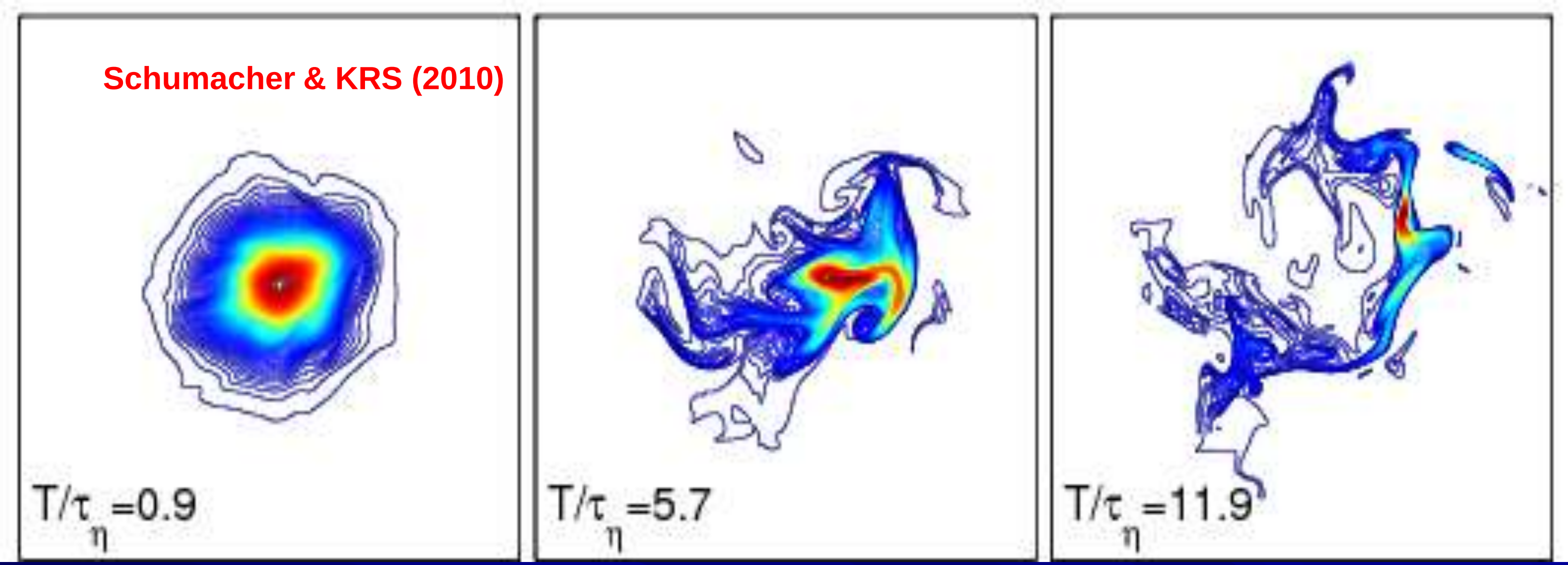
Problems in mixing additives



Prasad & KRS, *Phys. Fluids A* **2**, 792 (1990);
P. Constantin, I. Procaccia & KRS, *Phys. Rev. Lett.* **67**,1739 (1991)

Phys. Fluids **14**, 4178-4191, 2002; **15**, 84-90, 2003; **17**, 081703-6, 2005; **17**, 125107-1-9, 2005; **20**, 045108, 2008

J. Fluid Mech. **479**, 4178-4191, 2003; **531**, 113-122, 2005; **532**, 199-216, 2005

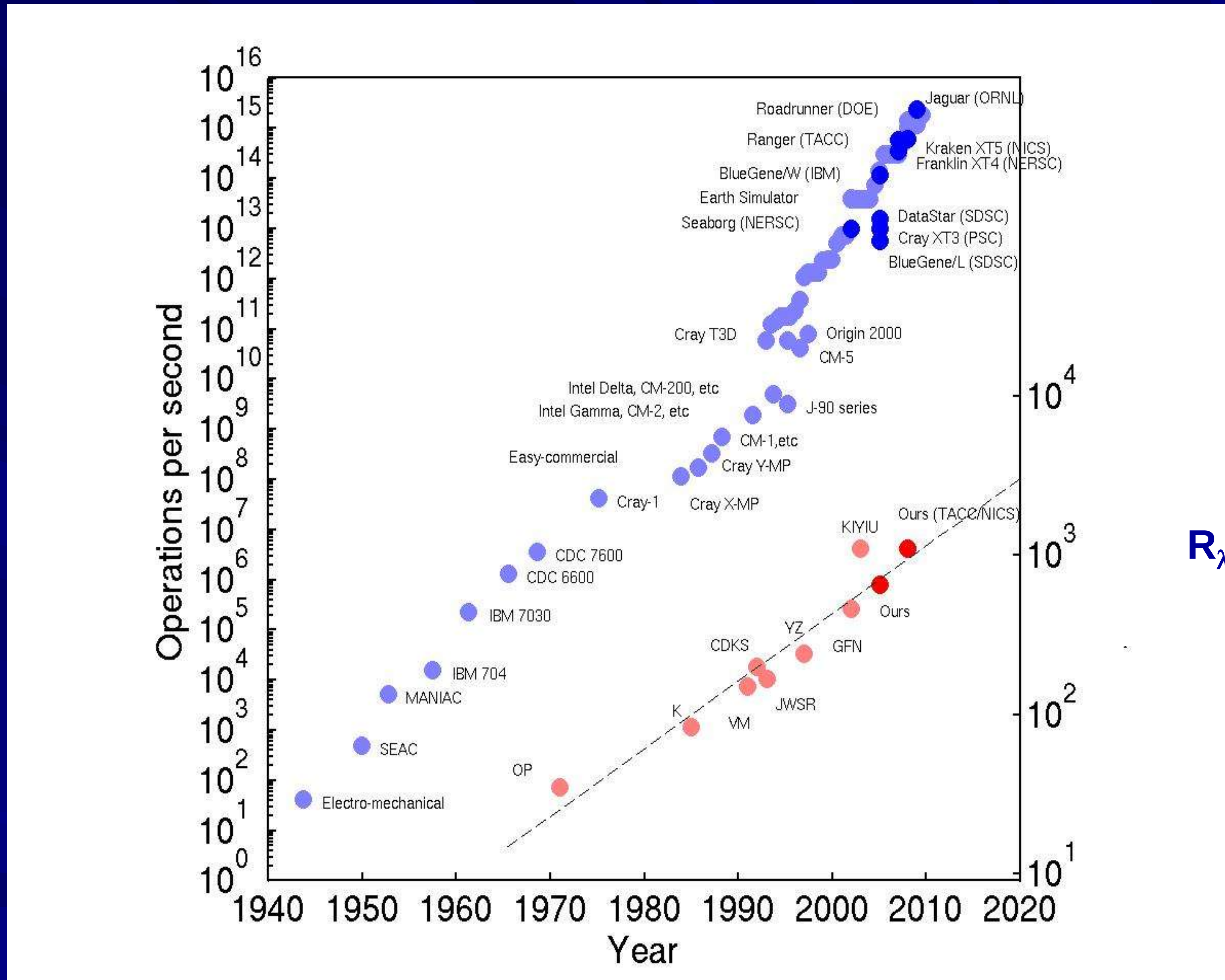
Phys. Rev. Lett. **91**, 174501-504, 2003

Flow, Turbulence and Combustion, **72**, 115-131, 2004; **72**, 333-347, 2004; 2010 (to appear)

P.K. Yeung
Georgia Tech.

Diego Donzis
Texas A&M

Jörg Schumacher
TU Ilmenau



Advection diffusion equation

$$\partial\theta/\partial t + \mathbf{u}.\nabla\theta = \kappa\nabla^2\theta$$

$\theta(\mathbf{x};t)$, the tracer; κ, its diffusivity (usually small); $\mathbf{u}(\mathbf{x};t)$, the advection velocity

Quite often $\mathbf{u}(\mathbf{x};t)$ does not depend on the additive: this is the case of the "passive scalar".

$\mathbf{u}(\mathbf{x};t)$ then obeys the same field equations as those without the additive: e.g., NS = 0.

Equation is then linear with respect to θ.

As a rule, BCs are also linear

(perhaps mixed)

Linearity holds for each realization but the equation is statistically nonlinear because of $\langle \mathbf{u}.\nabla\theta \rangle$, etc.

Langevin equation

$$d\mathbf{x} = \mathbf{u}(\mathbf{x}(t);t)\,dt + (2\kappa)^{1/2}\,d\boldsymbol{\chi}(t)$$

$\chi(t)$ = vectorial Brownian motion,

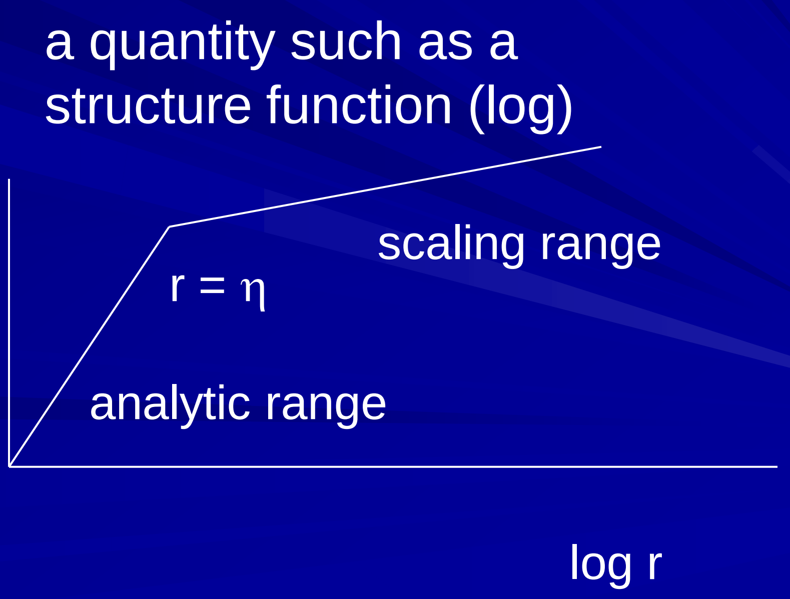
statistically independent in its three components

For smooth velocity fields, single-particle diffusion as well as two-particle dispersion are well understood.

The turbulent velocity field is analytic only in the range $r < \eta$, and Hölder continuous, or "rough," in the scaling range ($\Delta_r u \sim r^h$, $h < 1$), which introduces various subtleties.

$h = 1/3$ for Kolmogorov turbulence.

In practice, it has a distribution: "multiscaling"

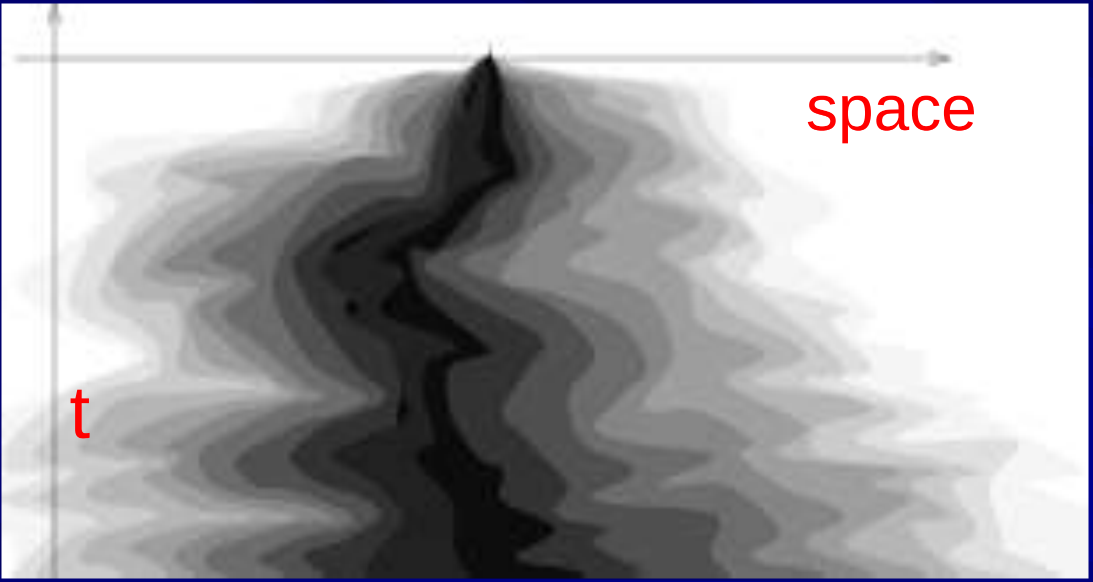


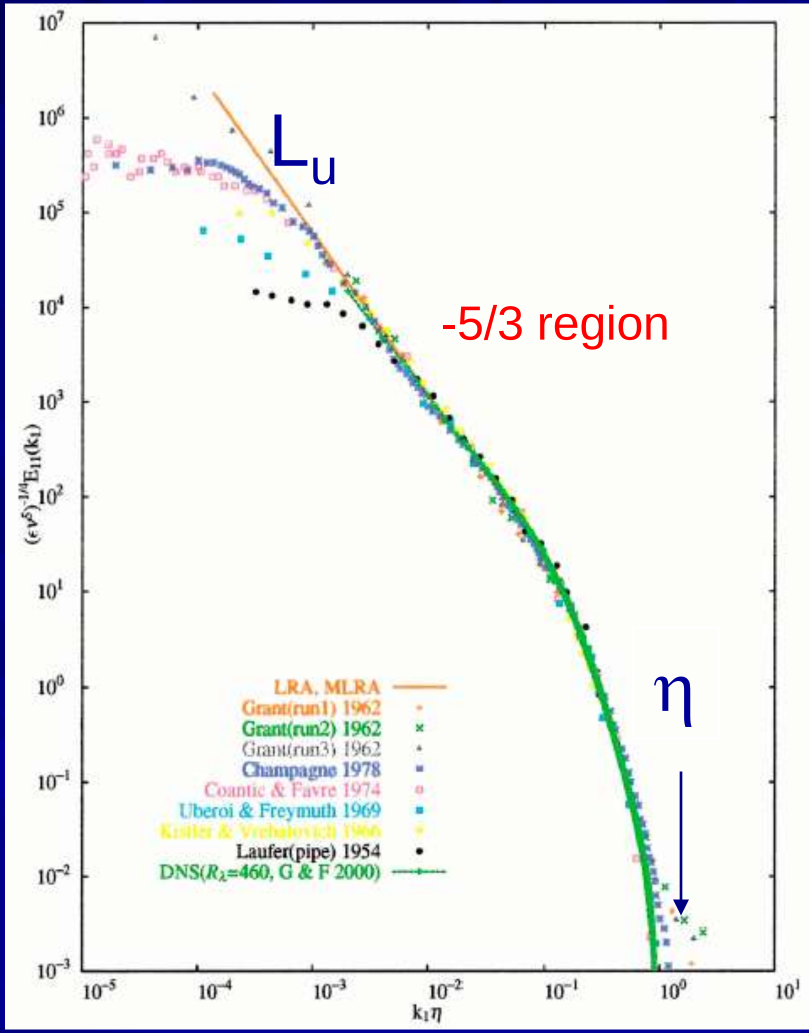
C. Meneveau & KRS, J. Fluid Mech. 224, 429 (1991)
KRS, *Annu. Rev. Fluid Mech*. **23**, 539 (1991)

If $\Delta_r u \sim r^h$, h <1

$\Delta_r u\ (t) \sim t^{1/(1-h)}$, and

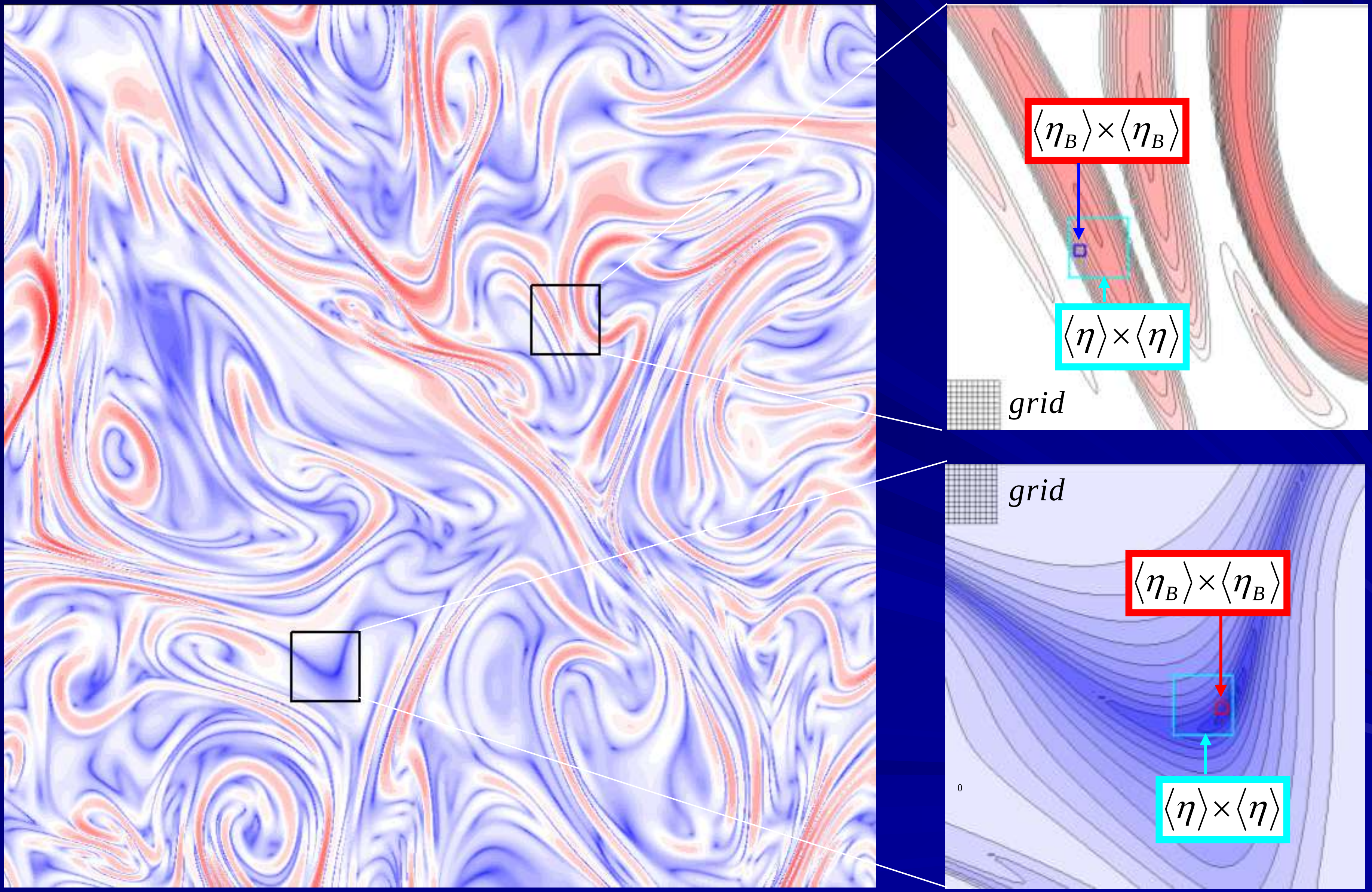
memory is lost rapidly.





Lagrangian trajectories are “not unique”

For short times, diffusion effects are additive. The finite time behavior is different.



Model studies

- Assume some artificial velocity field satisfying div **u** = 0
- see A.J. Majda & P.R. Kramer, *Phys. Rep.* **314**, 239 (1999)

Broad-brush summary of results

1. For smooth velocity fields (e.g., periodic and deterministic), homogenization is possible. That is,

   $$\langle \mathbf{u}(\mathbf{x};t)\, \nabla(\theta) \rangle = -(\boldsymbol{\kappa}_T \cdot \nabla(\theta(\mathbf{x};t)))$$

   where $\boldsymbol{\kappa}_T$ is an effective diffusivity (Varadhan, Papanicolaou, Majda, and others)
2. Velocity is a homogeneous random field, but a scale separation exists: $L_u/L_\theta \ll 1$. Homogenization is possible here as well.
3. Velocity is a homogeneous random field but delta correlated in time, $L_u/L_\theta = O(1)$; eddy diffusivity can be computed.
4. For the special case of shearing velocity (with and without transverse drift), the problem can be solved essentially completely: eddy diffusivity, anomalous diffusion, etc., can be calculated without any scale separation. See, e.g., G. Glimm, B. Lundquist, F. Pereira, R. Peierls, *Math. Appl. Comp.* **11**, 187 (1992); Avellaneda & Majda, *Phil. Trans. Roy. Soc. Lond. A* **346**, 205 (1994); G. Ben Arous & H. Owhadi, *Comp. Math. Phys.* **237**, 281 (2002)

II. Kraichnan model

R.H. Kraichnan, *Phys. Fluids* **11**, 945 (1968); *Phys. Rev. Lett.* **72,** 1016 (1994)

Review: G. Falkovich, K. Gawedzki & M. Vergassola, *Rev. Mod. Phys.* **73**, 913 (2001)

Surrogate Gaussian velocity field

$\langle v_i(\mathbf{x};t)v_j(\mathbf{y};t')\rangle = D_{ij}(\mathbf{x}-\mathbf{y})\delta(t-t')$

$D_{ij} \sim |\mathbf{x}-\mathbf{y}|^{2-\gamma}$, $\gamma = 2/3$ recovers Richardson's law of diffusion

Forcing for stationarity:

$\langle f_\theta(\mathbf{x};t)f_\theta(\mathbf{y};t')\rangle = C(r/L)\delta(t-t')$

C(r/L) is non-zero only on the large scale, decays rapidly to zero for smaller scale.

OUTSTANDING CHALLENGES

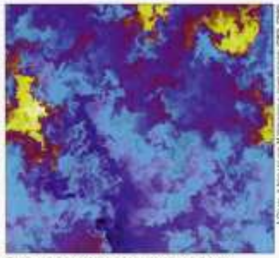
Turbulence nears a final answer

From **Uriel Frisch** at the Observatoire de la Côte d'Azur, Nice, France

The great Italian scientist Leonardo da Vinci was the first person to use the word "turbulence" (or *turbolenza*) to describe the complex motion of water or air. By carefully examining the turbulent wakes created behind obstacles placed in the path of a fluid, he found that there are three key stages to turbulent flow. Turbulence is first generated near an obstacle. Long-lived "eddies" – beautiful whirls of fluid – are then formed. Finally, the turbulence rapidly decays away once it has spread far beyond the obstacle.

However, it was not until the early 19th century that Claude Navier was able to write the basic equations governing how the velocity of a turbulent fluid evolves with time. Navier realized that the earlier equations of Leonhard Euler for ideal flow had to be supplemented by a diffusion term that took into account the viscosity of the fluid.

A few decades later, Adhemar de Saint-Venant noticed that turbulent flow – for example in a wide channel – has a much higher "effective" viscosity than the laminar flow found, for instance, in a capillary. It turns out that the turbulent transport of momentum, heat and pollutants can be 3–15 orders of magnitude more efficient than the predictions based on Maxwell's kinetic theory for transport in laminar flow. Indeed, if it were not for turbulence, pollution in our cities would linger for millennia, the heat generated by nuclear reactions



Concentration of a passive scalar, such as a pollutant, advected by a turbulent flow of the type found in the atmosphere or oceans, simulated numerically on a 2048 × 2048 grid. The scalar displays strong "intermittency" and has anomalous scaling properties that cannot be predicted by simple dimensional analysis. Low concentrations are dark, high ones are light

stand what is known as fully developed turbulence (FDT) in the case of a high Reynolds number – a non-dimensional parameter that essentially describes the relative sizes of the fluid's internal and viscous forces. The molecular viscosity then acts only at scales much smaller than those at which the instabilities drive the turbulence.

A coherent picture of FDT first emerged around the middle of this century, thanks to work by Kolmogorov, Lewis Fry Richardson, Lars Onsager and, again, many others. In the language of modern physics, it was pos-

invariance is actually broken and that fully developed turbulence is "intermittent". In other words, the exponents have anomalous values that cannot be predicted by dimensional analysis – they are instead universal, being independent of how the turbulence is produced. The intermittency also means that the small-scale turbulent activity looks "spotty", and the dissipation of energy has fractal properties – in other words energy is dissipated in a cascade of energy transfers to smaller and smaller scales. Roberto Benzi, Benoît Mandelbrot, Steven Orszag, Patrick Tabeling and many others have been involved in the development of such work.

For many years, only models that were rather loosely connected with the traditional equations of fluid dynamics were available to describe this intermittency. Early models were developed by Kolmogorov and colleagues in the 1960s, while in the 1980s the concept of "multifractal" was introduced by Giorgio Parisi and the author.

A few years ago Robert Kraichnan predicted that intermittency and anomalous scaling are already present in a much simpler problem where the governing dynamics are linear – namely for a passive scalar, such as a pollutant advected by a scale-invariant turbulent velocity, which is not intermittent itself. This phenomenon can be studied by numerical simulations (see figure).

Methods borrowed in part from modern field theory have recently led to a real breakthrough for Kraichnan's problem. For the first time we have a theory of intermittency

For a number of outstanding and unanswered issues, see:
KRS & J. Schumacher, *Phil. Trans. Roy. Soc. Lond. A* **368**, 1561 (2010)

the type found in aeronautical applications or in turbomachinery have been developed by Ludwig Prandtl, Andrei Kolmogorov, Brian Spalding and many others.

This progress has depended on an ever-increasing theoretical and experimental understanding of the physics of turbulence, and I can do no more than point to the crucial contributions of Lord Kelvin, Osborne Reynolds, Geoffrey Ingram Taylor, Jean Leray, Theodor von Kármán and many others. I will thus turn to one of the major challenges in the field, which is to under-

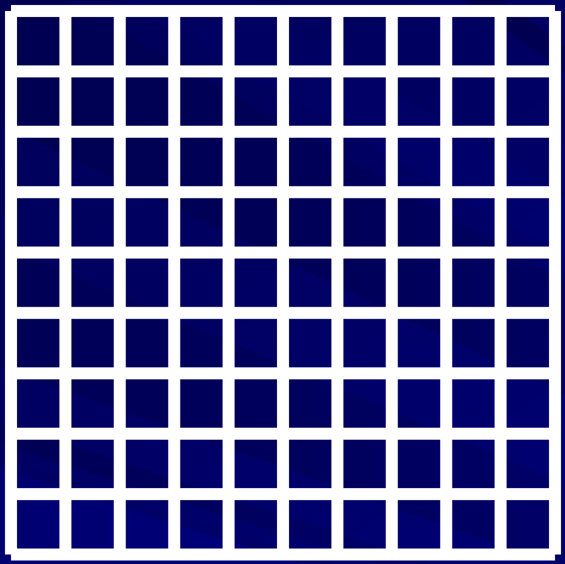
ments have been carried out to study FDT. These started with work by George Batchelor and Alan Townsend in the 1940s, right through to new table-top facilities that use low-temperature helium flowing between counter-rotating disks. New data-processing techniques that can measure scaling exponents with good accuracy have also been developed, as have advanced numerical simulations, the importance of which was first perceived by the mathematician John von Neumann.

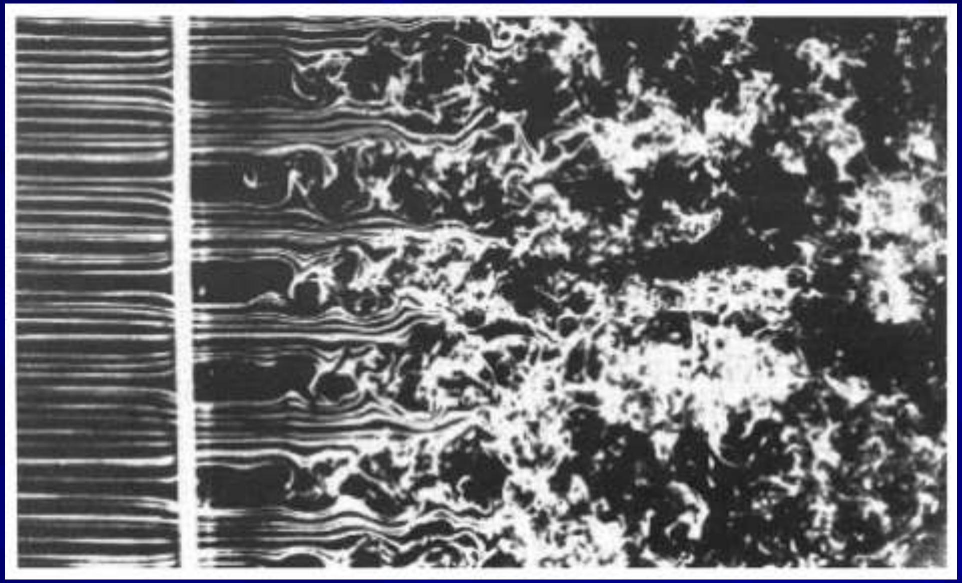
The evidence is that the assumed scale

ory Falkovich, Vladimir Lebedev and Igor Kolokolov). Non-perturbative calculations are also possible in some cases.

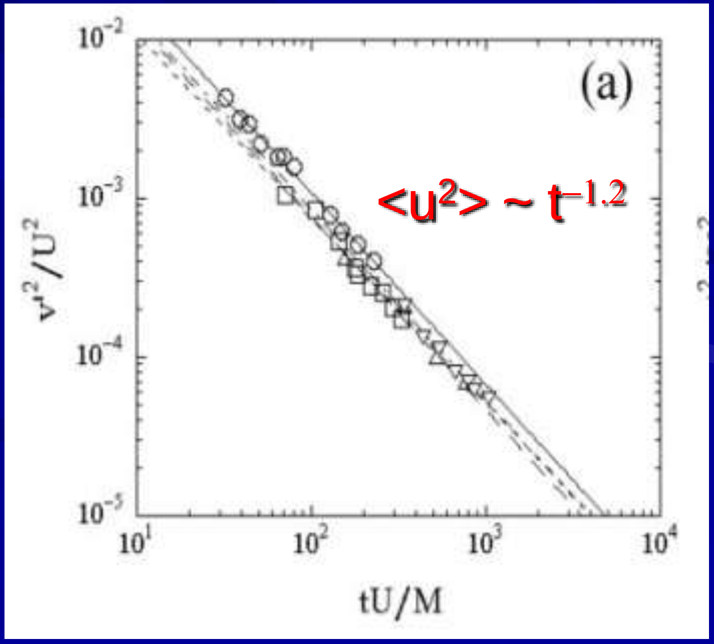
The extension of such ideas to the non-linear problem of intermittency in FDT is being actively pursued. Optimists predict that fully developed turbulence will be understood in a few years' time. But many more years may be needed to truly understand all of the complexity of turbulent flow – a problem that has been challenging physicists, mathematicians and engineers for at least half a millennium.

Decaying fields of turbulence and scalar





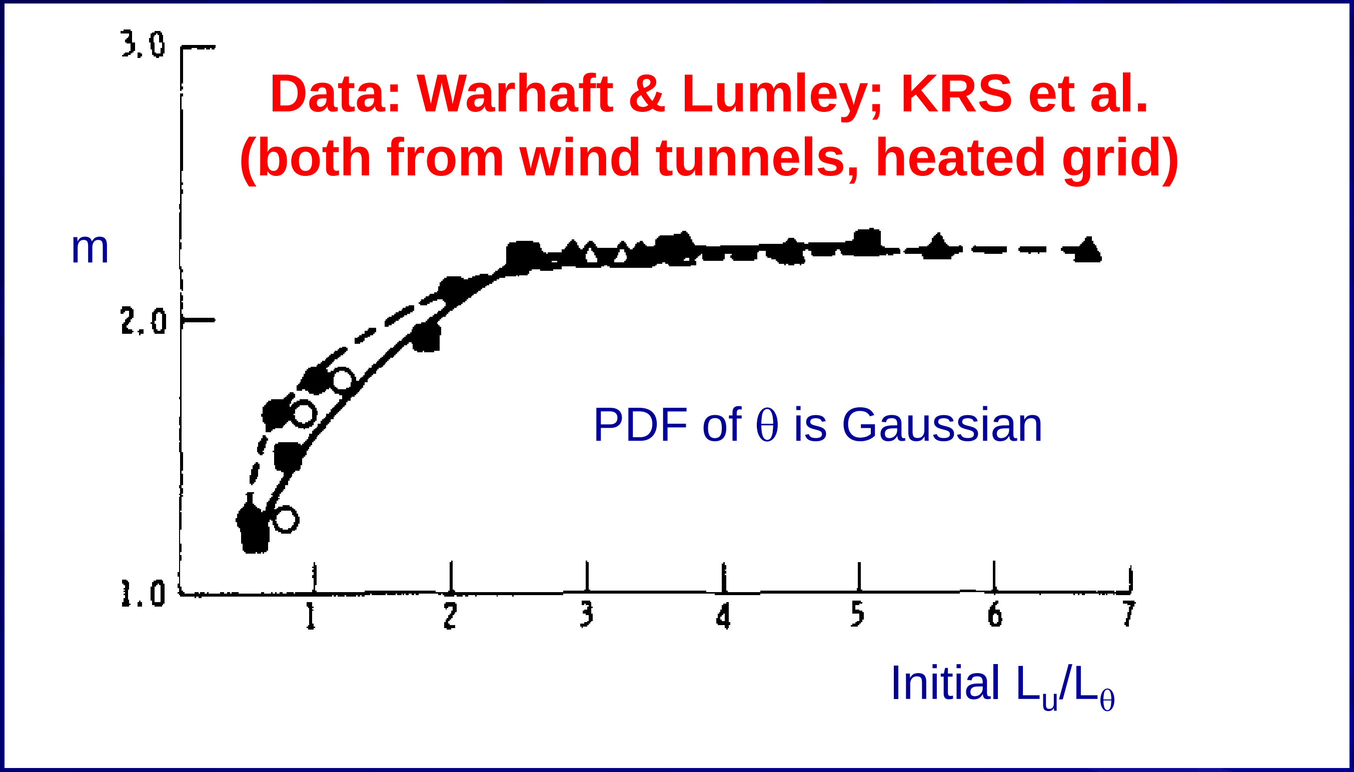
- L_u is set by the mesh size
- L_θ can be set independently and L_u/L_θ can be varied
- Diffusivity of the scalar can be varied: i.e., Pr or Sc is variable



$\langle\theta^2\rangle \sim t^{-m}$ (variable m)

$m - m_0 = f(Re; Sc; L_u/L_\theta)$?

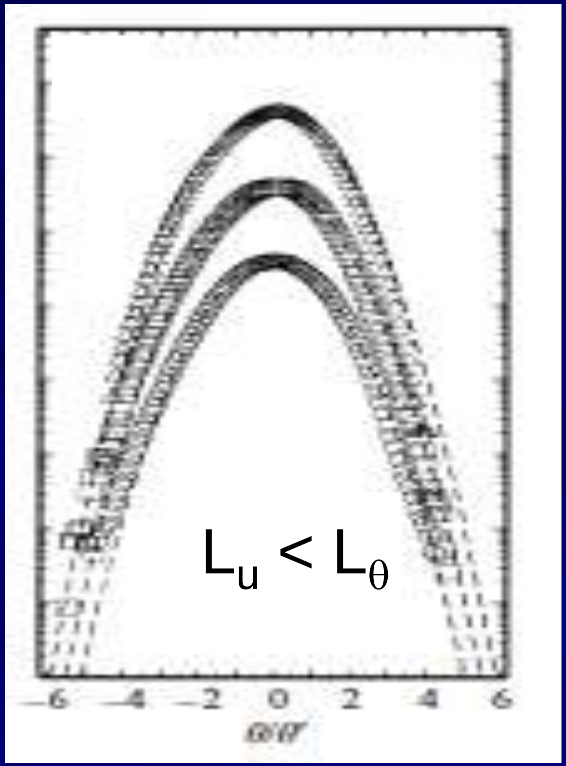
m_0: asymptotic m for large values of the arguments

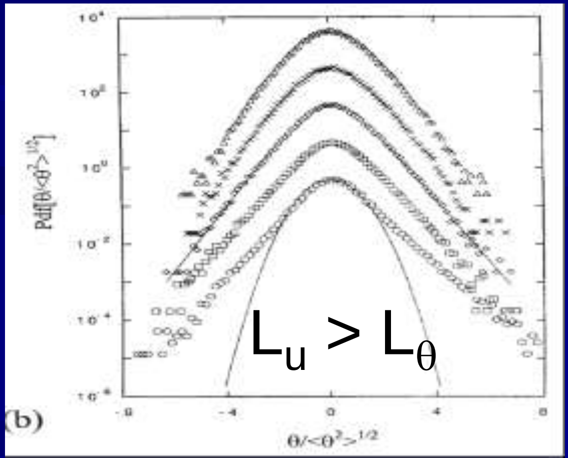


Durbin, Phys. Fluids 25, 1328 (1982)

A proper theory is needed!

Effect of length-scale ratio (stationary turbulence)



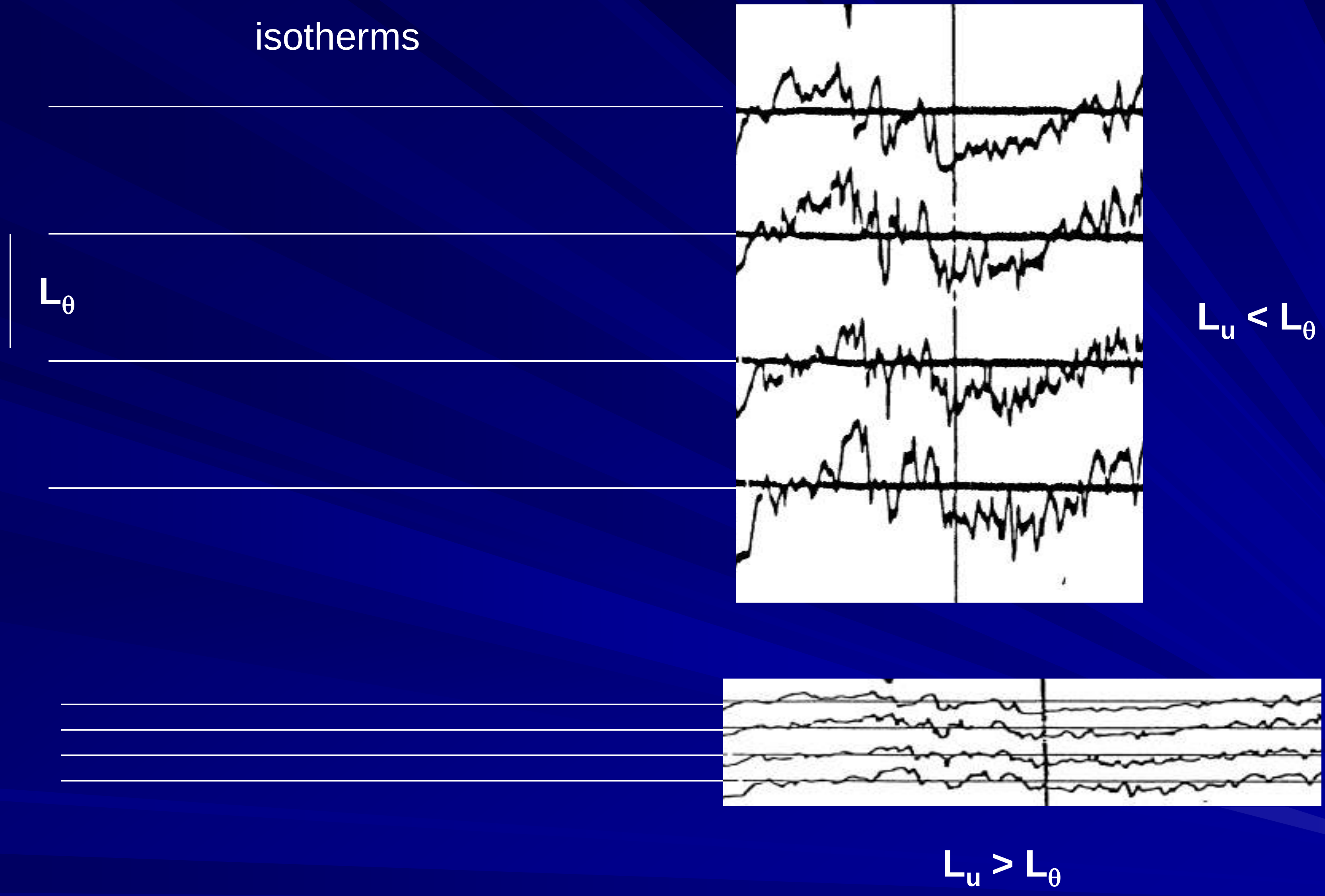


Both PDFs are for stationary velocity and scalar fields, under comparable Reynolds and Schmidt numbers.

Passive scalars in homogeneous flows most often have Gaussian tails, but long tails are observed also for column-integrated tracer distributions in horizontally homogeneous atmospheres.

Models of Bourlioux & Majda, *Phys. Fluids* **14**, 881 (2002), closely connected with models studied by Avellaneda & Majda

Probability density function of the passive scalar
Top: Ferchichi & Tavoularis (2002)
Bottom: Warhaft (2000)



Large-scale features depend on details of forcing, initial conditions and perhaps geometry. Only a few of these features are understood precisely, and our qualitative understanding rests on the models of the sort mentioned.

$$\langle \Delta_r \theta^2 \rangle \sim r^{\zeta_2}$$

$$\langle \Delta_r \theta^4 \rangle \sim r^{\zeta_4}$$

Dimensional analysis: $\zeta_4 = 2\zeta_2$

Flatness, $\langle \Delta_r \theta^4 \rangle / \langle \Delta_r \theta^2 \rangle^2 \sim r^0$, a constant

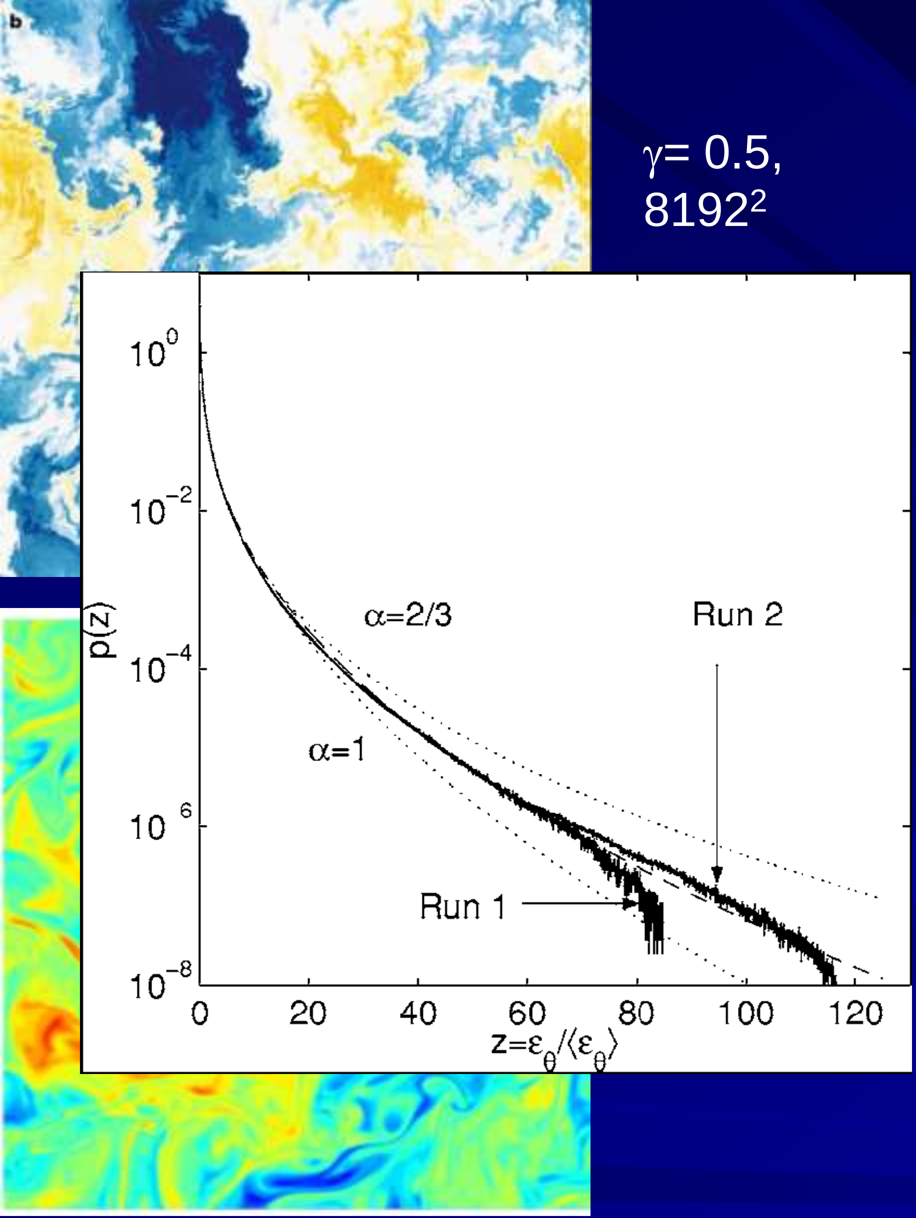
Measurements show that the flatness $\rightarrow \infty$ as $r \rightarrow 0$

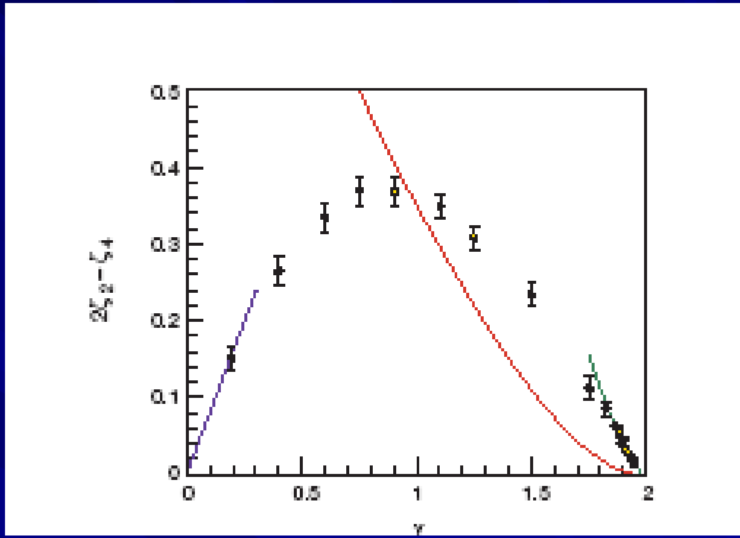
(because $\zeta_4 = 2\zeta_2$ (or generally $\zeta_{2n} < n\zeta_2$)

“Anomalous exponents”

$2\zeta_2 - \zeta_4$

$\gamma = 0.5$,
8192^2





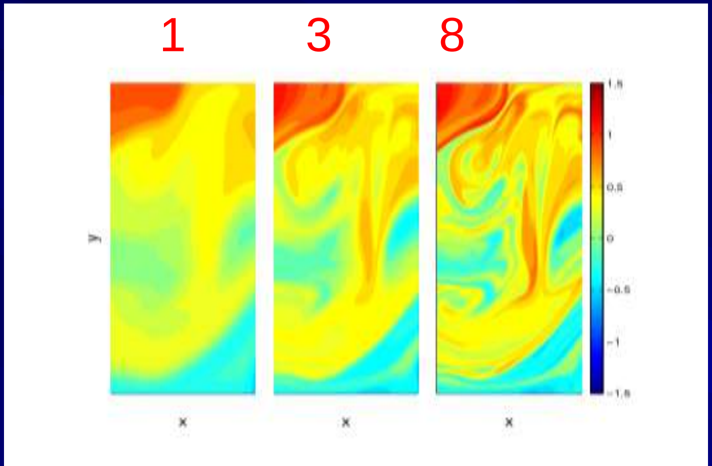
A measure of anomalous scaling, $2\zeta_2 - \zeta_4$, versus the index γ, for the Kraichnan model. The circles are obtained from Lagrangian Monte Carlo simulations. The results are compared with analytic perturbation theories (blue, green) and an ansatz due to Kraichnan (red).

Mixing process itself imprints large-scale features independent of the velocity field!

The case of large Schmidt number

Schmidt number, $Sc = \nu/\kappa \sim O(1000)$

$Sc >> 1$



$N = Re^3 Sc^2$

L

η

η_B

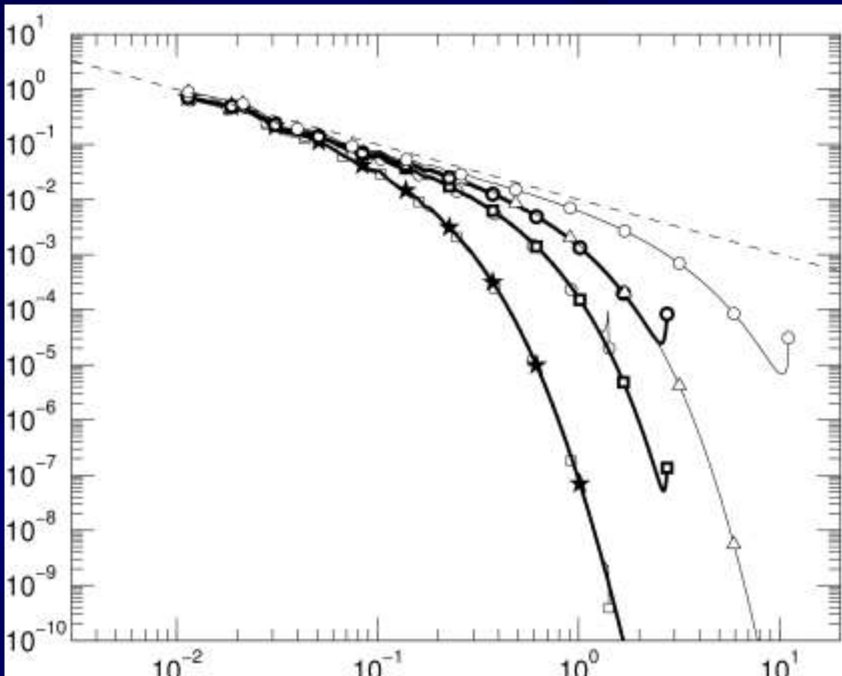
as for the velocity

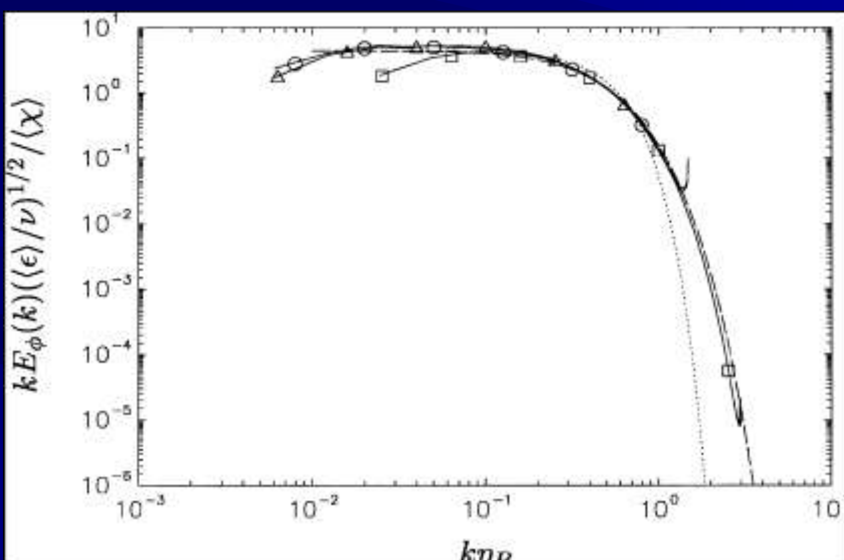
Batchelor regime

$\phi_\theta(k) \sim qk^{-1}$

$q = O(1)$

The Batchelor regime





Reynolds number: Re >>1
Schmidt number, Sc = ν/κ >>1

In support of the -1 power law

Gibson & Schwarz, JFM 16, 365 (1963)

KRS & Prasad, Physica D 38, 322 (1989)

Expressing doubts

Miller & Dimotakis, JFM 308, 129 (1996)

Williams et al. Phys. Fluids, 9, 2061 (1997)

Simulations in support

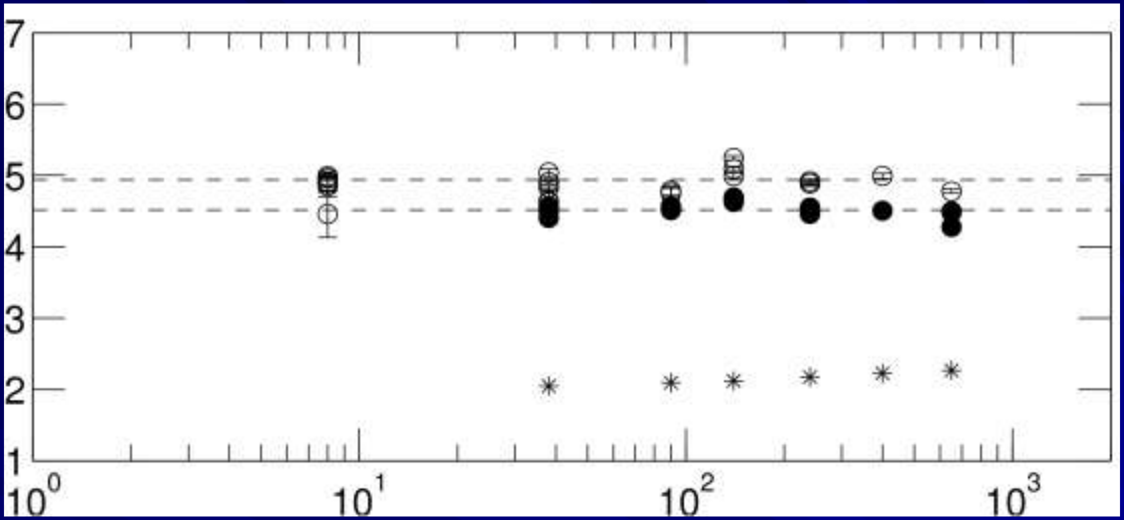
Holzer & Siggia, Phys. Fluids 6, 1820 (1994)

Batchelor (1956)

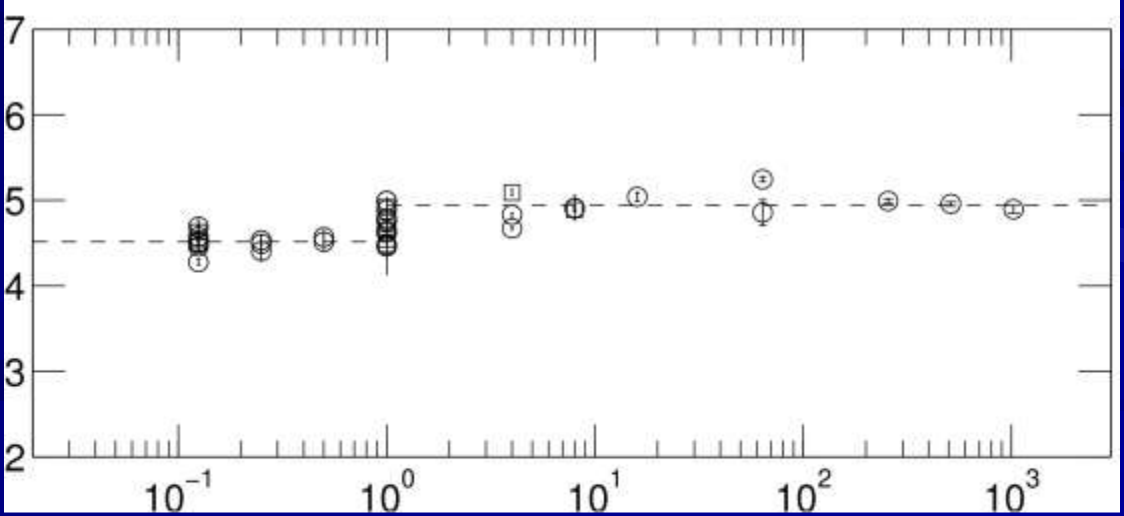
$E_\theta(k) = q\kappa(\nu/\varepsilon)^{1/2}k^{-1}\exp[-q(k\eta_B)^2]$

Kraichnan (1968)

$E_\theta(k) = q\kappa(\nu/\varepsilon)^{1/2}k^{-1}\,[1+(6q)^{1/2}k\eta_B \times \exp(-(6q)^{1/2}k\eta_B)]$



7
6
5
4
3
2
1
10^0
10^1
10^2
10^3



7
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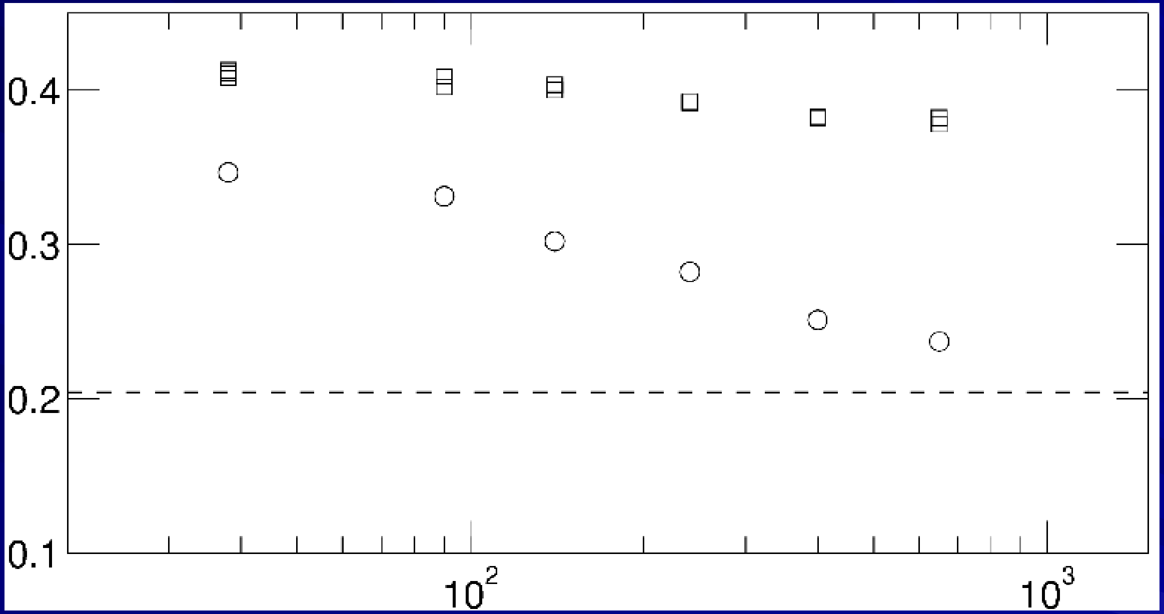
10^{-1}
10^0
10^1
10^2
10^3


0.4
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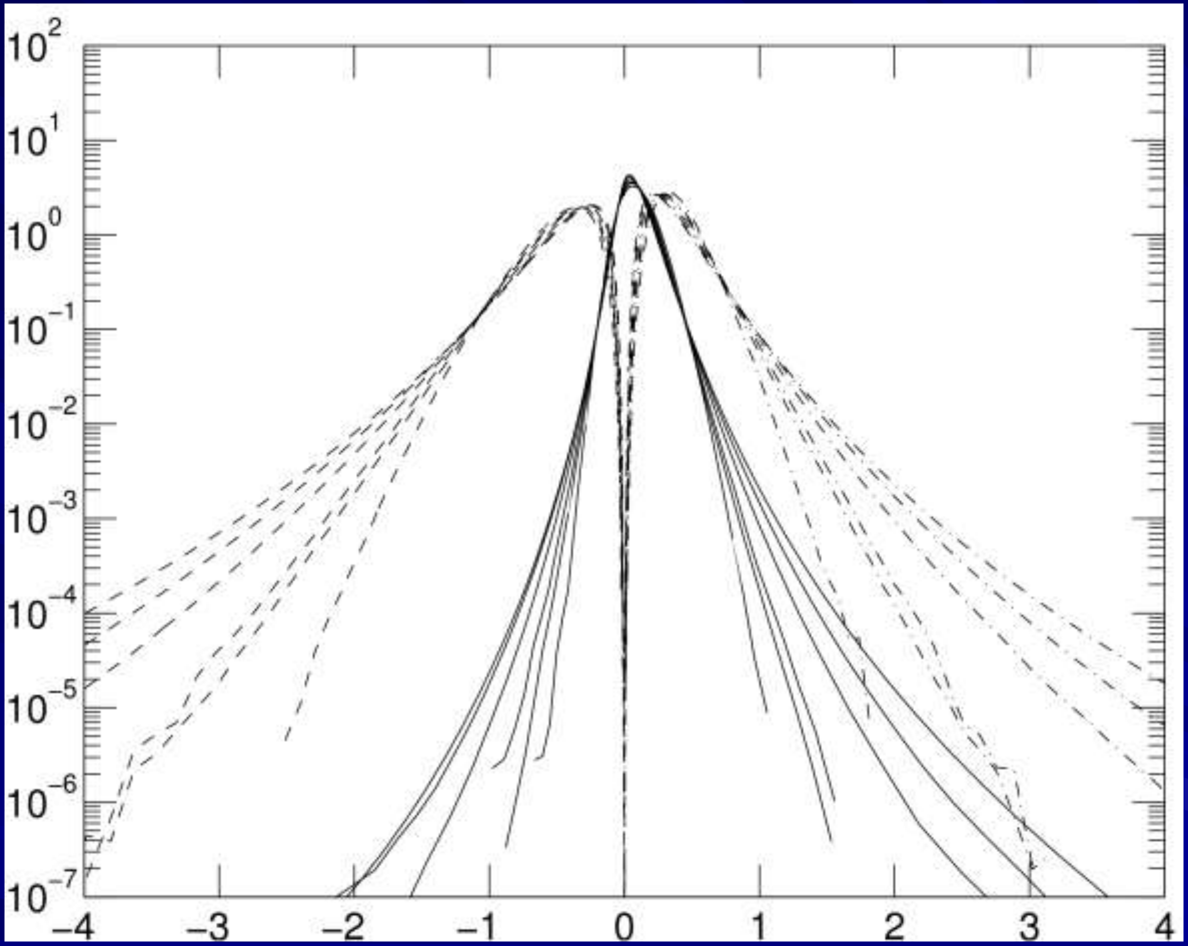
0.1
10^2
10^3



10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
−4
−3
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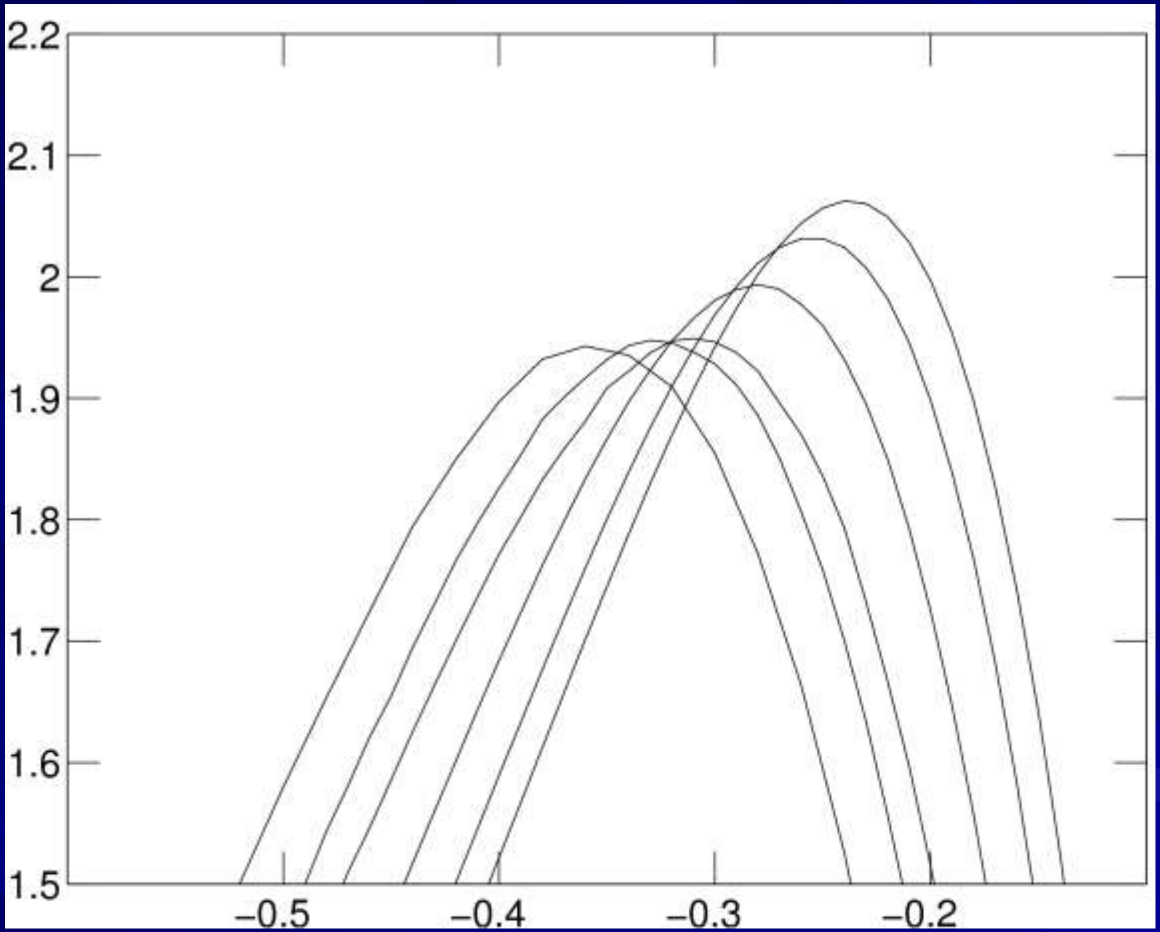
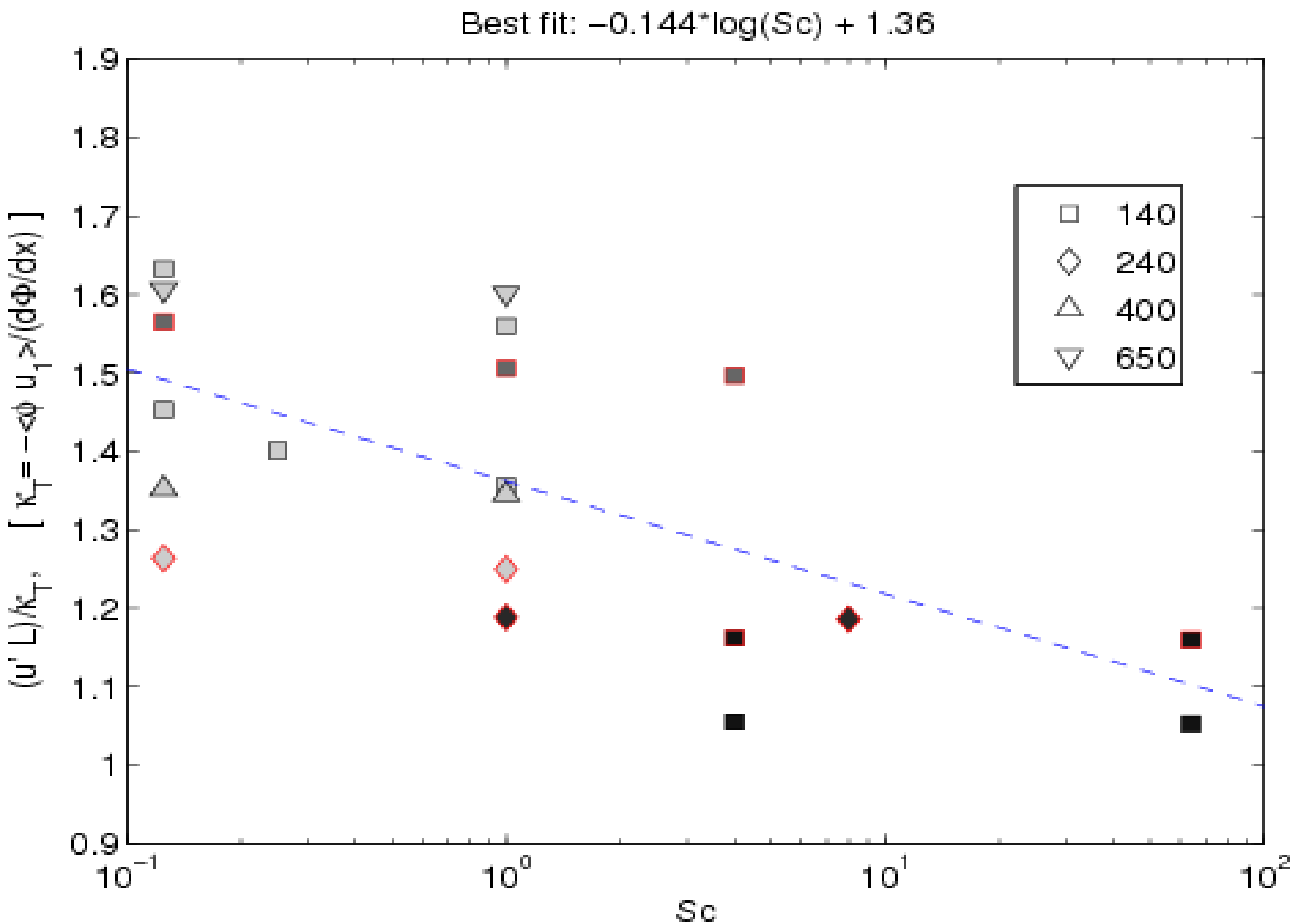
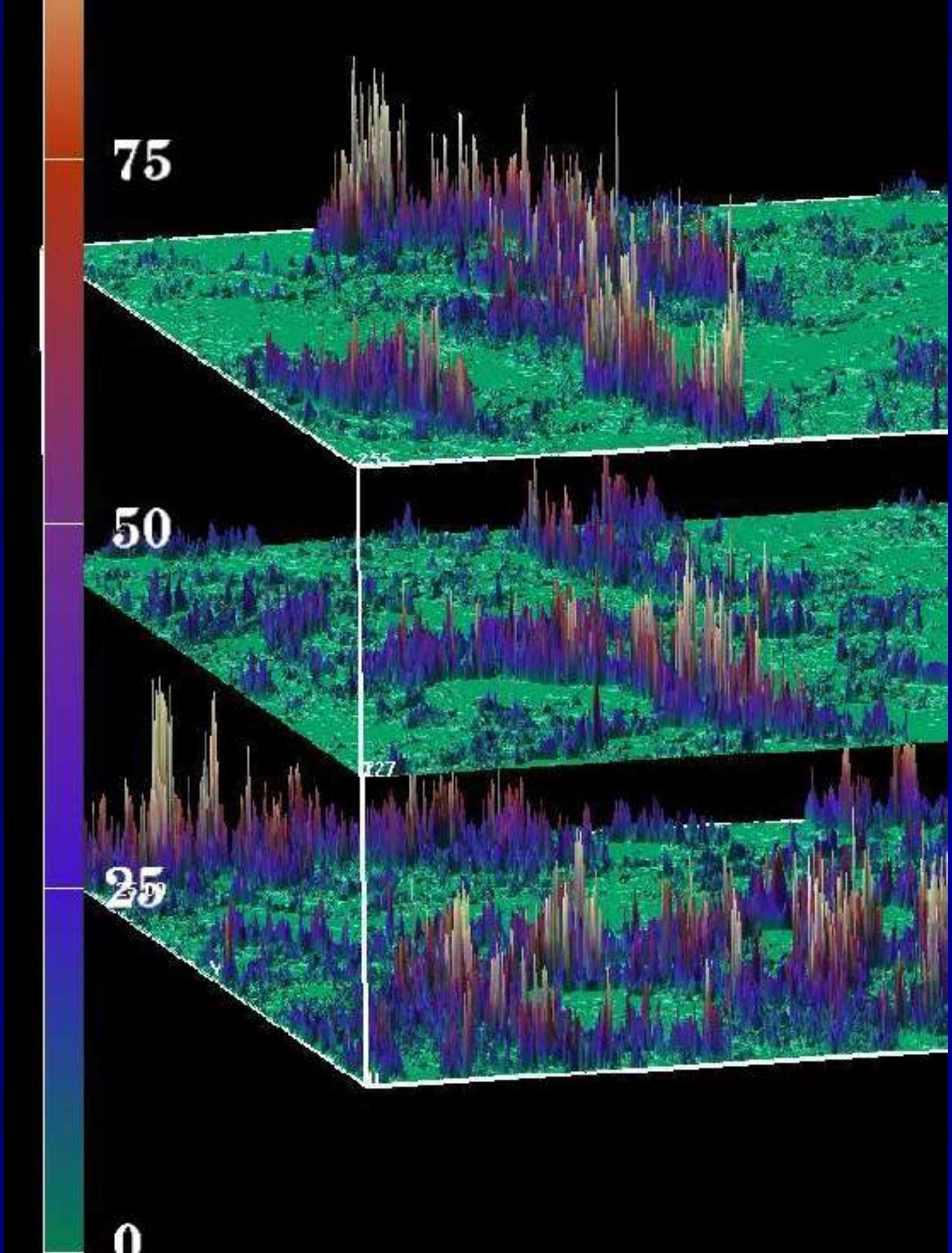
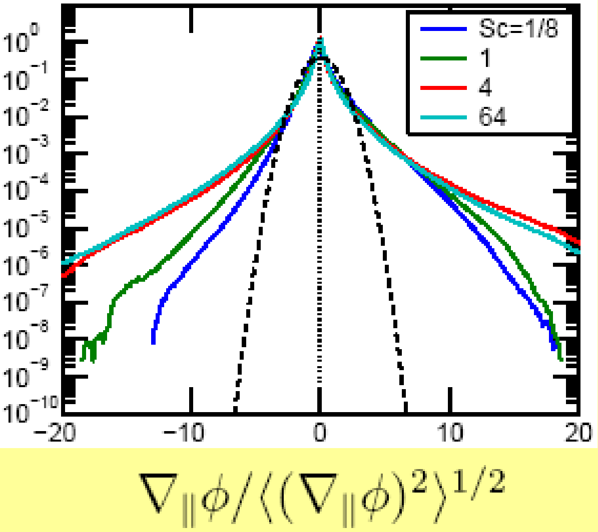
0
1
2
3
4

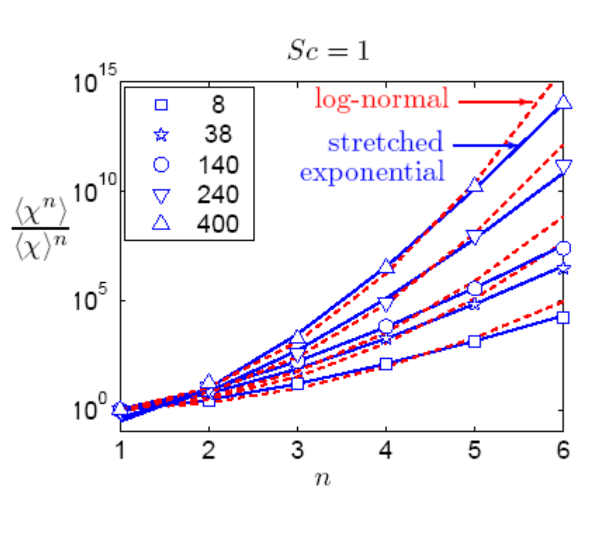

2.2
2.1
2
1.9
1.8
1.7
1.6
1.5
−0.5
−0.4
−0.3
−0.2

Effective diffusivity









Intermittency effects

Some consequences of fluctuations

0. Traditional definitions

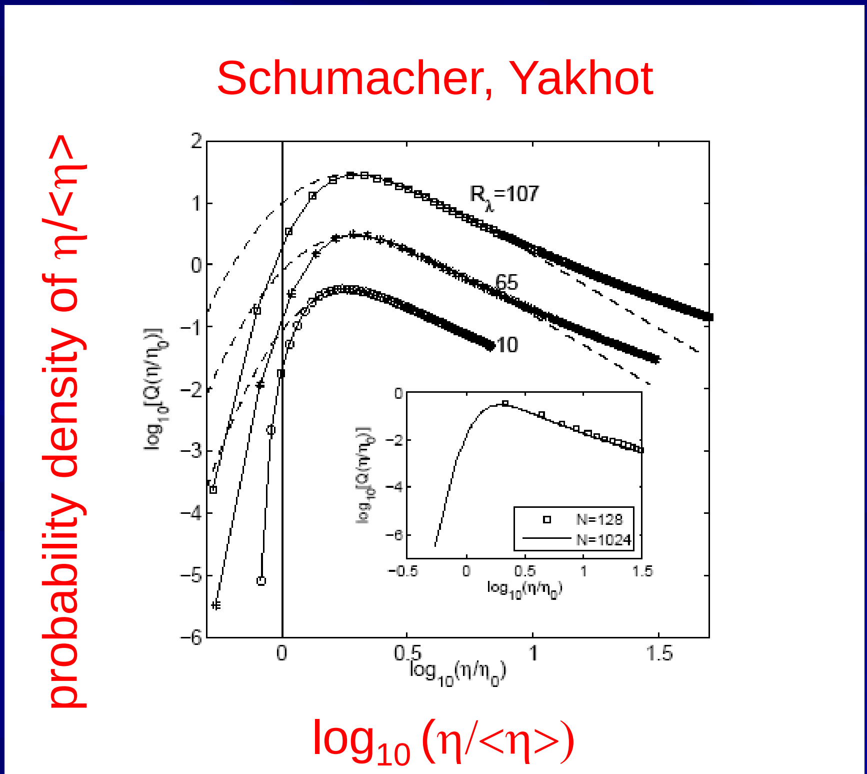
$\langle\eta\rangle = (\nu^3/\langle\varepsilon\rangle)^{1/4}$, $\langle\eta_B\rangle = \langle\eta\rangle/Sc^{1/2}$, $\langle\tau_d\rangle = \langle\eta_B\rangle^2/\kappa$

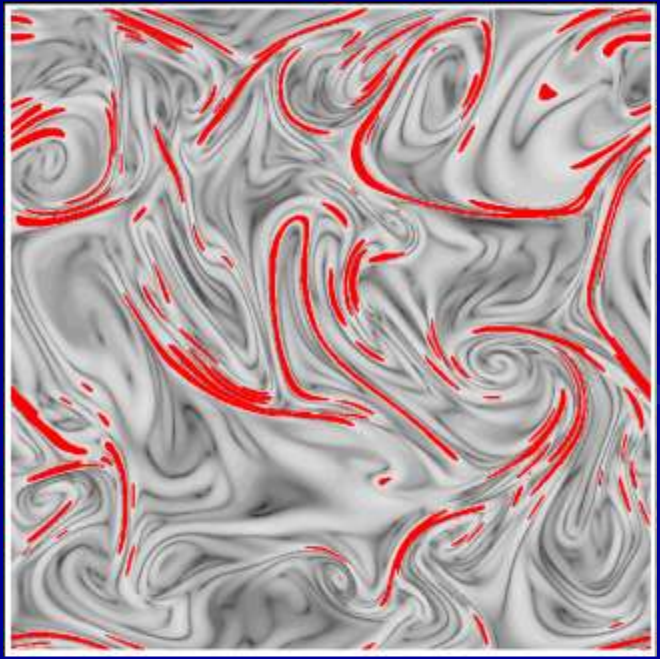
1. Local scales

$\eta = (\nu^3/\varepsilon)^{1/4}$, or define η through $\eta\delta_\eta u/\nu = 1$

$\eta_B = \eta/Sc^{1/2,}$, $\tau_d = \eta_B{}^2/\kappa$

2. Distribution of length scales





3. The velocity field is analytic only in the range $r < \eta$ (and the scalar field only for $r < \eta_B$)

4. Minimum length scale $\eta_{min} = \langle\eta\rangle Re^{-1/4}$ (Schumacher, KRS and Yakhot 2007)

5. Average diffusion time scale $\langle\tau_d\rangle = \langle\eta_B^2\rangle/\kappa$, not $\langle\tau_d\rangle = \langle\eta_B\rangle^2/\kappa$

6. From the distribution of length scales, we have $\langle\tau_d\rangle = \langle\eta_B^2\rangle/\kappa \approx 10 \langle\eta_B\rangle^2/\kappa$

7. Eddy diffusive time/molecular diffusive time $\approx Re^{1/2}/100$;exceeds unity only for $Re \approx 10^4$ (mixing transition advocated by Dimotakis, short-circuit in cascades of Villermaux, etc)

Classes of mixing problems

- Passive mixing
- Mixing of fluids of different densities, where the mixing has a large influence on the velocity field (e.g., thermal convection, Rayleigh-Taylor instability)
- Those accompanied by changes in composition, density, enthalpy, pressure, etc. (e.g., combustion, detonation, supernova)

Active scalars

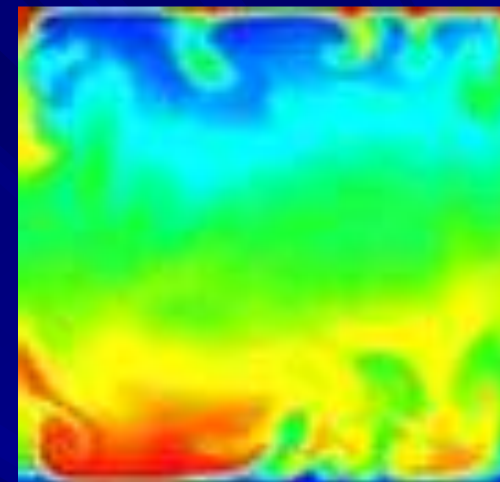
$$\partial_t a = \mathbf{v}.\nabla a + \boldsymbol{\kappa}\Delta a + \mathbf{F}_a$$

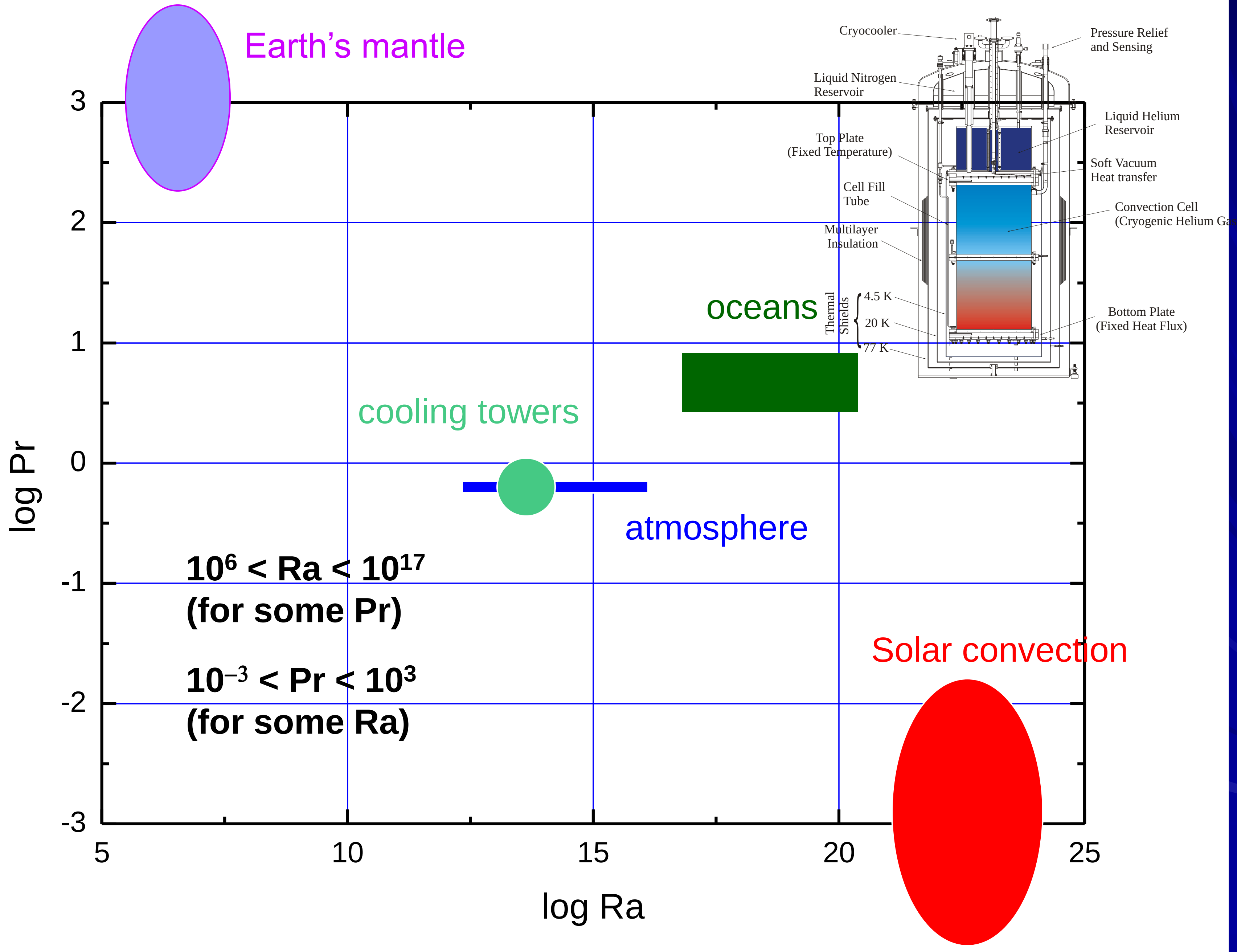
$$V_i(\mathbf{x};t) = \int d\mathbf{y}\; G_i\mathbf{(x,y)}\; a(\mathbf{y,}t)$$

Simple case: Boussinesq approximation

$$NS = -\beta g a$$

Filmato.wmv

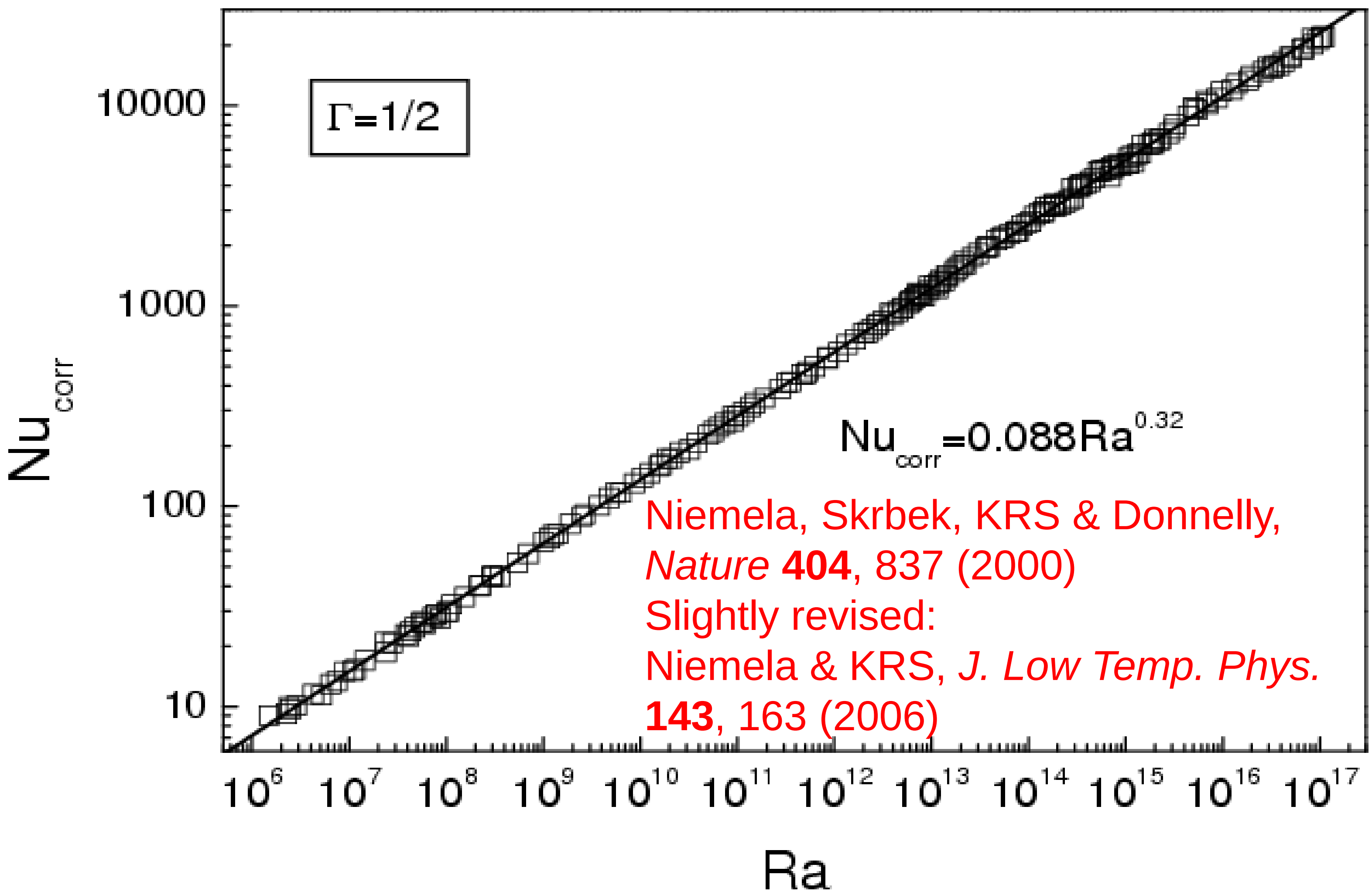




$10^6 < Ra < 10^{17}$
(for some Pr)

$10^{-3} < Pr < 10^3$
(for some Ra)

Helium gas convection (with and without rotation)



Niemela, Skrbek, KRS & Donnelly, *Nature* **404**, 837 (2000)
Slightly revised:
Niemela & KRS, *J. Low Temp. Phys.* **143**, 163 (2006)

[Pioneers: Threlfall (Cambridge); Libchaber, Kadanoff and coworkers (Chicago)]

Latest theoretical bound for the exponent (X. Wang, 2007): 1/3 for Pr/Ra = O(1)

Upperbound results in the limit of Ra $\to \infty$

1. *Arbitrary Prandtl number*

   Nu < $Ra^{1/2}$ for all Pr (Constantin).

   Rules out, for example, $Pr^{1/2}$ and $Pr^{-1/4}$.

2. *Large but finite Prandtl numbers*

   For Pr > c Ra, Nu < $Ra^{1/3}(\ln Ra)^{2/3}$ (Wang)

   For higher Rayleigh numbers, the ½ power holds.

3. *Infinite Prandtl number*

   Nu < $CRa^{1/3}(\ln Ra)^{1/3}$ (Doering et al., exact)

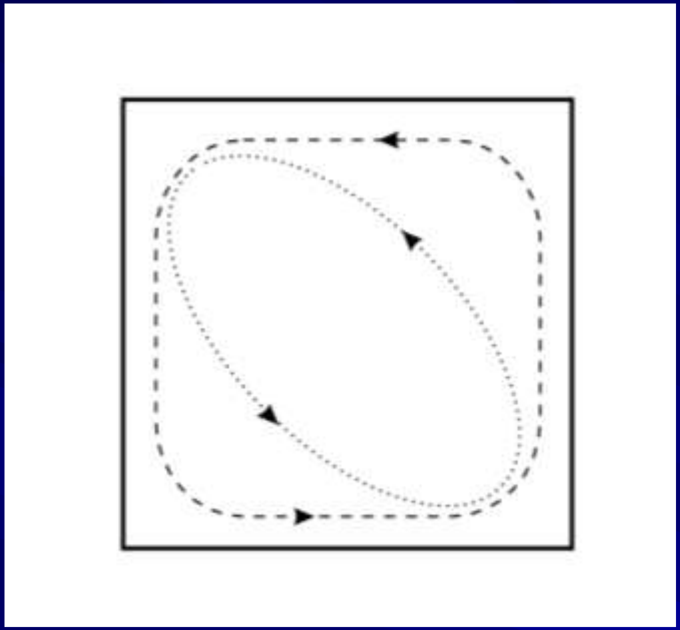
   Nu < $aRa^{1/3}$ (Ierley et al., “almost exact”)

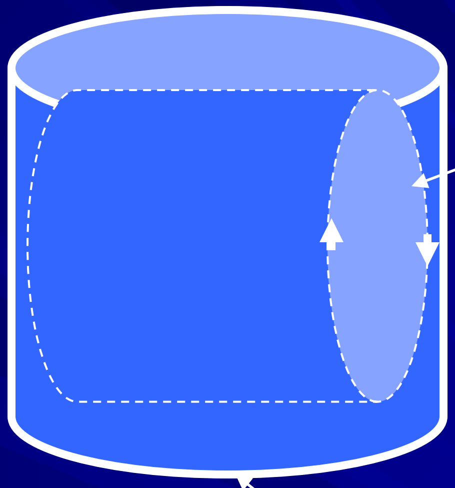
(Early work by Howard and Malkus gave 1/3 for all Pr.)

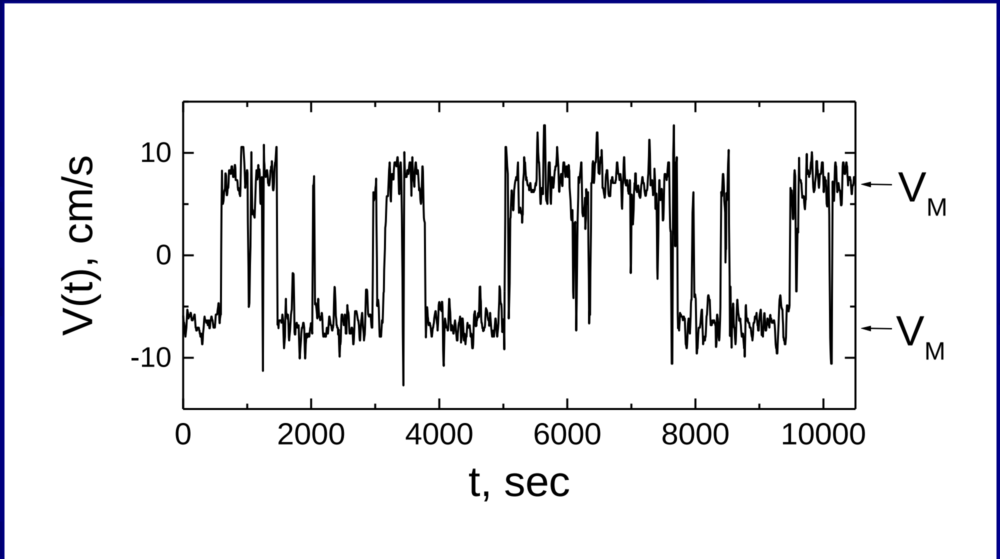
2 questions: Pr. 1/3 (2 views)

The mean wind

The "mean wind" breaks symmetry, with its own consequences







E:\videoplayback(2).wmv

Segment of 120 hr record

E:\videoplayback(2).wmv

How are the reversals distributed?

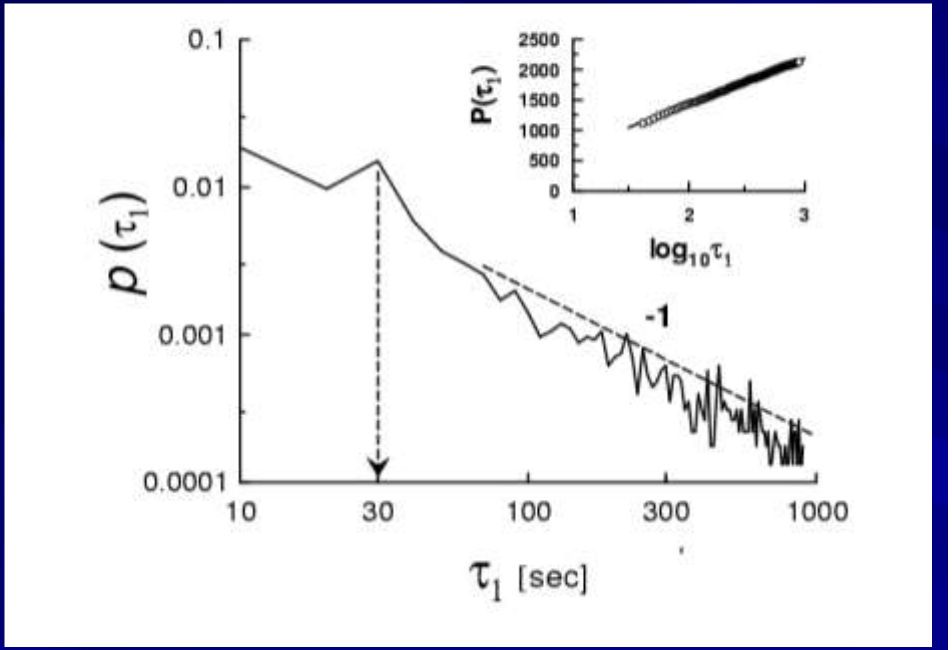
τ_1 = time between subsequent switches in the velocity signal

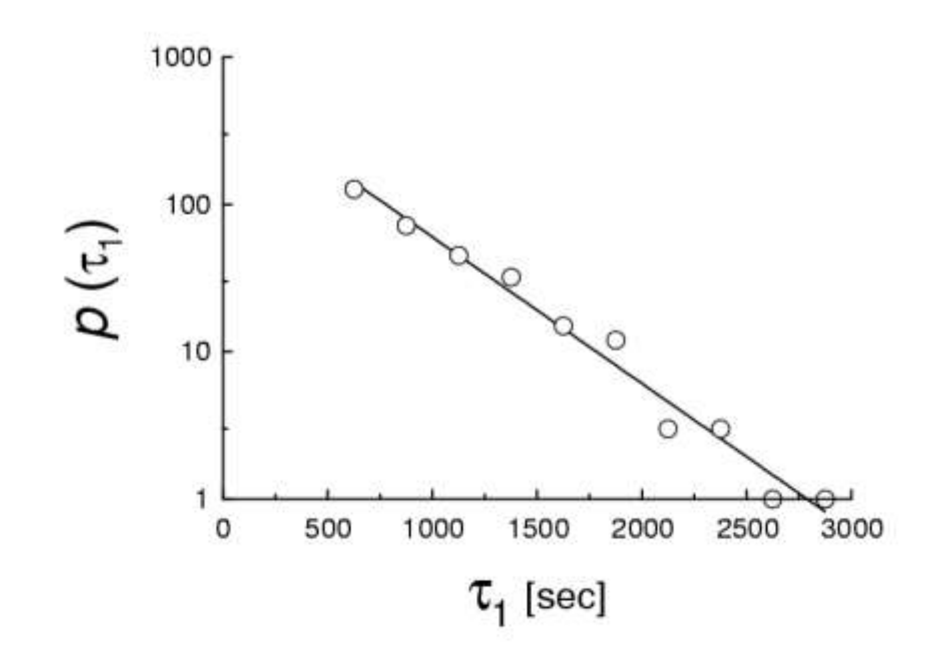
$\tau_1 \equiv T_{n+1} - T_n$

power-law scaling of the probability density function for small τ_1

for large τ_1: $p(\tau_1) \sim \exp[-(\tau_1/\tau_m)]$

τ_m = 400 s





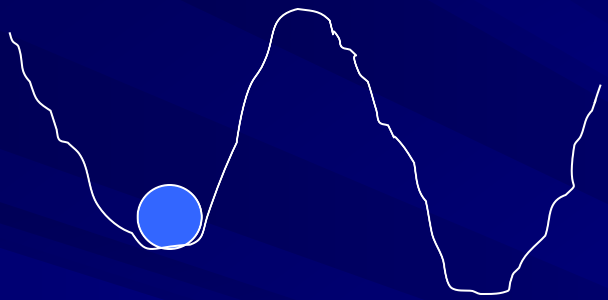
Sreenivasan, Bershadskii & Niemela, *Phys. Rev. E* **65**, 056306 (2002)

-1 power law scaling characteristic of SOC systems
(see papers in *Europhys. Lett., Physica A and PRE*)

Dynamical model

Balance between buoyancy and friction, forced by stochastic noise

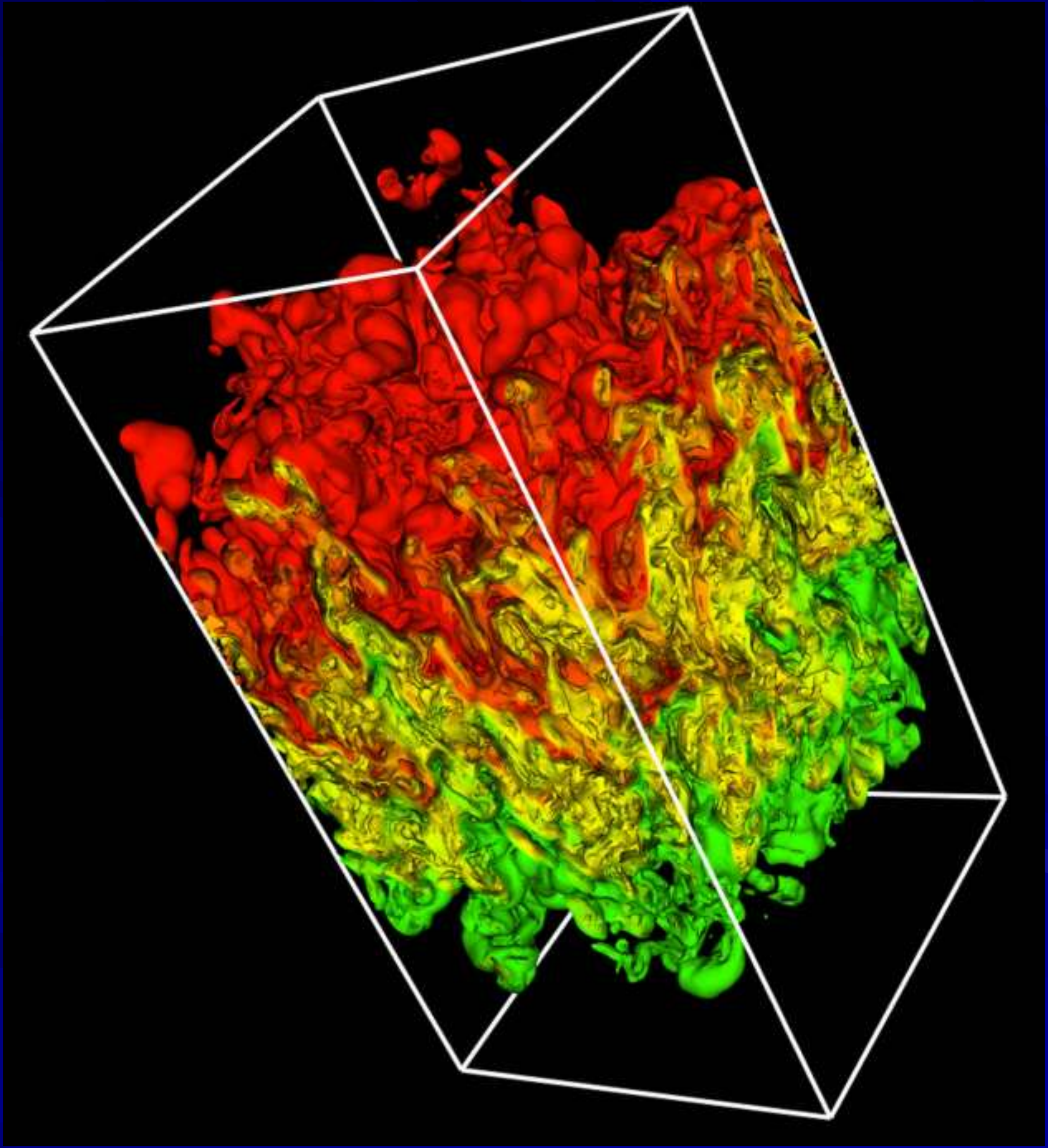
For certain combinations of parameters, one obtains power-law for small times and exponential distribution for large times.



double-well potential

$$p(\tau_1) : \exp[-(\tau_1 / \tau_m)]$$

Sreenivasan, Bershadskii & Niemela, *Phys. Rev. E* **65**, 056306 (2002)



Summary of major points

• Despite the enormous importance of the problem of mixing, there are numerous problems (which can be posed sharply) for which there are no sharp answers. There is an enormous opportunity here.

• The large scale features of the scalar depend on initial and boundary conditions, and each of them has to be understood on its own merits. In the absence of full-fledged theory, models are very helpful to understand the essentials.

• The Kraichnan model explains the appearance of anomalous scaling.

• The best-understood part corresponds to large Sc, for which classical predictions of the past have been confirmed (e.g., those relating to the -1 power). There is, however, no theory for the numerical value of the spectral constant and its behavior for $Sc < 1$ remains unexplained.

thanks

DISSIPATIVE ANOMALY

Non-dimensional parameters and scales

Reynolds number: Re $\equiv uL/\nu >> 1$

$\eta = (\nu^3/\varepsilon)^{1/4}$: Re based on η = 1

Schmidt number, Sc = ν/κ

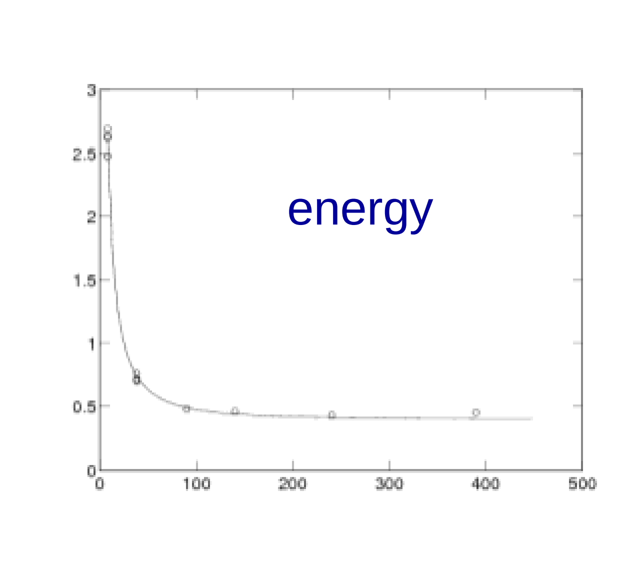
L

η

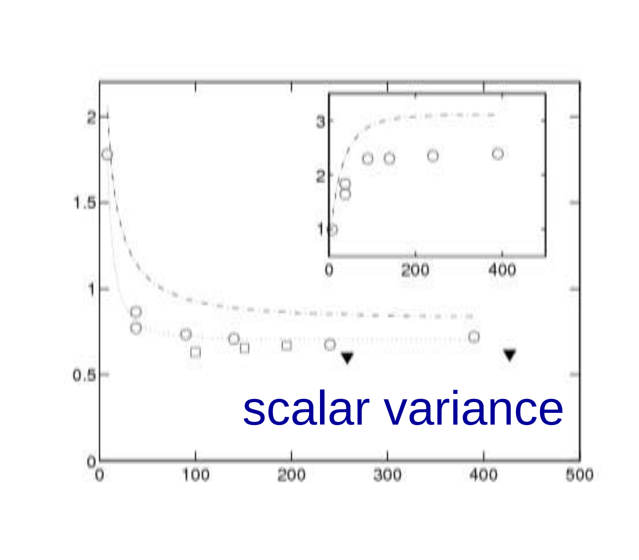
inertial range
$\phi(k) = C_K k^{-5/3}$
$C_K \approx 0.5$
[PoF, 7, 2778 (1995)]

For Sc = O(1),
$\phi_\theta(k) = C_{OC} k^{-5/3}$
$C_{OC} \approx 0.35$
[PoF, 8, 189 (1996)]

normalized dissipation rate



microscale Reynolds number

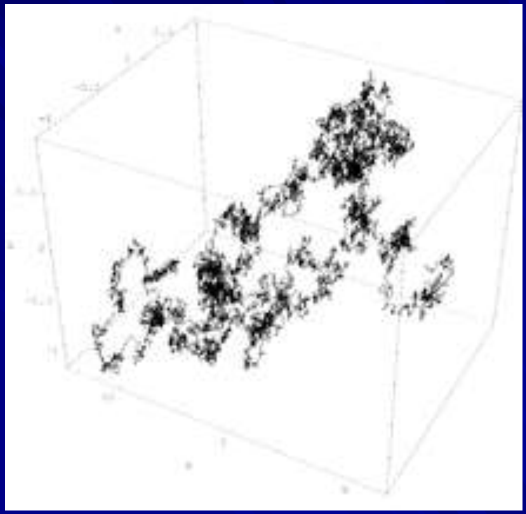


Brownian motion

Robert Brown, a botanist, discovered in 1827, that pollen particles suspended in a liquid execute irregular and jagged motion, as shown.

Einstein 1905 and Smoluchovski 1906 provided the theory.

- The Brownian motion of pollen grain is caused by the exceedingly frequent impacts of the incessantly moving molecules of the liquid.
- The motion of the molecules is quite complex but the effect on the pollen occurs via exceedingly frequent and statistically independent impacts.



simulation in three dimensions

Langevin's derivation

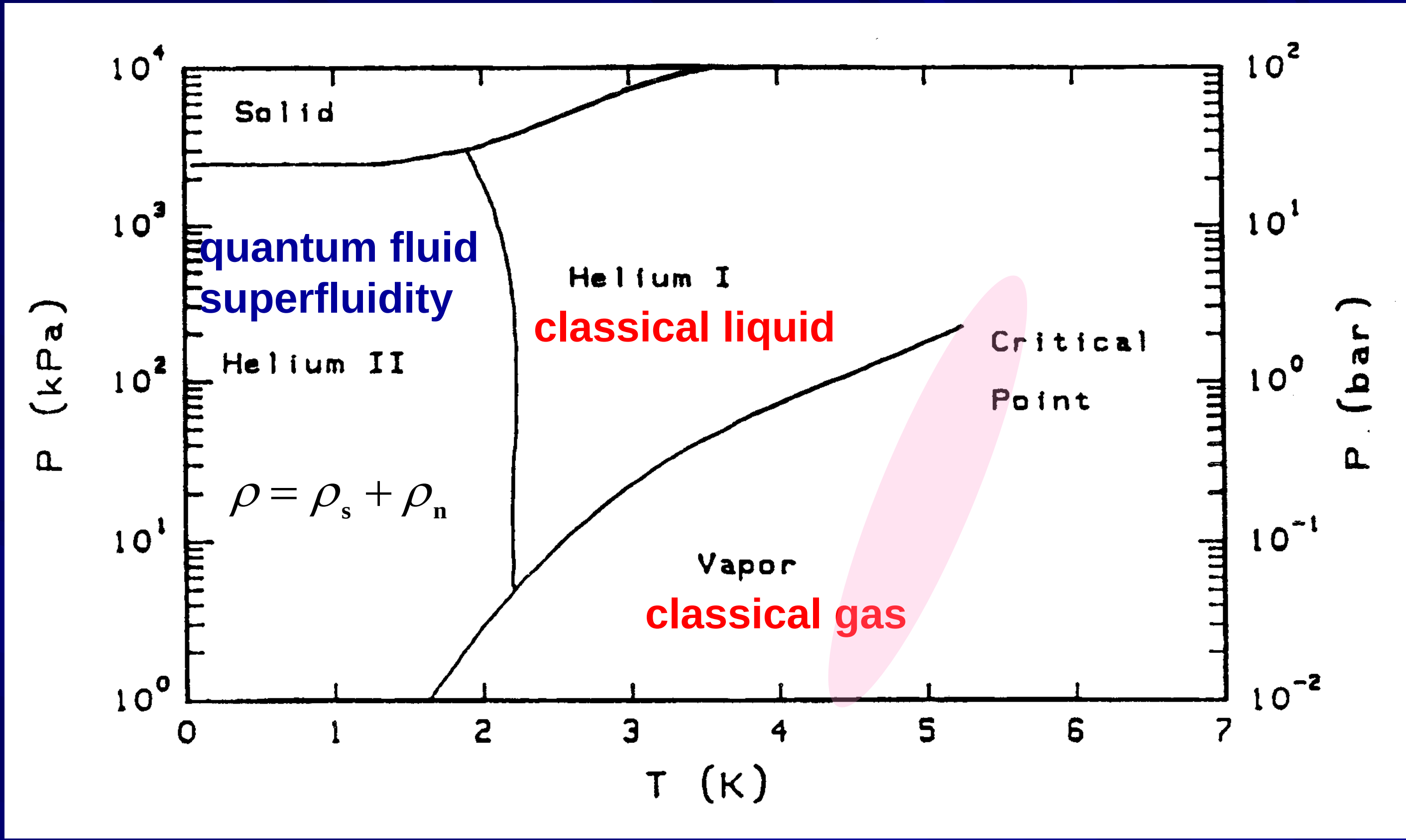
- Consider a small spherical particle of diameter 'a' and mass 'm' executing Brownian motion.
- Equipartition: $\langle \frac{1}{2}mv^2 \rangle = \frac{1}{2}kT$; $v = dx/dt$

  Two forces: viscous (Stokes) drag = $6\pi\eta av$ and the fluctuating force X due to bombardment of molecules; X is negative and positive with equal probability.
- Newton's law: $m\, d^2x/dt^2 = -6\pi\eta a(dx/dt) + X$
- Multiply by x
- $(m/2)\, d^2(x^2)/dt^2 = -6\pi\eta a(dx^2/dt) + Xx$

  Average over a large number of different particles
- $(m/2)\, d^2\langle x^2\rangle/dt^2 + 6\pi\eta a(d\langle x^2\rangle/dt) = kT$

  We have put $\langle Xx\rangle = 0$ because x fluctuates too rapidly on the scale of the motion of the Brownian particle.
- Solution: $d\langle x^2\rangle/dt = kT/3\pi\eta a + C\exp(-6\pi\eta at/m)$

  The last term approaches zero on a time scale of the order 10^{-8} s.
- We then have: $d\langle x^2\rangle/dt = kT/3\pi\eta a$
- Or, $\langle x^2\rangle - \langle x_0^2\rangle = (kT/3\pi\eta a)t$
- Comparing with the result:
- Mean square displacement $\langle x^2\rangle^{1/2} = (2\kappa t)^{1/2}$, we have $\kappa = kT/6\pi\eta a$



Helium I: $\nu = 2 \times 10^{-8}$ m²/s (water: 10^{-6} m²/s, air: 1.5×10^{-5} m²/s) obvious interest in model testing.

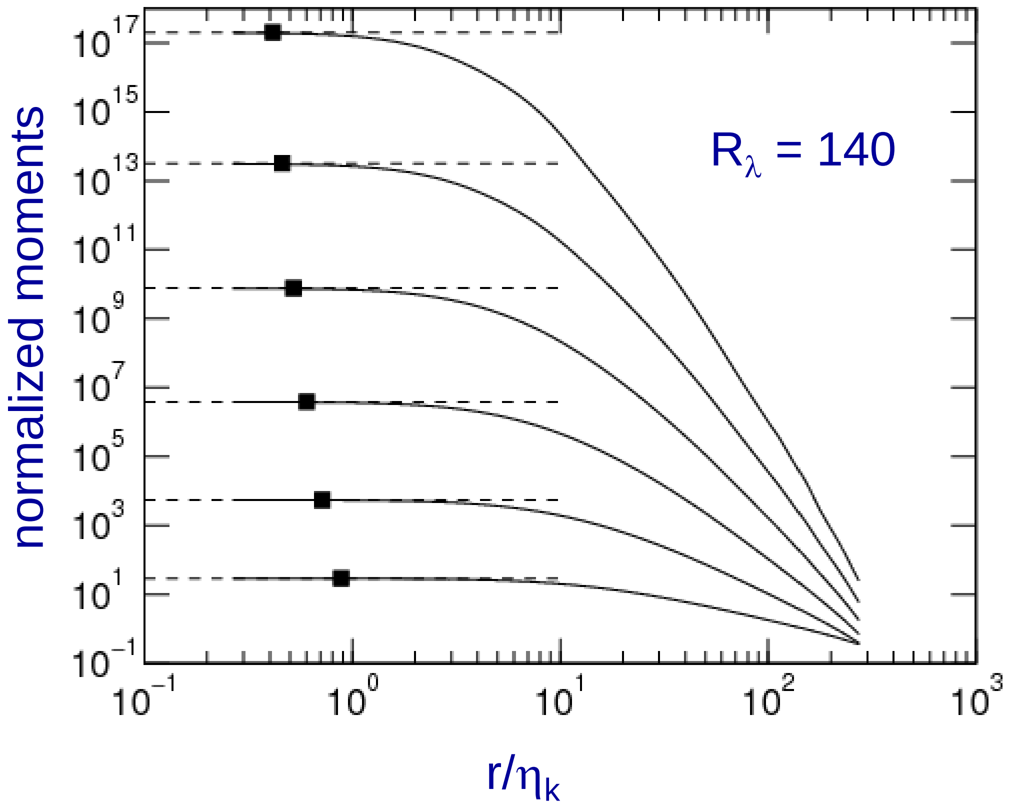
$$Ra = g \cdot \left(\frac{\alpha}{\nu\kappa}\right) \cdot \Delta T \cdot H^3$$

4.4 K, 2 mbar: $\alpha/\nu\kappa = 6.5 \times 10^9$

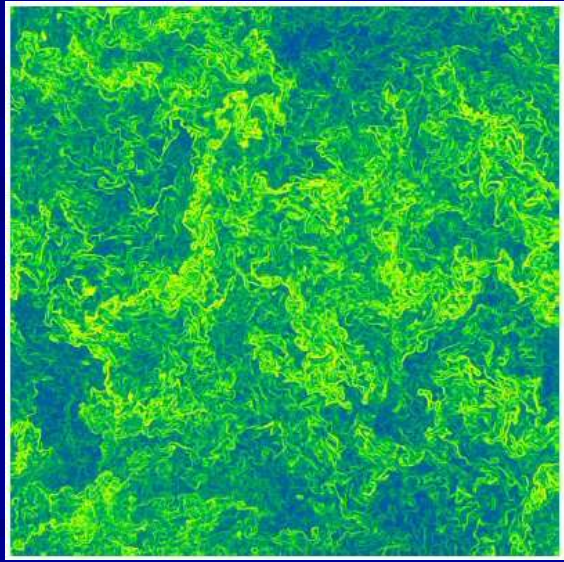
5.25 K, 2.4 bar: $\alpha/\nu\kappa = 5.8 \times 10^{-3}$

Superfluids flow without friction and transport heat without temperature gradients.

$k_{max}\eta$ = 1.5 -33.6
R_λ = 10 -690
Sc = 1-1024
$k_{max}\eta_B$ = 1.5-6
box-size: 512-2048
(some preliminary results for 4096)



moment orders 2, 4, 6, 8,10, 12; $k_{max}\eta$ =11



plumes…

… and their self-organization into a large scale flow in a confined apparatus

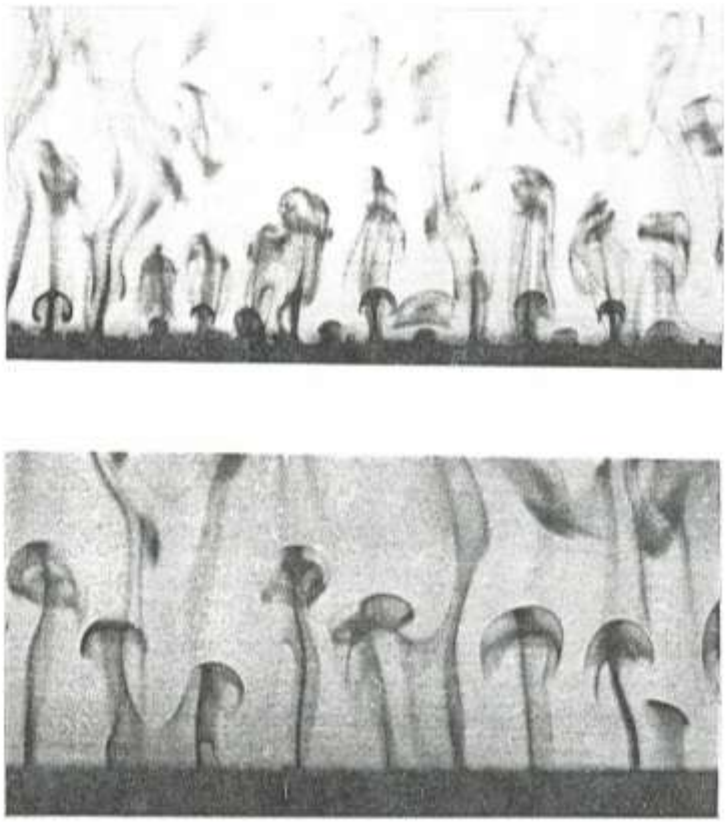
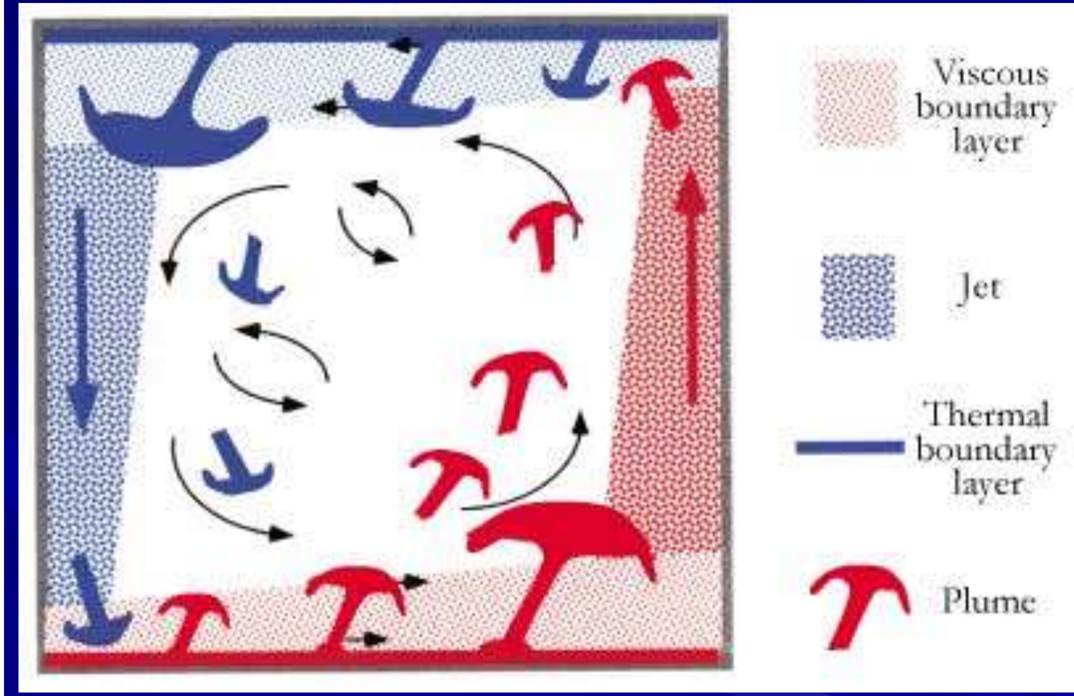


FIGURE 1. Photographs of thermals rising from a heated horizontal surface.

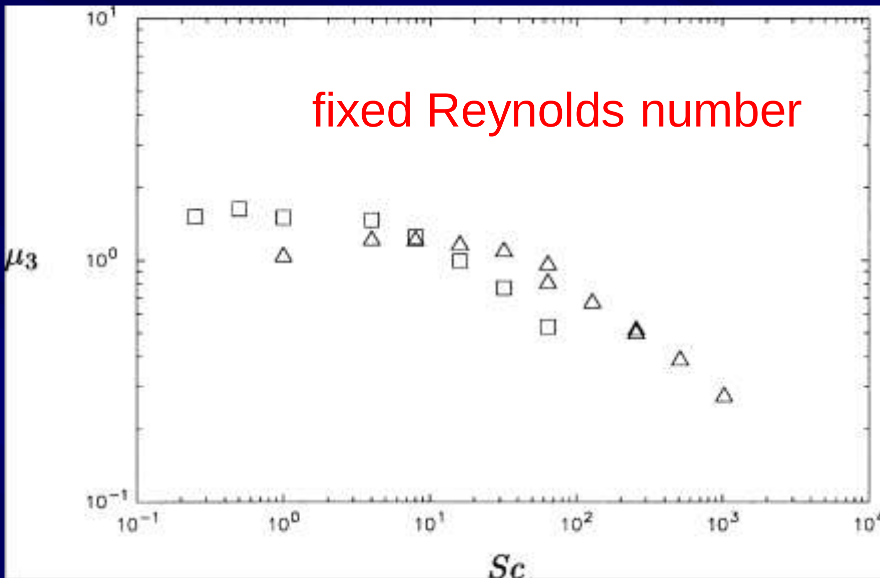
Sparrow, Husar & Goldstein

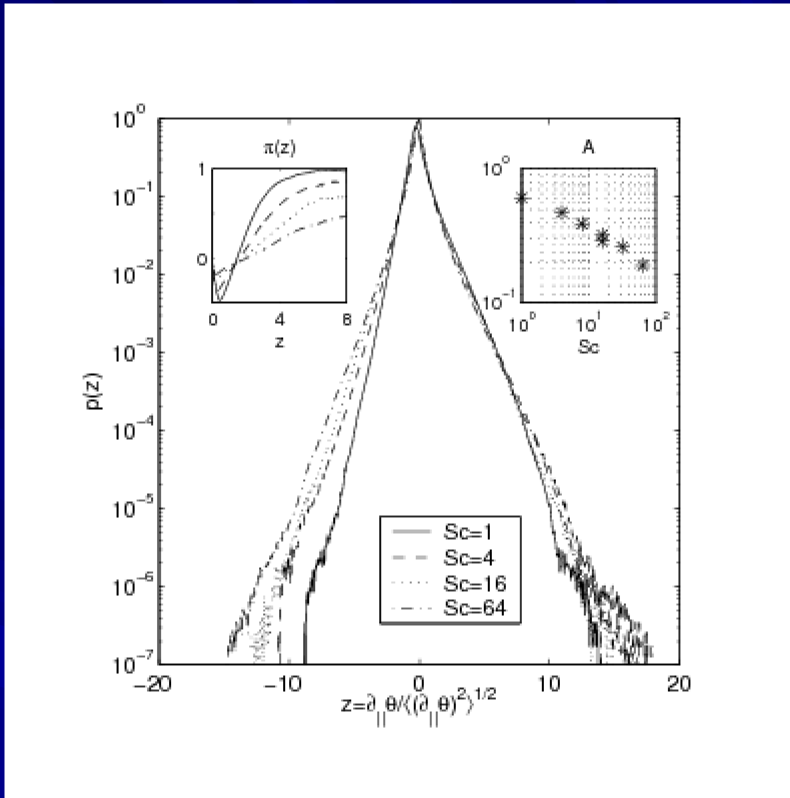
J. Fluid Mech. **41,** 793 (1970)



L. Kadanoff, *Phys. Today*, August 2001
(for flow visualization and quantitative work, see K.-Q. Xia et al. from Hong Kong)

4. Schmidt number effects on anisotropy

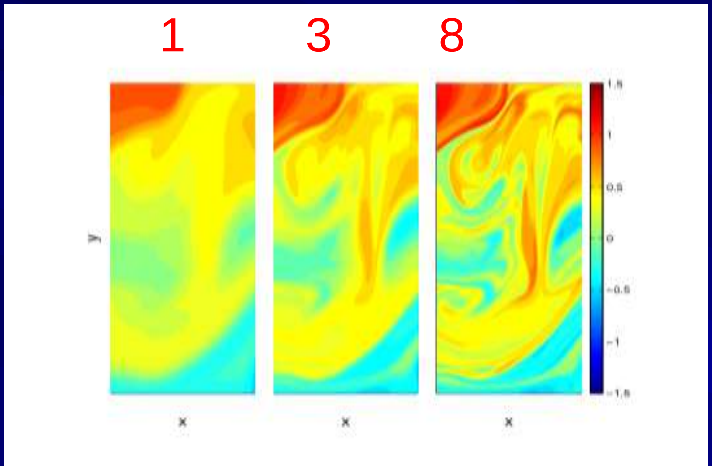




The case of large Schmidt number

Schmidt number, $Sc = \nu/\kappa \sim O(1000)$

$Sc >> 1$



$N = Re^3 Sc^2$

L

as before

η

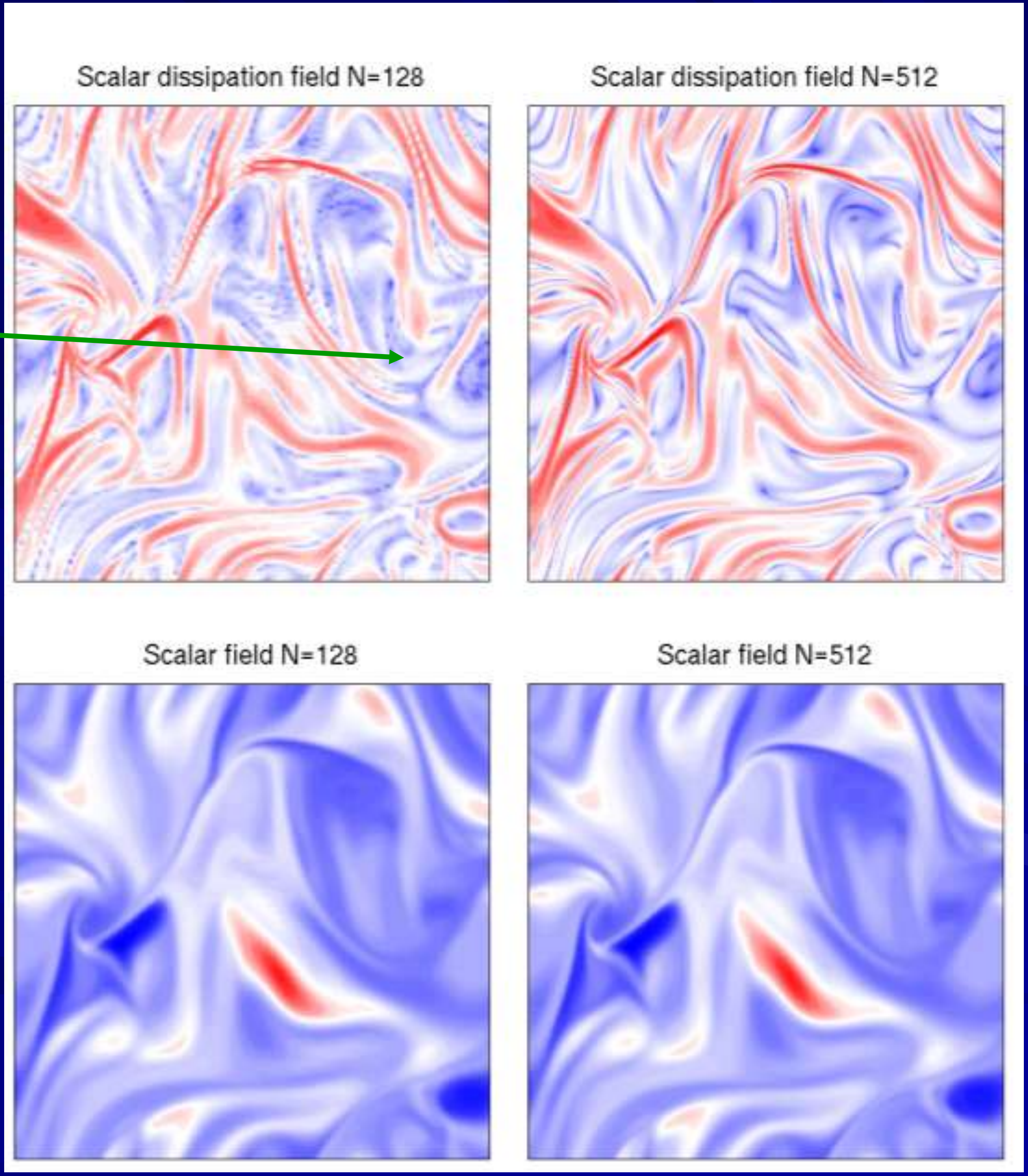
Batchelor regime

$\phi_\theta(k) \sim qk^{-1}$

$q = O(1)$

η_B

Resolution matters!



Low scalar dissipation

Not much difference

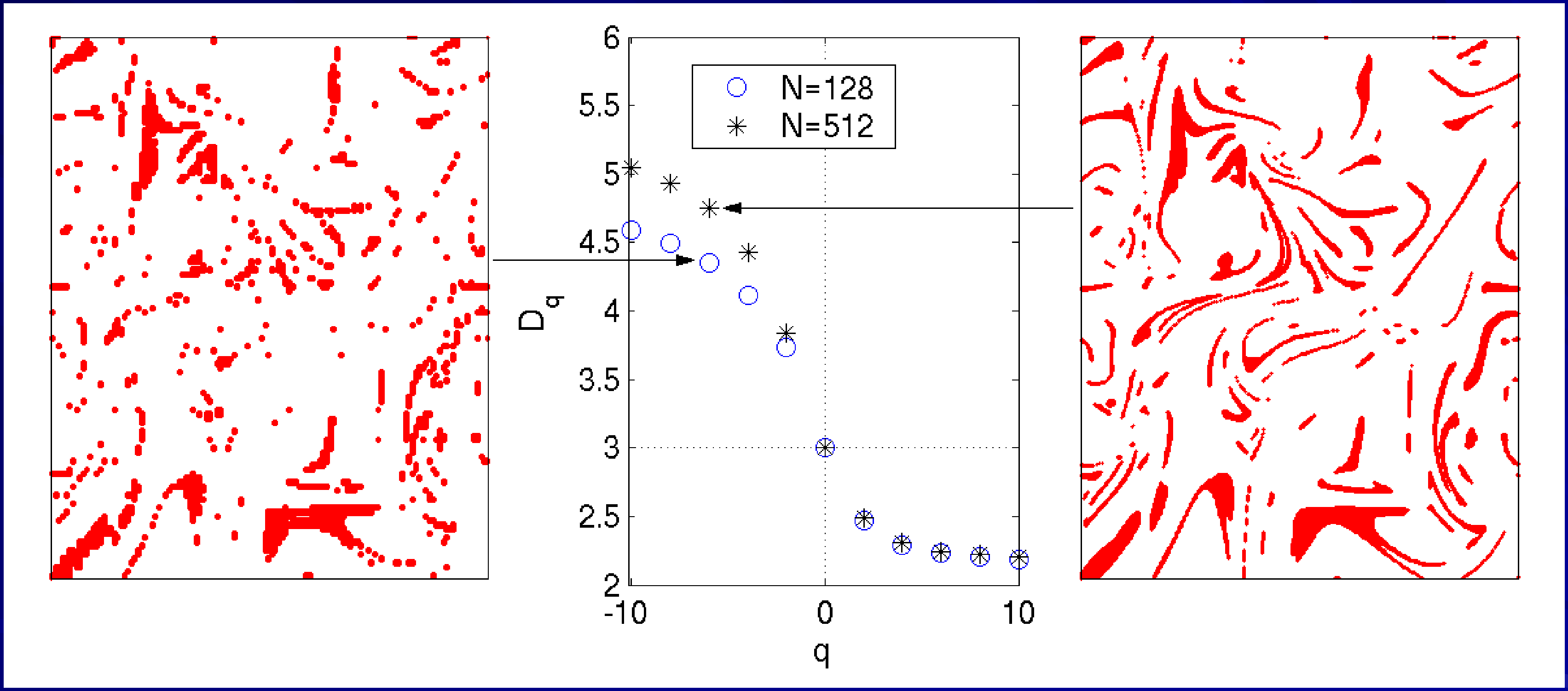
$k_{max}\eta_B = 1.5$

$k_{max}\eta_B = 6$

Sensitivity of low dissipation regions

(Schumacher, KRS & Yeung, JFM 2005)

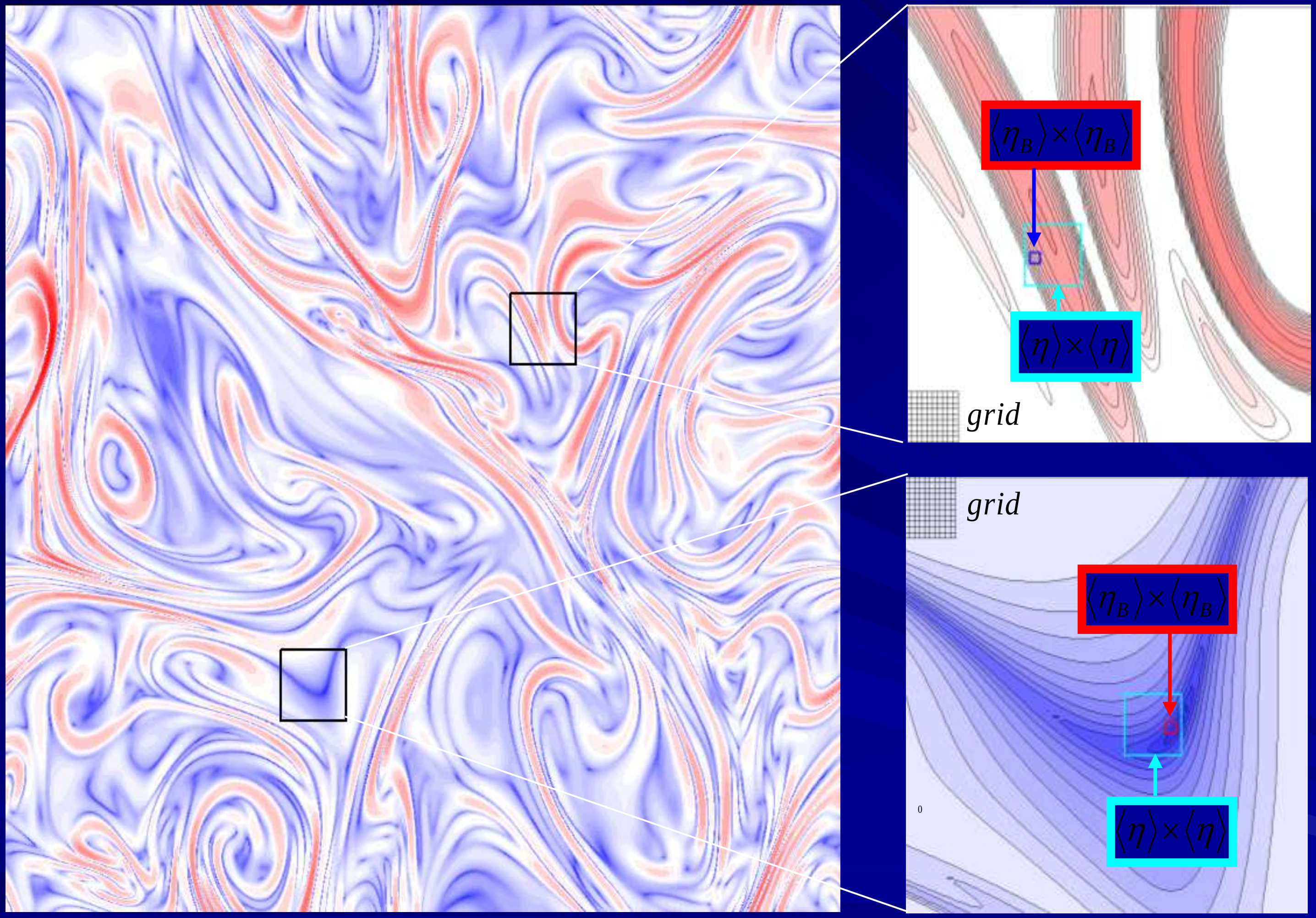
Regular resolution High resolution



128^3

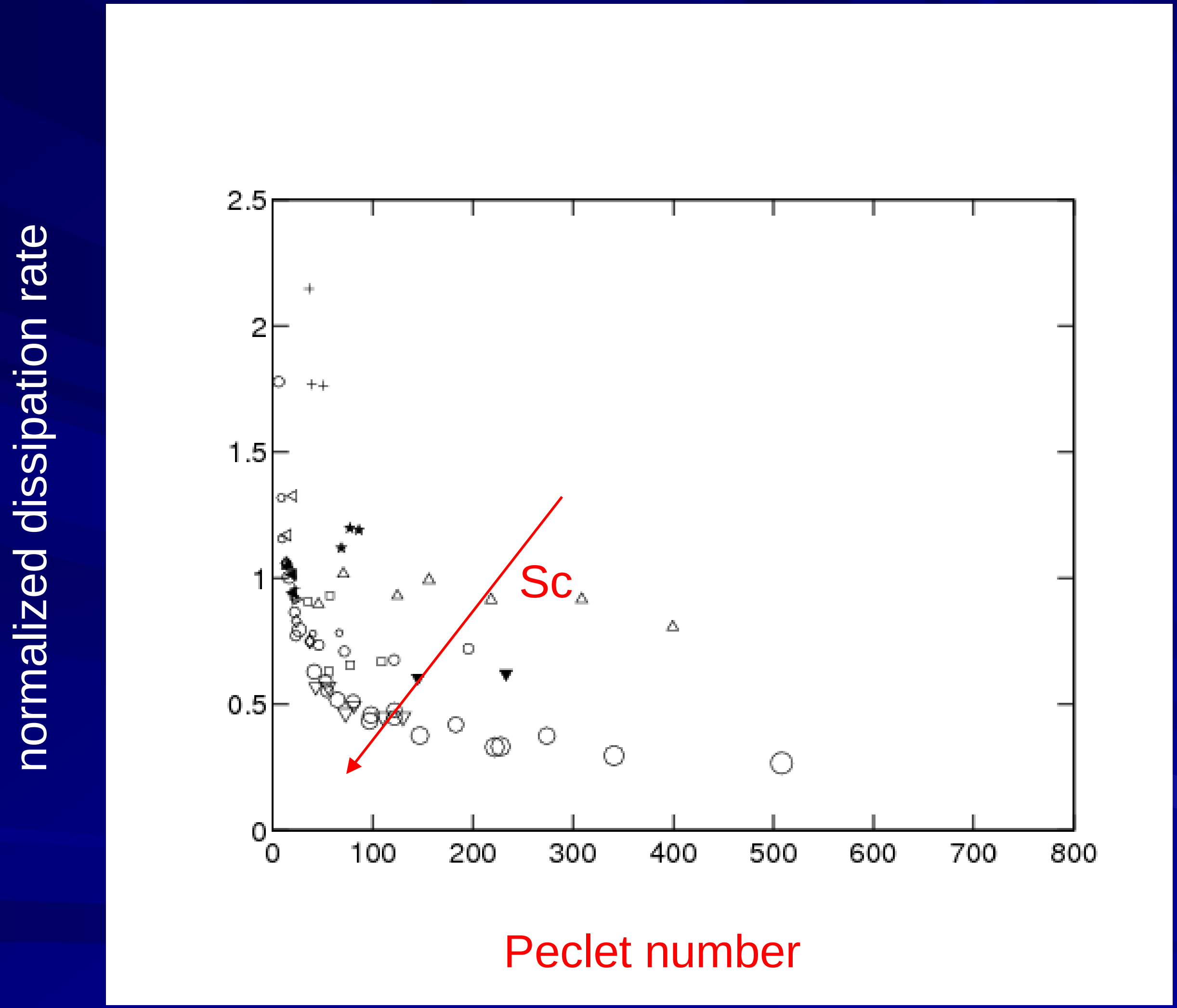
$$\frac{\varepsilon_\theta}{\langle \varepsilon_\theta \rangle} < \frac{1}{40}$$

512^3

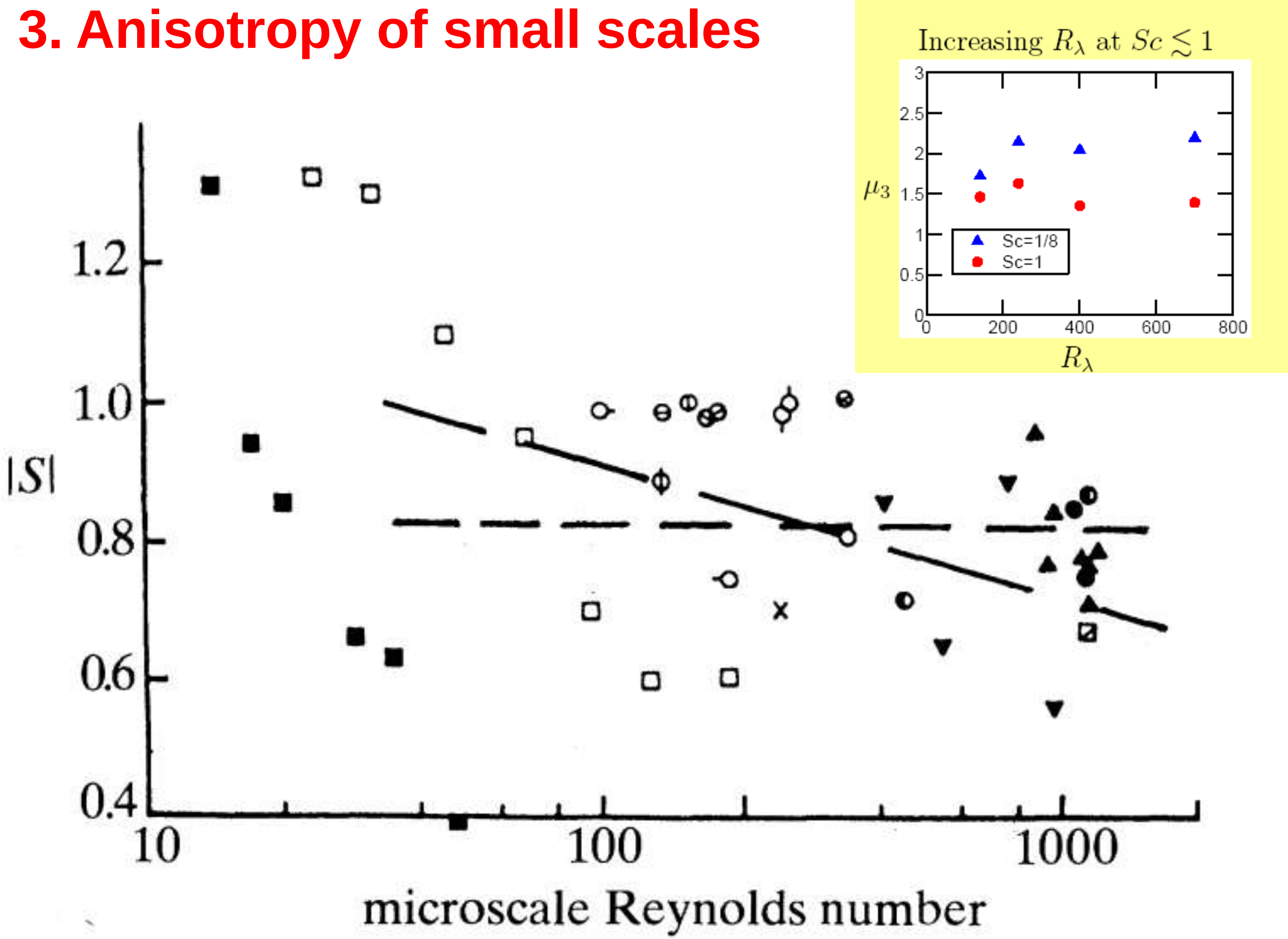


$\langle\eta_B\rangle\times\langle\eta_B\rangle$
$\langle\eta\rangle\times\langle\eta\rangle$
grid
grid
$\langle\eta_B\rangle\times\langle\eta_B\rangle$
$\langle\eta\rangle\times\langle\eta\rangle$

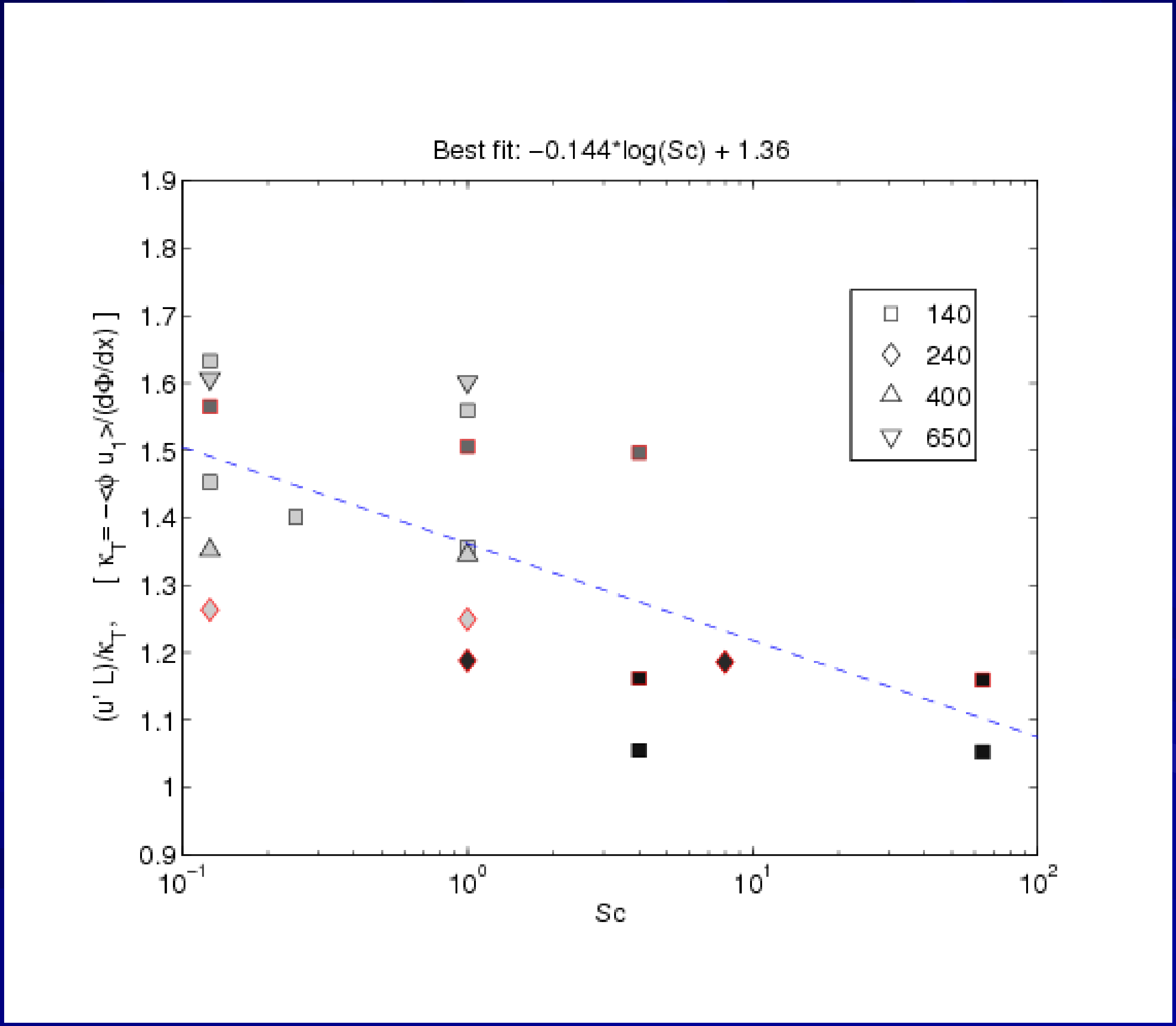
2. The effect of Schmidt number on dissipative anomaly



3. Anisotropy of small scales



5. Effective diffusivity



Turbulence nears a final answer

From Uriel Frisch at the Observatoire de la Côte d'Azur, Nice, France

The great Italian scientist Leonardo da Vinci was the first person to use the word "turbulence" (or *turbolenza*) to describe the complex motion of water or air. By carefully examining the turbulent wakes created behind obstacles placed in the path of a fluid, he found that there are three key stages to turbulent flow. Turbulence is first generated near an obstacle. Long-lived "eddies" – beautiful whirls of fluid – are then formed. Finally, the turbulence rapidly decays away once it has spread far beyond the obstacle.

However, it was not until the early 19th century that Claude Navier was able to write the basic equations governing how the velocity of a turbulent fluid evolves with time. Navier realized that the earlier equations of Leonhard Euler for ideal flow had to be supplemented by a diffusion term that took into account the viscosity of the fluid.

A few decades later, Adhemar de Saint-Venant noticed that turbulent flow – for example in a wide channel – has a much higher "effective" viscosity than the laminar flow found, for instance, in a capillary. It turns out that the turbulent transport of mon [illegible] 3-15 [illegible] than [illegible] kinet [illegible] Indeed, if it were not for turbulence, pollution in our cities would linger for millennia, the heat generated by nuclear reactions deep inside stars would not be able to escape within an acceptable time, and the weather could be predicted far into the future.

Modelling turbulent transport thus became – and remains to this day – a major challenge. The first attempt goes back to a student of Saint-Venant called Joseph Boussinesq, who introduced what is now known as the "mixing-length approach". He assumed that transport of fluid elements proceeds as a random walk, in which the typical step is the size of eddies.

Since then, vastly improved models that can deal with increasingly complex flows of the type found in aeronautical applications or in turbomachinery have been developed by Ludwig Prandtl, Andrei Kolmogorov, Brian Spalding and many others.

This progress has depended on an ever-increasing theoretical and experimental understanding of the physics of turbulence, and I can do no more than point to the crucial contributions of Lord Kelvin, Osborne Reynolds, Geoffrey Ingram Taylor, Jean Leray, Theodor von Kármán and many others. I will thus turn to one of the major challenges in the field, which is to understand what is known as fully developed turbulence (FDT) in the case of a high Reynolds number – a non-dimensional parameter that essentially describes the relative sizes of the fluid's internal and viscous forces. The molecular viscosity then acts [illegible] work by Kolmogorov, Lewis Fry Richardson, Lars Onsager and, again, many others. In the language of modern physics, it was postulated that the Navier–Stokes equations – which describe the hydrodynamic properties of a fluid – have solutions that would display the same scale invariance as the equations themselves, but in a statistical sense. For example, the average of the velocity difference across a certain distance raised to a certain power would be proportional to that distance raised to an exponent proportional to the power. The actual exponents can then be obtained by a simple dimensional argument, rather than having to solve the Navier–Stokes equations themselves.

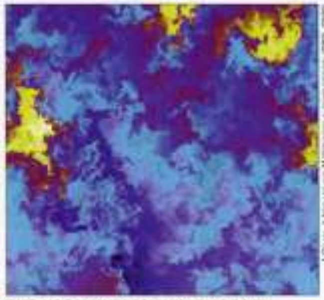
A range of increasingly accurate experiments have been carried out to study FDT. These started with work by George Batchelor and Alan Townsend in the 1940s, right through to new table-top facilities that use low-temperature helium flowing between counter-rotating disks. New data-processing techniques that can measure scaling exponents with good accuracy have also been developed, as have advanced numerical simulations, the importance of which was first perceived by the mathematician John von Neumann.

The evidence is that the assumed scale invariance is actually broken and that fully developed turbulence is "intermittent". In other words, the exponents have anomalous values that cannot be predicted by dimensional analysis – they are instead universal, being independent of how the turbulence is produced. The intermittency also means that the small-scale turbulent activity looks "spotty", and the dissipation of energy has fractal properties – in other words energy is dissipated in a cascade of energy transfers to smaller and smaller scales. Roberto Benzi, Benoît Mandelbrot, Steven Orszag, Patrick Tabeling and many others have been involved in the development of such work.

For many years, only models that were rather loosely connected with the traditional equations of fluid dynamics were available to describe this intermittency. Early models were developed by Kolmogorov and colleagues in the 1960s, while in the 1980s the concept of "multifractal" was introduced by Giorgio Parisi and the author.

A few years ago Robert Kraichnan predicted that intermittency and anomalous scaling are already present in a much simpler problem where the governing dynamics are linear – namely for a passive scalar, such as a pollutant advected by a scale-invariant [illegible] intermittent [illegible] e studied by [illegible] e). [illegible] om modern field theory have recently led to a real breakthrough for Kraichnan's problem. For the first time we have a theory of intermittency derived from first principles that can predict the values of the anomalous exponents. The anomalous corrections to what would be predicted by naive dimensional analysis arise through the presence of non-trivial elements (actually functions of several variables) in the "null space" of the operators governing the evolution of correlation functions. These can be calculated perturbatively, either using an exponent that characterizes the roughness of the prescribed velocity (as Krzysztof Gawedzki and Antti Kupiainen have done) or using the inverse of the space dimension (with the work of Mikhail Chertkov, Gregory Falkovich, Vladimir Lebedev and Igor Kolokolov). Non-perturbative calculations are also possible in some cases.

The extension of such ideas to the non-linear problem of intermittency in FDT is being actively pursued. Optimists predict that fully developed turbulence will be understood in a few years' time. But many more years may be needed to truly understand all of the complexity of turbulent flow – a problem that has been challenging physicists, mathematicians and engineers for at least half a millennium.



Concentration of a passive scalar, such as a pollutant, advected by a turbulent flow of the type found in the atmosphere or oceans, simulated numerically on a 2048 × 2048 grid. The scalar displays strong "intermittency" and has anomalous scaling properties that cannot be predicted by simple dimensional analysis. Low concentrations are dark, high ones are light

U. Frisch, *Physics World*, Dec. 1999

6. Frisch's excitement

a. Normal scaling

$S_n \sim (r/L)^{\zeta_n}$, where $\zeta_n = n/3$.

b. Anomalous scaling

$\zeta_n \neq n/3$, $2\zeta_n > \zeta_{2n}$

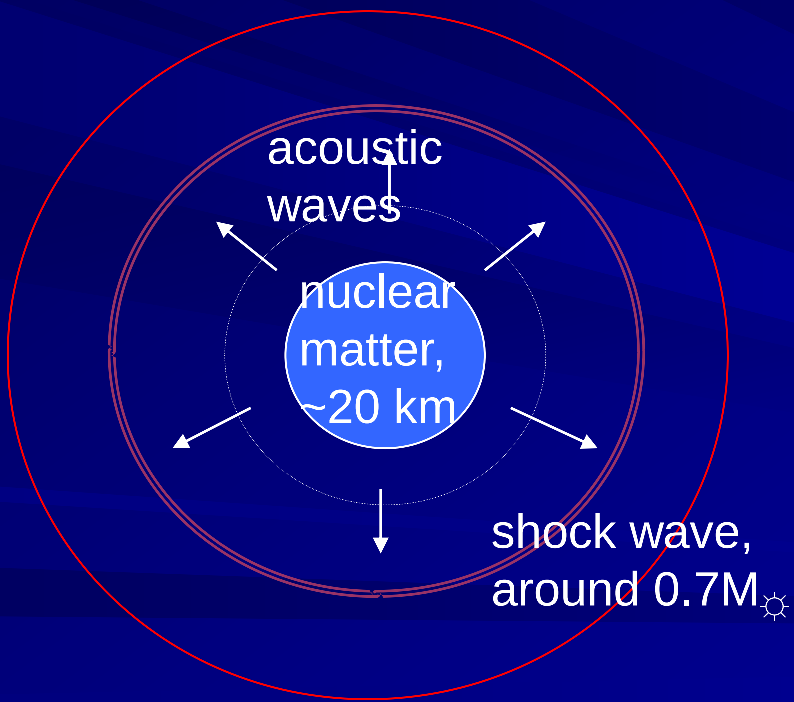
c. Importance

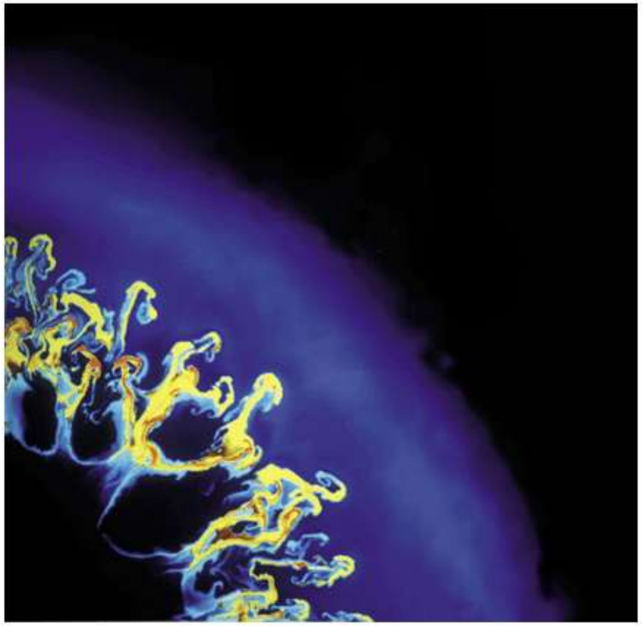
Contrast to critical scaling

d. $M_n \neq M_2^n$

The iron core becomes nuclear matter and cannot shrink anymore. The matter from outside continues to be attracted and rebounds off the nuclear matter. The acoustic waves created coalesce to an outward moving shock wave which stirs up and, eventually blasts out, the matter. This is the supernova.

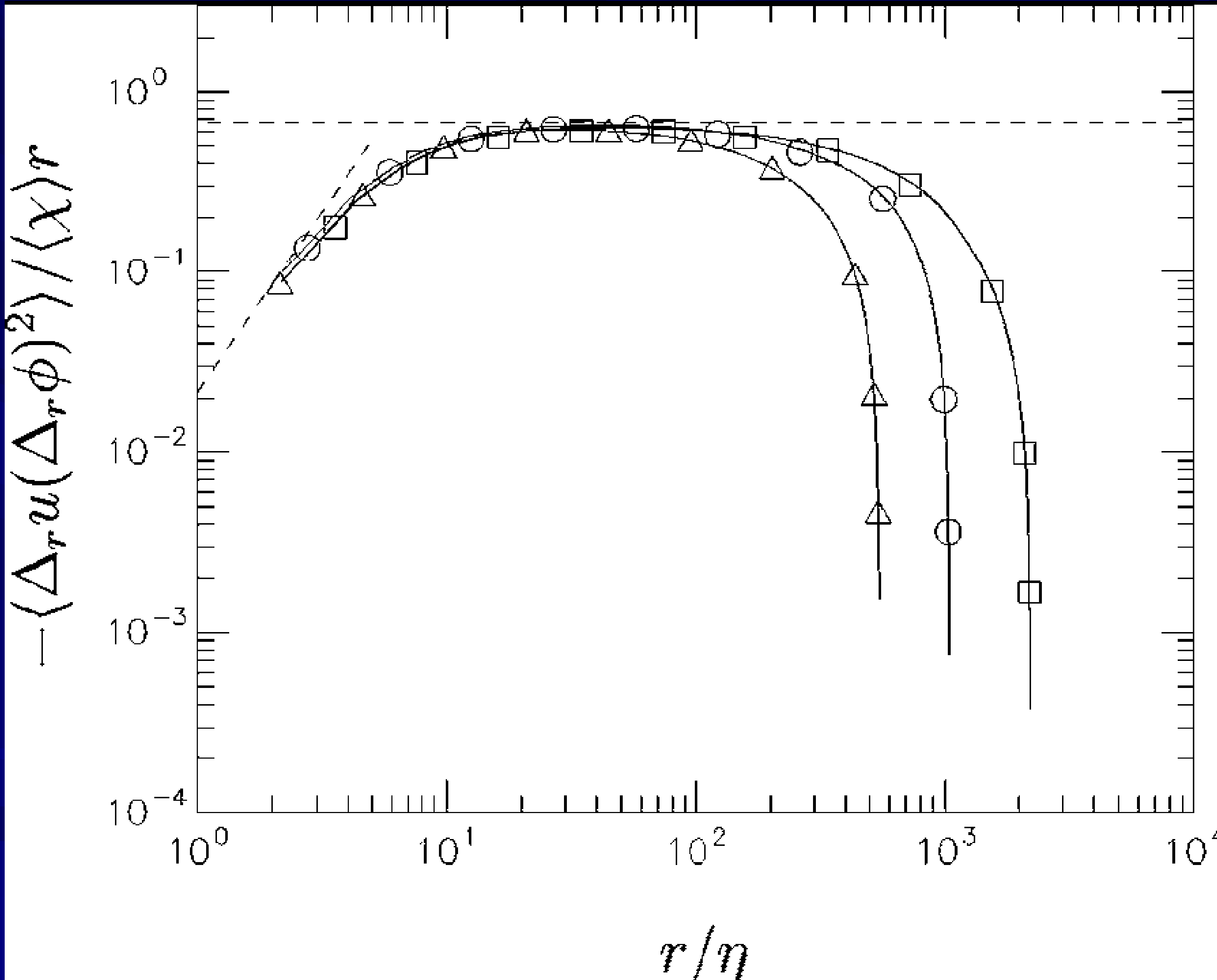
All calculations show that the shock wave stalls. We read from G.E. Brown, *Physics Today* **58**, 62 (2005): "To this day, calculated explosions have yet t
Investigat
about con
assumptio
the explos



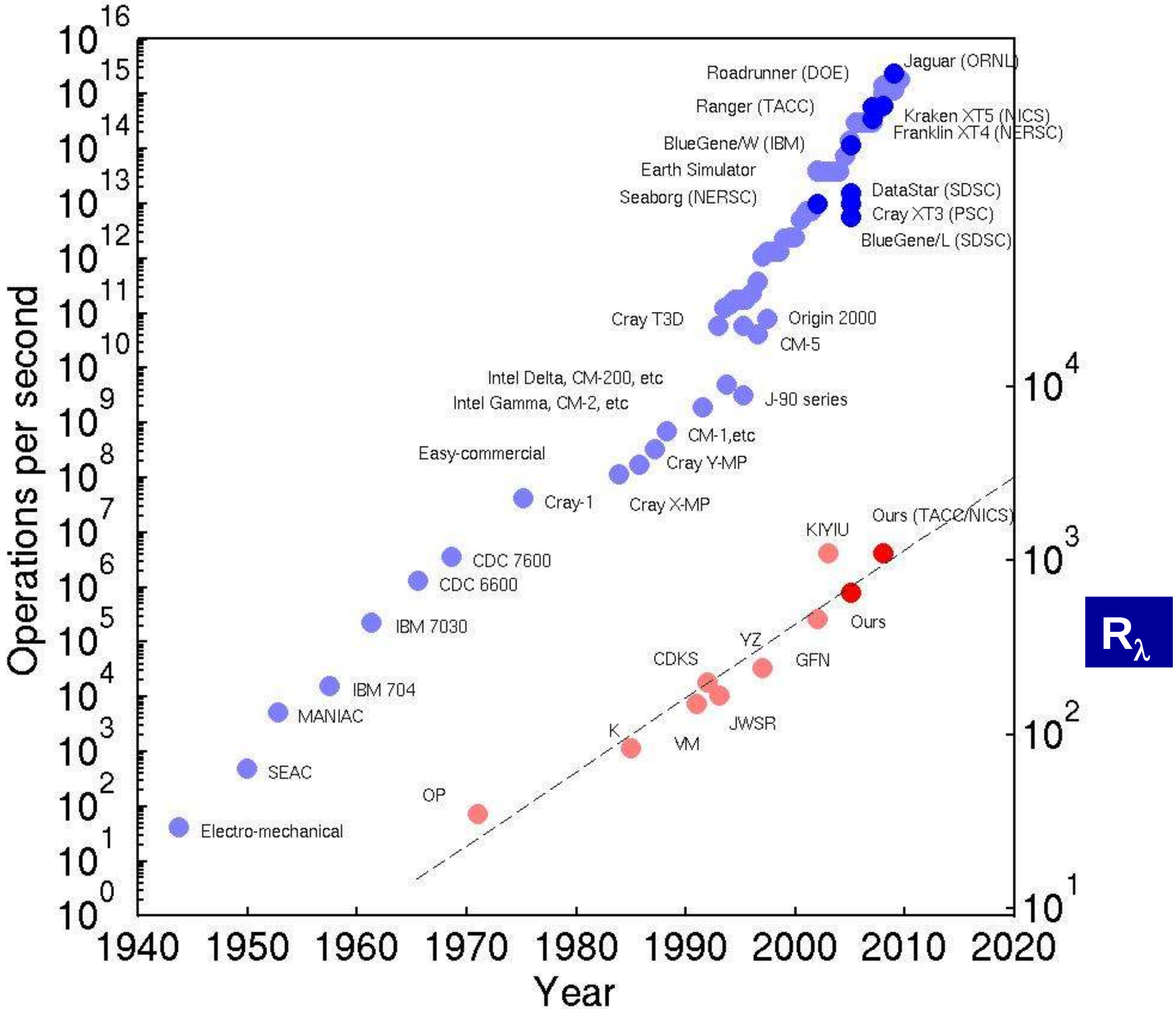


Supernova 1987A provided strong evidence of turbulence emanating from the core of the exploded star because core materials were observed well before they were predicted. The turbulence caused mixing among the layers and greatly complicated the tidy "onion" model of dying stars. [Image reproduced from Muller, Fryxell, and Arnett, Astronomy & Astrophysics 251, 505 (1991).]

- (i) dissipative anomaly for both low and high Sc
- (ii) clear inertial-convective scaling for low and moderate Sc
- (iii) viscous-convective k^{-1} for scalars of high Sc, which has received mixed support from the experiments and simulations
- (iv) clear tendency to isotropy with Sc to a lesser degree with Re which may be a big issue now that we found that with high resolution the latter appears to be true
- (v) saturation of moments of scalar gradients with Sc; I also used a very simple model for large gradient formation to explain saturation of intermittency. This analysis, leads to (R_\lambda^2*Sc) as the important parameter and the data show a high degree of universality when normalized by this parameter (see the paper I submitted to Physica D)
- (vi) systematic study of resolution effects for scalars and derivation of analytic expressions to estimate errors.

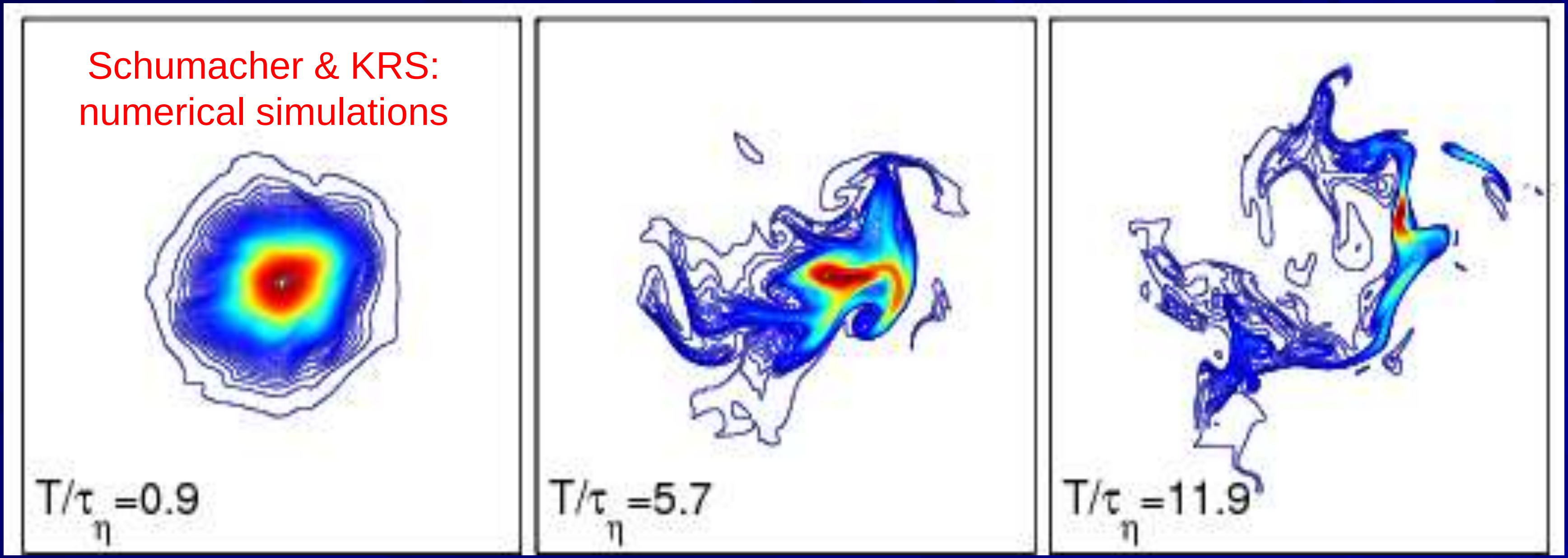


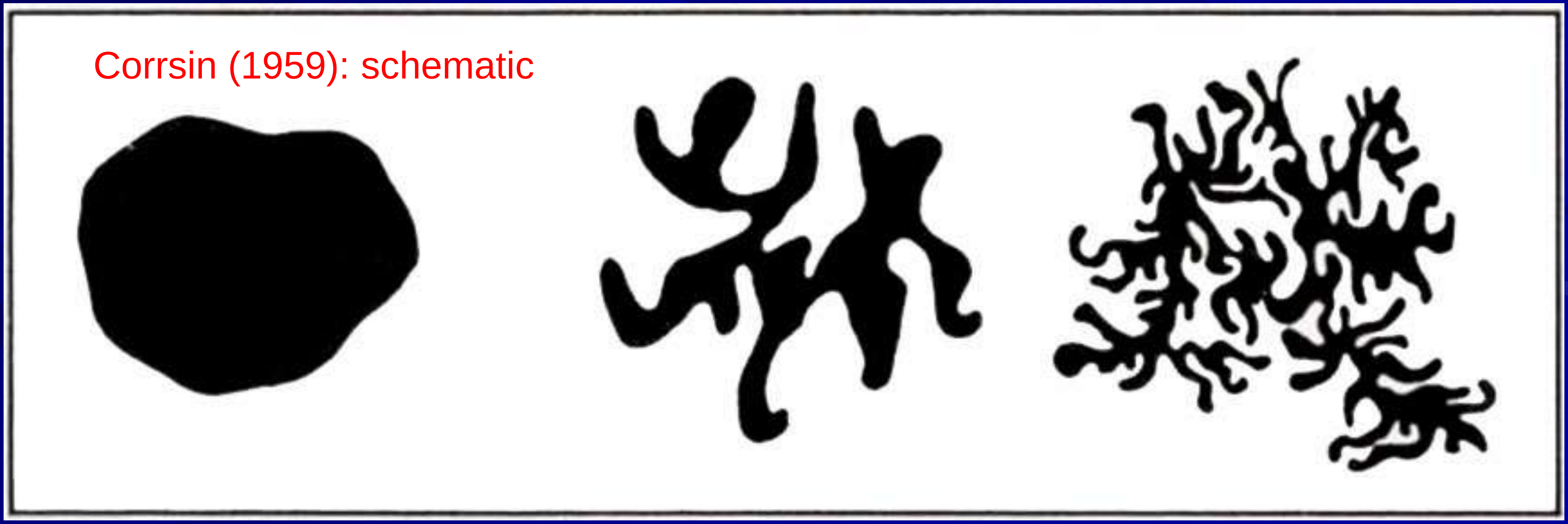
$-\langle \Delta_r u (\Delta_r \phi)^2 \rangle / \langle \chi \rangle r$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^0
10^1
10^2
10^3
10^4
r/η



Operations per second
Year
Jaguar (ORNL)
Roadrunner (DOE)
Ranger (TACC)
Kraken XT5 (NICS)
Franklin XT4 (NERSC)
BlueGene/W (IBM)
Earth Simulator
Seaborg (NERSC)
DataStar (SDSC)
Cray XT3 (PSC)
BlueGene/L (SDSC)
Cray T3D
Origin 2000
CM-5
Intel Delta, CM-200, etc
Intel Gamma, CM-2, etc
J-90 series
CM-1,etc
Easy-commercial
Cray Y-MP
Cray-1
Cray X-MP
CDC 7600
CDC 6600
IBM 7030
IBM 704
MANIAC
SEAC
Electro-mechanical
KIYIU
Ours (TACC/NICS)
Ours
YZ
GFN
CDKS
JWSR
K
VM
OP

R_λ


Schumacher & KRS:
numerical simulations
T/τη =0.9
T/τη =5.7
T/τη =11.9


Corrsin (1959): schematic

$$<\mathbf{u}(\mathbf{x};t)\ \nabla(\theta)> = -(\boldsymbol{\kappa}_T \cdot \nabla(\theta(\mathbf{x};t))$$

$\infty \approx >< \pm \Sigma \partial \times \quad \equiv \chi \Delta \nabla$

$$\partial_t a = \mathbf{v}.\nabla a + \boldsymbol{\kappa}\Delta a + \mathbf{F}_a$$

$$V_i(\mathbf{x};t) = \int dy\ G_i(\mathbf{x},\mathbf{y})\ a(\mathbf{y},t)$$

Other cases

1. Velocity field stationary, scalar field decaying

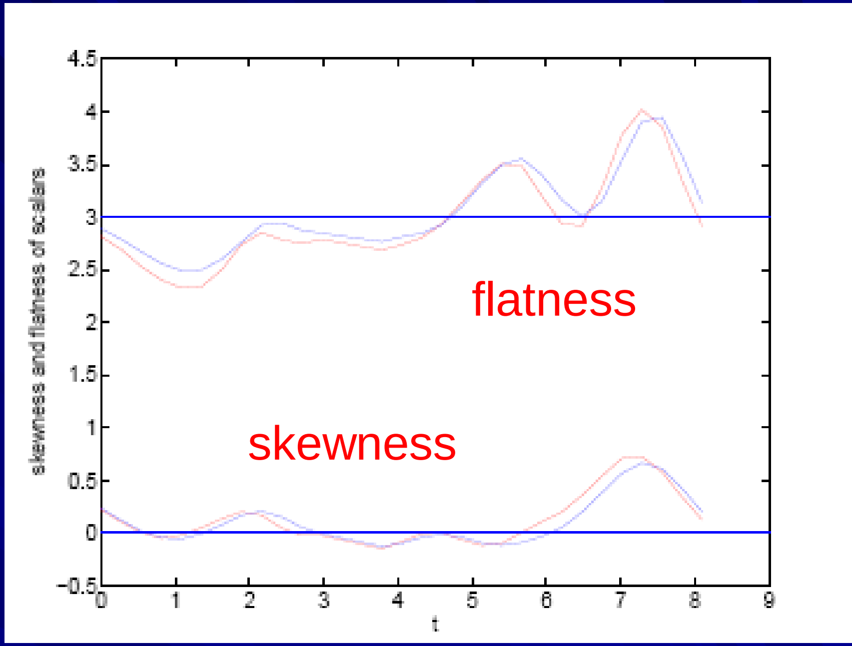
   Main result known: initially non-G PDFs tend to a Gaussian (Yeung & Pope)

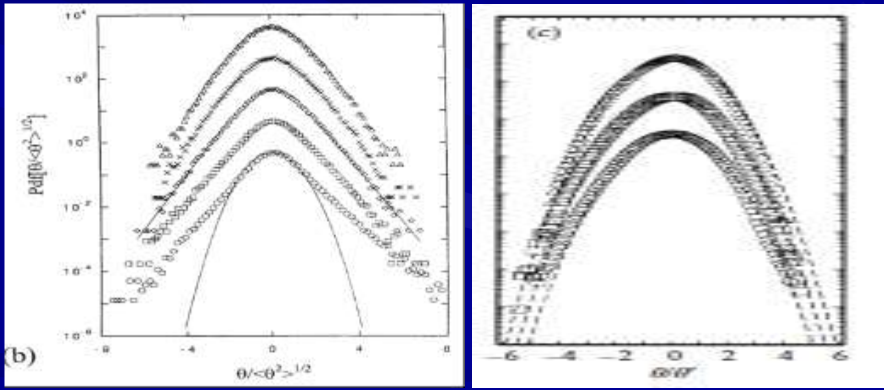
2. Velocity field decaying, scalar field stationary: unlikely to be practical, nothing known

3. Both velocity and scalar fields are stationary: some results are the same for the scalar whether sustained by random forcing or through mean gradients, but there are differences as well.

Large-scale features depend on details of forcing, initial conditions and perhaps geometry. Only some of these features are understood well.

Are small-scales universal?





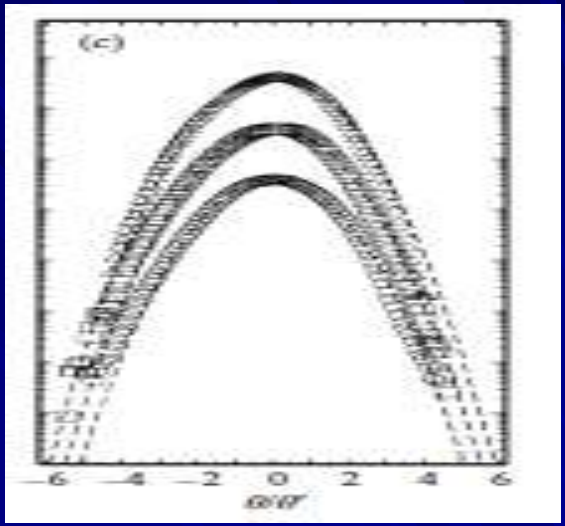
Length-scale ratio?
(autocorrelation times?)

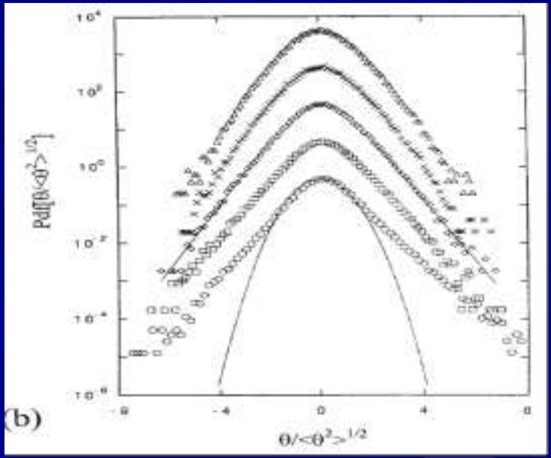
Other cases

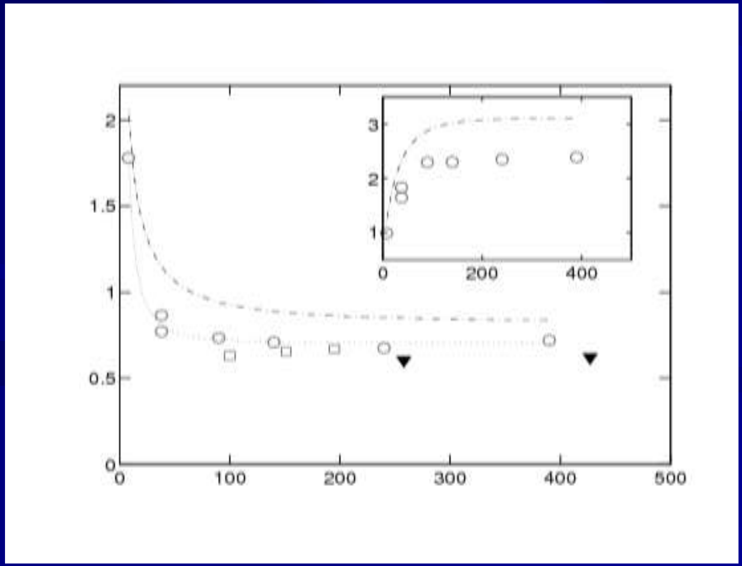
1. Velocity field stationary, scalar field decaying
   Numerical result: initially non-G PDFs tend to a Gaussian (Yeung & Pope)

2. Velocity field decaying, scalar field stationary: unlikely to be practical, nothing known

3. Both velocity and scalar fields are stationary: some results are the same whether the scalar is sustained by random forcing or through mean gradients (dissipation).





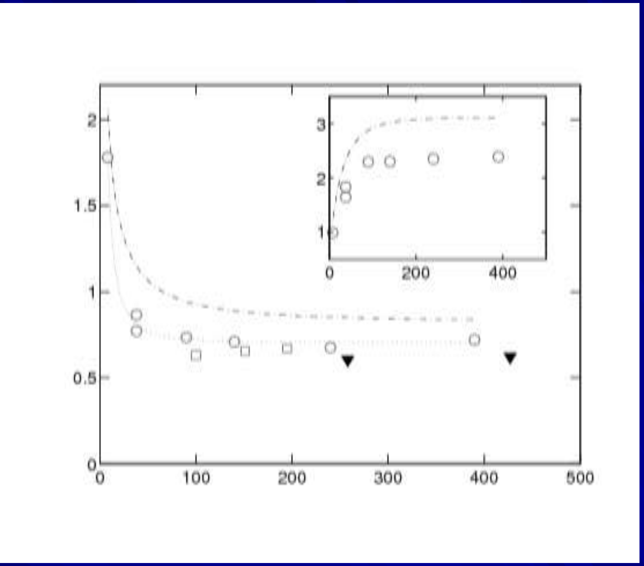


Length-scale ratio?
(autocorrelation times?)

Shear flow ref

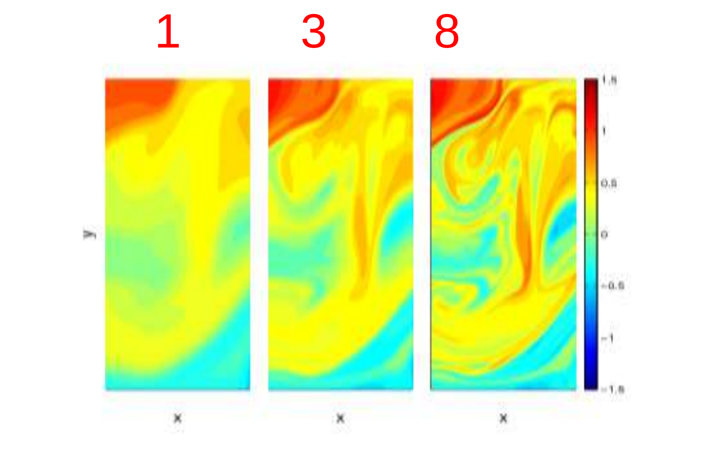
DISSIPATIVE ANOMALY

normalized dissipation rate



microscale Reynolds number

Sc > 1



The problem is simple if the velocity field is simple (e.g., **u** = constant, or periodic in 2d)

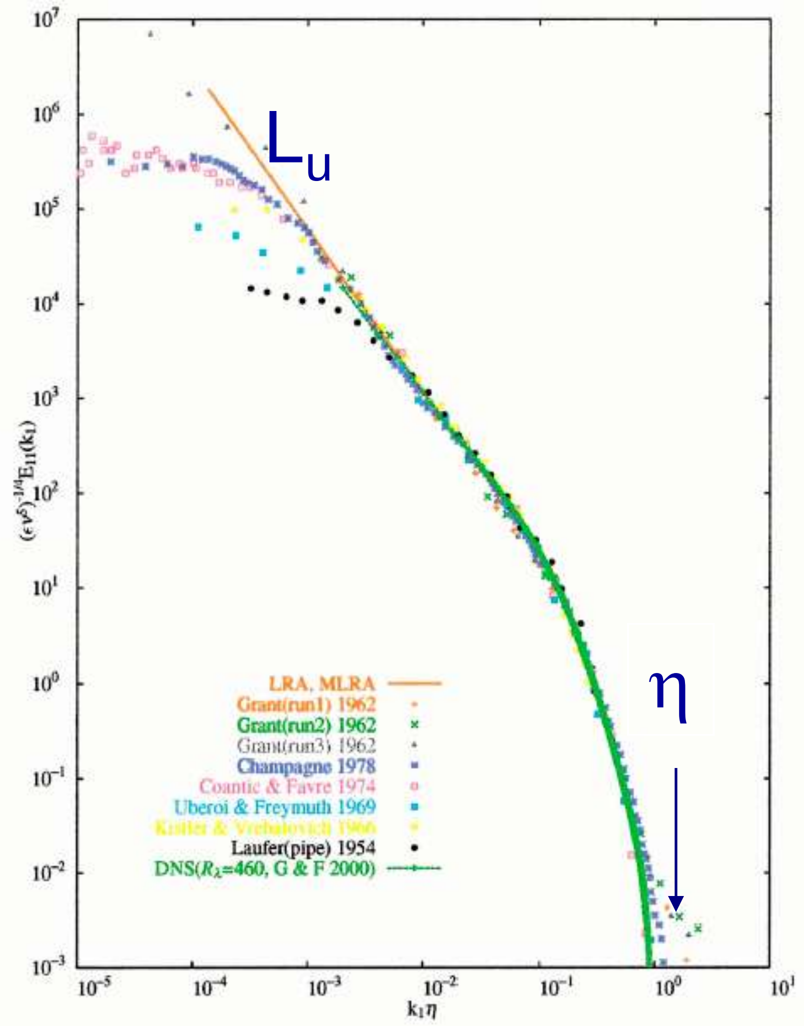
Not many results are known if **u** is turbulent in 3d, but this is what we consider here: the equation is linear for each realization but statistically nonlinear because of $\langle \mathbf{u}.\nabla\theta \rangle$.

The turbulent velocity field is analytic only in the range $r < \eta$, and only Hölder continuous, or “rough,” ($\Delta_r u \sim r^h$, $h < 1$), in the scaling range, which introduces various subtleties

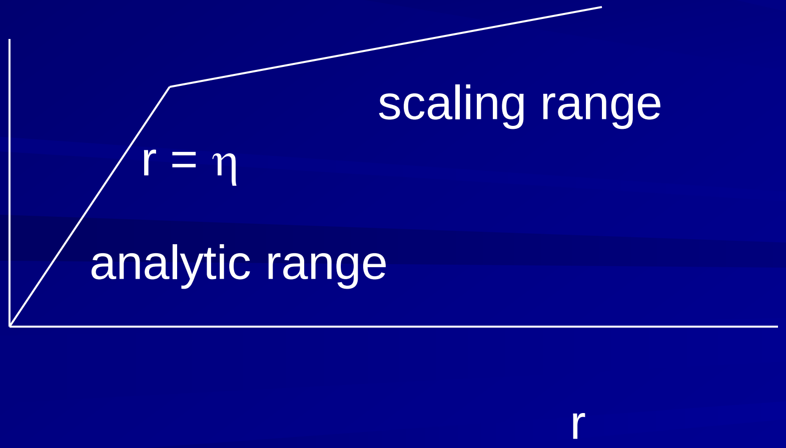
$h = 1/3$ for Kolmogorov turbulence,

in practice, h has a distribution: multiscaling

KRS, *Annu. Rev. Fluid Mech*. **23**, 539 (1991)

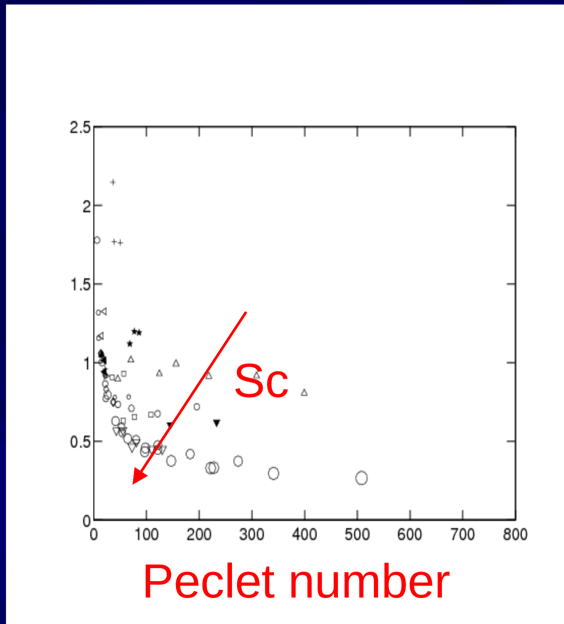


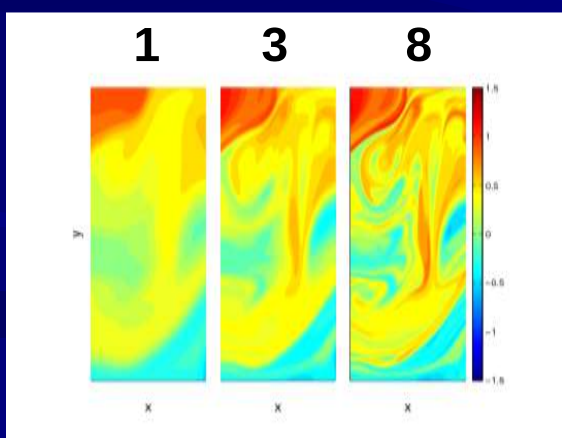
a quantity such as a structure function



DISSIPATIVE ANOMALY

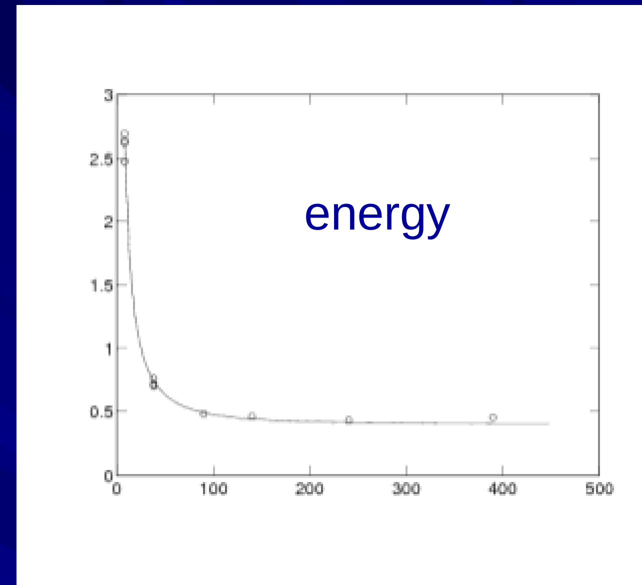
normalized dissipation rate



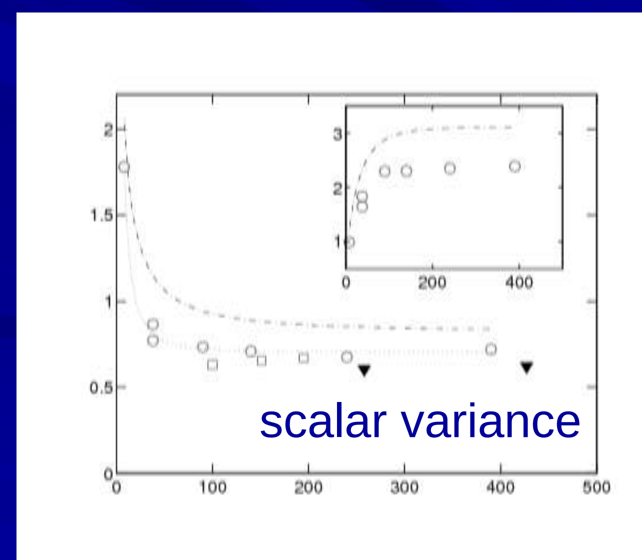


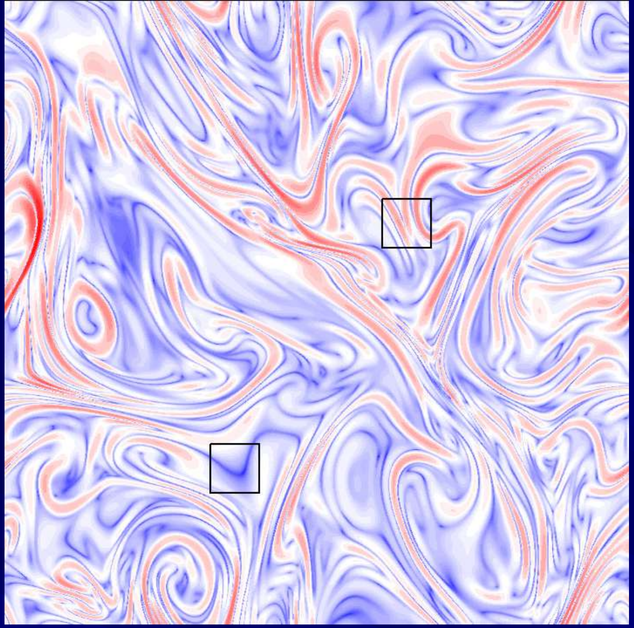
No theory exists!

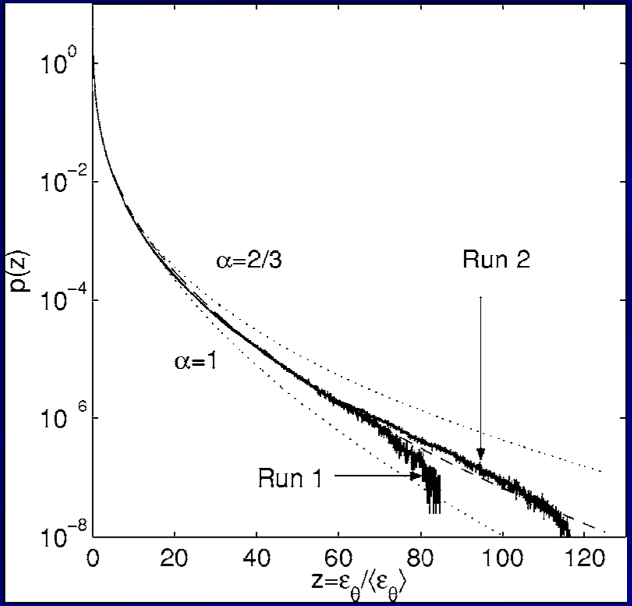
normalized dissipation rate



microscale Reynolds number







p(z)
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
α=2/3
α=1
Run 2
Run 1
0
20
40
60
80
100
120
$z=\varepsilon_\theta/\langle\varepsilon_\theta\rangle$