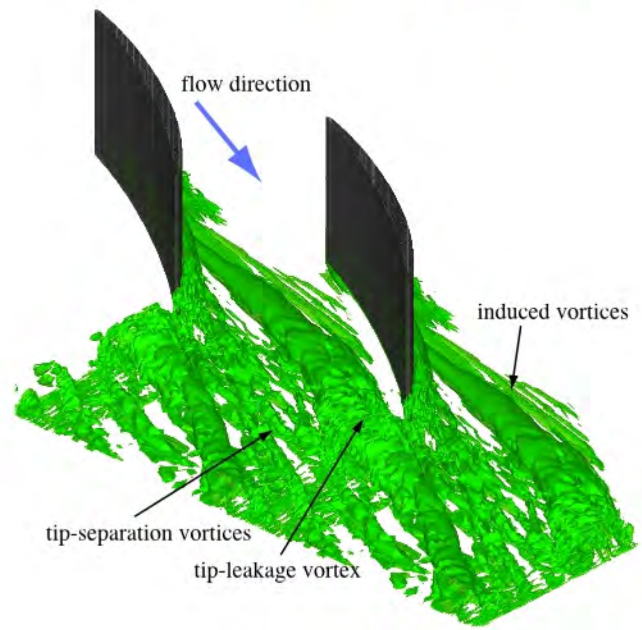


Examples:

LES of flow in turbomachinery
Zou, Wang, Moin, Mittal. (2007)
Journal of Fluid Mechanics.

(a)



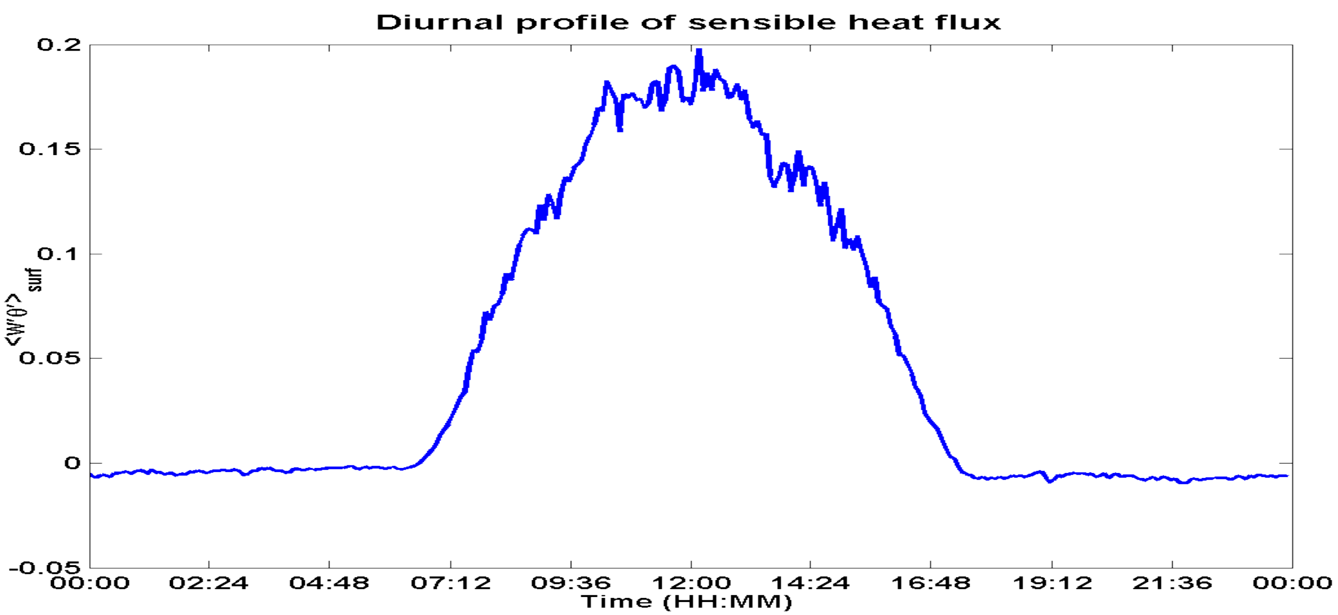


Examples:

LES of convective atmospheric boundary layer:
Kumar, M. & Parlange (Water Resources Research, 2006)

- Transport equation for temperature
- Boussinesq approximation
- Coriolis forcing
- Lagrangian dynamic model with assumed $\beta = C_s(2\,\Delta)/C_s(\Delta)$
- Constant (non-dynamic) SGS Prandtl number $Pr_{sgs}=0.4$
- Imposed surface flux of sensible heat on ground
- Diurnal cycle: start stably stratified, then heating….

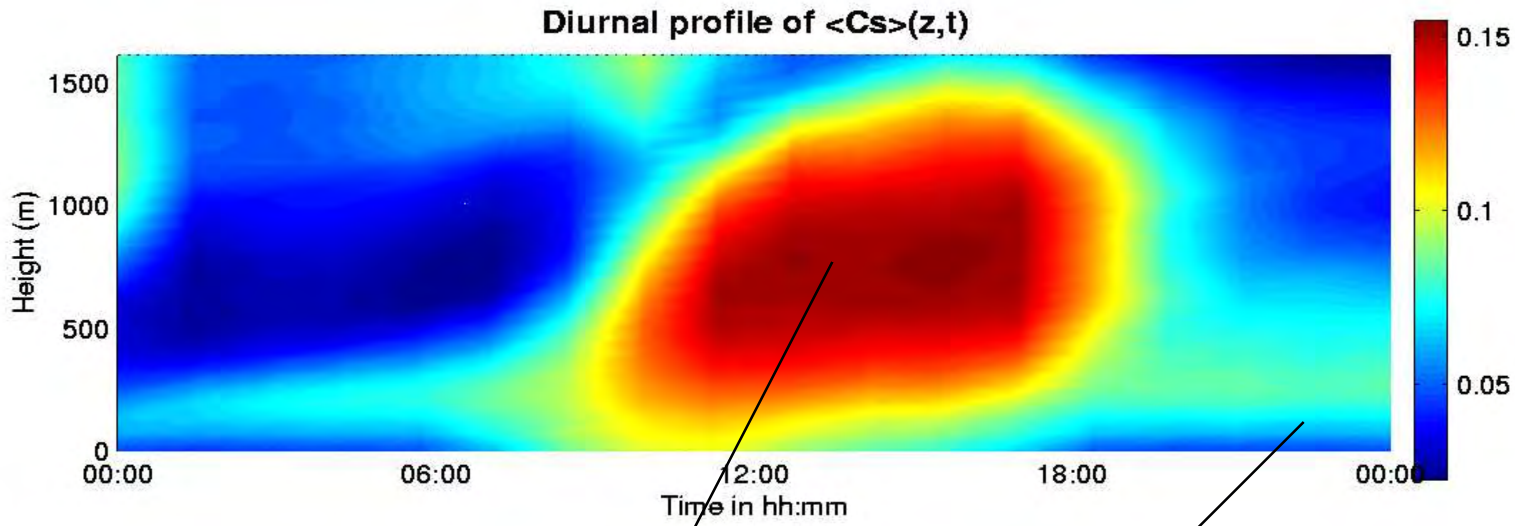
Imposed ground heat flux during day:



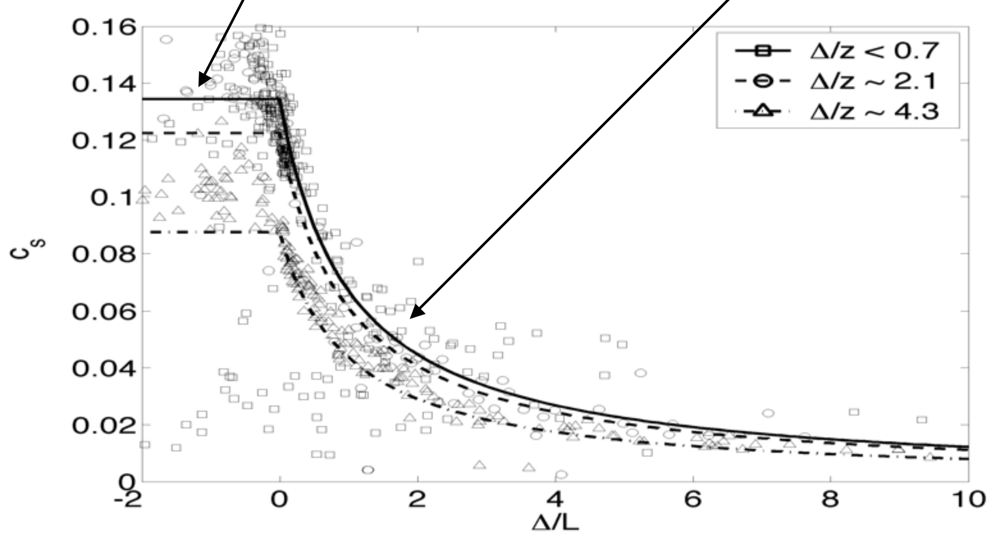
Examples:

- Diurnal cycle: start stably stratified, then heating….

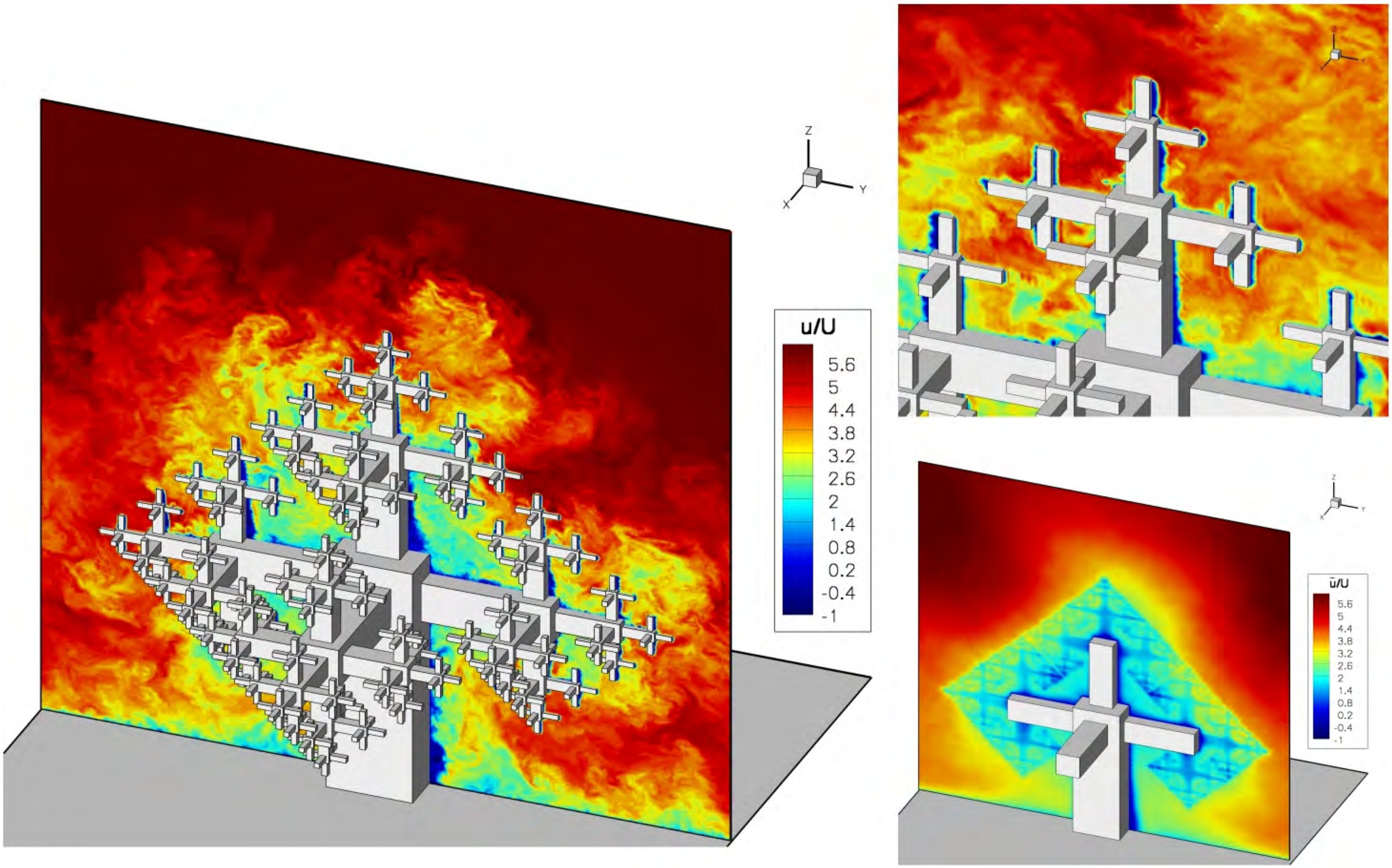
Resulting dynamic coefficient (averaged):



Consistent with HATS field measurements:

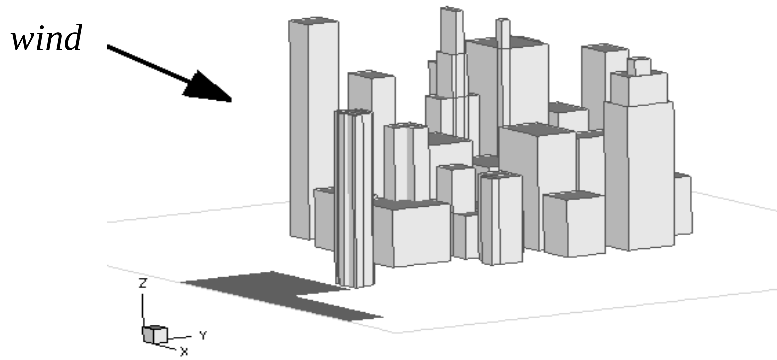


Large-eddy-Simulation of atmospheric flow over fractal trees:

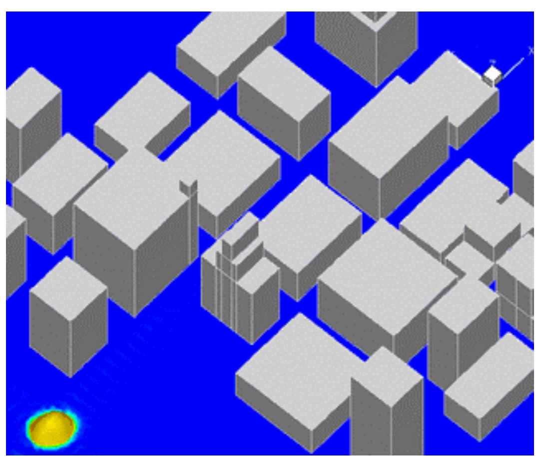


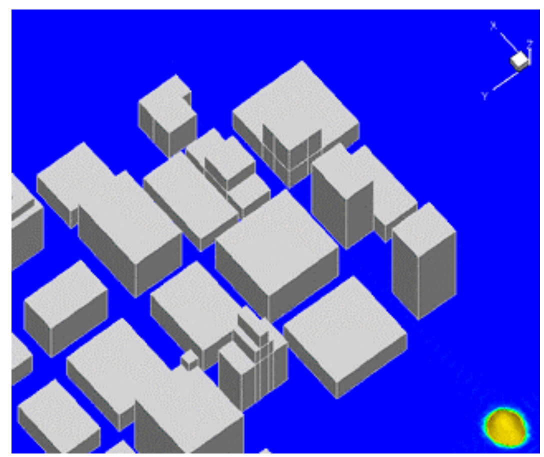
URBAN CONTAMINATION AND TRANSPORT

Downtown Baltimore:



Momentum and scalar transport equations solved using LES and Lagrangian dynamic subgrid model. Buildings are simulated using immersed boundary method.





Yu-Heng Tseng, C. Meneveau & M. Parlange, 2006 (Env. Sci & Tech. **40**, 2653-2662)

Useful references on LES and SGS modeling:

- P. Sagaut: “Large Eddy Simulation of Incompressible Flow” (Springer, 3rd ed., 2006)
- U. Piomelli, Progr. Aerospace Sci., 1999
- C. Meneveau & J. Katz, Annu Rev. Fluid Mech. **32**, 1-32 (2000)
- C. Meneveau, Scholarpedia **5**, 9489 (2010).