

A SHARP LOCAL BLOW-UP CONDITION FOR EULER-POISSON EQUATIONS WITH ATTRACTIVE FORCING

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ABSTRACT. We improve the recent result of [2] proving a one-sided threshold condition which leads to finite-time breakdown of the Euler-Poisson equations in arbitrary dimension n .

1. INTRODUCTION

The pressure-less Euler-Poisson (EP) equations in dimension $n \geq 1$ are

$$(1.1a) \quad \rho_t + \operatorname{div}(\rho \mathbf{u}) = 0$$

$$(1.1b) \quad \mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u} = k \nabla \Delta^{-1}(\rho - c),$$

governing the unknown density $\rho = \rho(t, x) : \mathbb{R}_+ \times \mathbb{R}^n \mapsto \mathbb{R}_+$ and velocity $\mathbf{u} = \mathbf{u}(t, x) : \mathbb{R}_+ \times \mathbb{R}^n \mapsto \mathbb{R}^n$ subject to initial conditions $\rho(0, x) = \rho_0(x)$ and $\mathbf{u}(0, x) = \mathbf{u}_0(x)$. They involve two constants: a fixed background state $c > 0$ such that $\int(\rho - c)dx = 0$, and the constant k which parameterizes the repulsive $k > 0$ or attractive $k < 0$ forcing, caused by the Poisson potential $\Delta^{-1}(\rho - c)$. The EP system appears in numerous applications including semiconductor, plasma physics ($k > 0$) and collapse of interstellar cloud ($k < 0$).

This paper is restricted to the *attractive case*, $k < 0$. For simplicity, we set $c = 1$, $k = -1$ in (1.1a), (1.1b) to arrive at the unit-free EP system,

$$(1.2a) \quad \rho_t + \operatorname{div}(\rho \mathbf{u}) = 0,$$

$$(1.2b) \quad \mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \Delta^{-1}(\rho - 1).$$

All discussion in this paper remains valid for EP system with physical parameters $c > 0$, $k < 0$ upon a simple rescaling argument — see Corollary 1.1 below.

We are concerned here with the persistence of C^1 regularity for solutions of the attractive EP system. Our main theorem reveals a *pointwise* criterion on the initial data, a so-called critical threshold criterion [5, 8, 10], that leads to finite time blow-up of $\nabla \mathbf{u}$. It is a sharp, nonlinear quantification of balance between $\operatorname{div} \mathbf{u}$ and ρ , two competing mechanisms that dictate the C^1 regularity of EP flows. Our result also stands out as a generalization of several existing results [5, 2, 9, 10] for which further discussion is given after the Main Theorem and its corollary.

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Main Theorem 1.1. *Consider the n -dimensional, attractive Euler-Poisson system (1.2a), (1.2b) subject to initial data ρ_0, \mathbf{u}_0 . Then, the solution will lose C^1 regularity at a finite time $t = t_c < \infty$, if there exists a non-vacuum initial state $\rho_0(\bar{x}) > 0$ with vanishing initial vorticity, $\nabla \times \mathbf{u}_0(\bar{x}) = 0$, such that the following sup-critical condition is fulfilled,*

$$(1.3a) \quad \operatorname{div} \mathbf{u}_0(\bar{x}) < \operatorname{sgn}(\rho_0(\bar{x}) - 1) \sqrt{nF(\rho_0(\bar{x}))},$$

where

$$(1.3b) \quad F(\rho) := \begin{cases} 1 + \frac{2\rho}{n-2} - \frac{n\rho^{2/n}}{n-2}, & n \neq 2, \\ 1 - \rho + \rho \ln \rho, & n = 2. \end{cases}$$

In particular, $\min_x \operatorname{div} \mathbf{u}(t, x) \rightarrow -\infty$ and $\max_x \rho(t, x) \rightarrow \infty$ as $t \uparrow t_c$.

Proof. Combine Lemma 3.1 and Lemma 4.2 below, while noting that the curve

$$\operatorname{div} \mathbf{u} = \operatorname{sgn}(\rho - 1) \sqrt{nF(\rho)},$$

is the separatrix along the boundary of the blow-up region $\Omega = \Omega_1 \cup \Omega_2$ defined in (4.3) and illustrated in Figure 4.1. \square

We note by passing that, by classical arguments, the force-free Euler system $\mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u} = 0$ exhibits finite time blow-up if and only if there exists at least one *negative* eigenvalue of $\nabla \mathbf{u}_0(\bar{x})$. In the above theorem, however, finite-time blow-up can occur solely depending on the initial profile of $\operatorname{div} \mathbf{u}_0$ and ρ_0 regardless of individual eigenvalues of $\nabla \mathbf{u}_0$.

We also note that, by rescaling ρ to ρ/c , x to $\sqrt{-kc}x$ and t to $\sqrt{-kc}t$, the Main Theorem immediately applies to the EP system (1.1a), (1.1b) with physical parameters. Since the EP system with $k < 0$ models the collapse of interstellar cloud, the following corollary reveals a pointwise condition for mass concentration, $\rho \rightarrow \infty$, which interestingly preludes the birth of new stars.

Corollary 1.1. *Consider the Euler-Poisson system (1.1a), (1.1b) with $c > 0$, $k < 0$ subject to initial data ρ_0, \mathbf{u}_0 . Then, the solution will lose C^1 regularity at a finite time $t_c < \infty$, if there exists a non-vacuum initial state $\rho_0(\bar{x}) > 0$ with vanishing initial vorticity, $\nabla \times \mathbf{u}_0(\bar{x}) = 0$, such that the sup-critical condition is fulfilled,*

$$(1.4) \quad \operatorname{div} \mathbf{u}_0(\bar{x}) < \operatorname{sgn}(\rho_0(\bar{x}) - c) \sqrt{-nkcF\left(\frac{\rho_0(\bar{x})}{c}\right)}$$

where $F(\cdot)$ is given in (1.3b). In particular, $\min_x \operatorname{div} \mathbf{u}(t, x) \rightarrow -\infty$ and $\max_x \rho(t, x) \rightarrow \infty$ as $t \uparrow t_c$.

The concept of Critical Threshold and associated methodology is originated and developed in a series of papers by Engelberg, Liu and Tadmor [5], Liu and Tadmor [10, 8] and more. It first appears in [5] regarding pointwise criteria for C^1 solution regularity of 1D EP system. The key argument in that paper is based on the convective derivative along particle paths $' = \partial_t + \mathbf{u} \cdot \nabla$. It makes possible to obtain a 2-by-2 ODE system for u_x and ρ along particle paths — the so-called Lagrangian formulation. Phase plane analysis is then employed to study the finiteness of the ODE solutions and therefore C^1 regularity of the PDE solution. Similar results stay valid for Euler-Poisson systems with

geometric symmetry in higher dimensions [1, 8]. To treat genuinely multi-D cases, Liu and Tadmor introduce in [8] the method of spectral dynamics which relies on the ODE system governing eigenvalues of

$$M := \nabla \mathbf{u},$$

which is the velocity gradient matrix, along particle paths. They identify if-and-only-if, pointwise conditions for global existence of C^1 solutions to *restricted* Euler-Poisson systems. Chae and Tadmor [2] further extend the Critical Threshold argument to multi-D full Euler-Poisson systems (1.2a), (1.2b) with attractive forcing $k < 0$. Their result, however, offers a blow-up region $\nabla \times \mathbf{u}_0 = 0$, $\text{div } \mathbf{u}_0 < -\sqrt{-nkc}$ which is only a subset of the blow-up region in (1.4). This subset is to the left of the solid line $d \leq d^- := -\sqrt{-nkc}$ depicted in figure 4.1. Finally, a recent paper by Tadmor and Wei [22] reveals the critical threshold phenomena in 1D Euler-Poisson system with pressure.

When tracking other results on well-posedness of Euler-Poisson equations, we find them commonly relying on (the vast family of) energy methods and thus fundamentally differ from our pointwise results obtained via the Lagrangian approach. With repulsive force $k > 0$, we refer to [7, 3] for global existence of classical solutions with small data and [19] for nonexistence of global solutions. With attractive force $k < 0$, see [15] for local regularity of classical solutions and [16, 17] for nonexistence results. Discussion on weak solutions of Euler-Poisson systems can be found in e.g. [25, 18, 20]. We also refer to [6, 4, 12, 14, 21] and references therein for steady-state solutions. Study of Euler-Poisson system with damping relaxation can be found in e.g. [23, 24, 13].

The rest of this paper is organized as following. In Section 2, we follow the idea of [2] to derive along particle paths an ODE system governing the dynamics of eigenvalues for $S := \frac{1}{2}(M + M^\top)$. This is a variation of the spectral dynamics for M introduced in [8]. We then derive in Section 3 a closed 2×2 ODE system (3.1) at the cost of turning one equation into inequality. By the comparison principle, this inequality is in favor of blow-up. Thus, with the inequality sign being replaced with equality sign, a modified ODE system is used to yield sub-solutions and to study blow-up scenario for the original system. Section 4, devoted to the modified system, reveals the Critical Threshold for such a system. Consequently, a pointwise blow-up condition for the original system is identified.

2. SPECTRAL DYNAMICS

We examine the gradient matrix $M = \nabla \mathbf{u}$ and its symmetric part, $S = \frac{1}{2}(\nabla \mathbf{u} + (\nabla \mathbf{u})^\top)$. Both matrices are used to study the spectral dynamics of Euler systems (see e.g. [8] for M and [2] for S). The relation between the spectra of M and S is described in the following.

Proposition 2.1. *Let $\{\lambda_M\}$ denote the eigenvalues of M and $\{\lambda_S\}$ for S . Then*

$$(2.1) \quad \sum_{\lambda_M} \lambda_M = \sum_{\lambda_S} \lambda_S = \text{div } \mathbf{u},$$

$$(2.2) \quad \sum_{\lambda_M} \lambda_M^2 = \sum_{\lambda_S} \lambda_S^2 - \frac{1}{2}|\omega|^2.$$

Here, ω is the $\frac{n(n-1)}{2}$ vorticity vector which consists of the off-diagonal entries of $A := \frac{1}{2}(\nabla \mathbf{u} - (\nabla \mathbf{u})^\top)$.

Proof. Use identity $M = S + A$ and the skew-symmetry of A ,

$$\sum_{\lambda_M} \lambda_M = \text{tr}(M) = \text{tr}(S + A) = \text{tr}(S) = \sum_{\lambda_S} \lambda_S.$$

Squaring the last identity we have $M^2 = S^2 + A^2 + AS + SA$ and therefore,

$$\sum_{\lambda_M} \lambda_M^2 = \text{tr}(M^2) = \text{tr}(S^2 + A^2 + AS + SA) = \sum_{\lambda_S} \lambda_S^2 + \text{tr}(A^2).$$

Note that $AS + SA$ is skew-symmetric and thus traceless. A simple calculation yields $\text{tr}(A^2) = -\frac{1}{2}|\omega|^2$. \square

Following [8], we turn to study the dynamics of M along particle paths. Take the gradient of (1.2b) to find

$$(2.3) \quad M' + M^2 \equiv M_t + \mathbf{u} \cdot \nabla M + M^2 = -R(\rho - 1),$$

where R stands for the *Riesz matrix*, $R = \{R_{ij}\} := \{\partial_{x_i x_j} \Delta^{-1}\}$.

The trace of (2.3) then yields that the divergence, $d := \text{tr}(M)$, is governed by

$$d' = -\sum_{\lambda_M} \lambda_M^2 - (\rho - 1),$$

and in view of (2.2),

$$(2.4) \quad d' = -\sum_{\lambda_S} \lambda_S^2 + \frac{1}{2}|\omega|^2 - (\rho - 1).$$

We now make the first observation regarding the invariance of the vorticity ω : taking the skew-symmetric part of the M -equation (2.3),

$$(2.5) \quad A' + AS + SA = 0.$$

It follows that if the initial vorticity vanishes, $\omega(\bar{x}) \mapsto \nabla \times \mathbf{u}(\bar{x}) = 0$, then by (2.5), $\omega \mapsto \nabla \times \mathbf{u}$ vanishes along the particle path which emanates from \bar{x} . This allows us to decouple the vorticity and divergence dynamics, and (2.4) implies

$$(2.6) \quad d' = -\sum_{\lambda_S} \lambda_S^2 - (\rho - 1), \quad \nabla \times \mathbf{u} = 0.$$

Finally, we use Cauchy-Schwartz $\sum \lambda_S^2 \leq \frac{1}{n} \left(\sum \lambda_S \right)^2 = \frac{1}{n} d^2$ and the fact that all λ_S are real (due to the symmetry of S), to deduce the *inequality*,

$$(2.7a) \quad d' \leq -\frac{1}{n} d^2 - (\rho - 1).$$

This, together with the mass equation (1.2a) which can be written along particle path

$$(2.7b) \quad \rho' = -d\rho,$$

give us the desired closed system which governs (ρ, d) along particle paths.

Remark 2.1. *The approach pursued in this paper will be based on the inequality (2.7a) and is therefore limited to derivation of a finite time breakdown. To argue the global regularity, one needs to study the underlying equality (2.6), and to this end, to study the trace $\sum \lambda_S^2$. In the two-dimensional case, for example, one can use $\sum \lambda_S^2 = d^2/2 + \eta^2/2$ to replace (2.7a) with*

$$d' = -\frac{1}{2}d^2 - \frac{1}{2}\eta^2 - (\rho - 1), \quad \eta := \lambda_{S,2} - \lambda_{S,1}.$$

In this framework, global 2D regularity is dictated by the dynamics of the spectral gap, $\eta = \lambda_{S,2} - \lambda_{S,1}$, which in turn requires the dynamics of the Riesz transform $R(\rho - 1)$.

3. A COMPARISON PRINCIPLE WITH A MAJORANT SYSTEM

The blow-up analysis, driven by the inequalities (2.7),

$$(3.1a) \quad d' \leq -\frac{1}{n}d^2 - (\rho - 1),$$

$$(3.1b) \quad \rho' = -d\rho.$$

is carried out by standard comparison with the majorant system

$$(3.2a) \quad e' = -\frac{1}{n}e^2 - (\zeta - 1),$$

$$(3.2b) \quad \zeta' = -e\zeta.$$

The following proposition guarantees the monotonicity of the solution operator associated with (3.1).

Lemma 3.1. *The following monotone relation between system (3.1) and system (3.2) is invariant forward in time,*

$$(3.3) \quad \begin{cases} d(0) < e(0) \\ 0 < \zeta(0) < \rho(0) \end{cases} \quad \text{implies} \quad \begin{cases} d(t) < e(t) \\ 0 < \zeta(t) < \rho(t) \end{cases} \quad \text{for } t \geq 0,$$

as long as all solutions remain finite on time interval $[0, t]$.

Proof. Invariance of positivity of ζ is a direct consequence of (3.2b) and finiteness of e . The rest can be proved by contradiction. Suppose t_1 is the earliest time when (3.3) is violated. Then,

$$(3.4) \quad \zeta(t_1) = \zeta(0) \exp\left(-\int_0^{t_1} e(t)dt\right) < \rho(0) \exp\left(-\int_0^{t_1} d(t)dt\right) = \rho(t_1).$$

Therefore, we are left with only one possibility $e(t_1) = d(t_1)$. However, subtracting (3.1a) from (3.2a),

$$(3.5) \quad (e - d)' \geq -\frac{1}{n}(e^2 - d^2) - (\zeta - \rho).$$

Setting $t = t_1$ in the above inequality, we find that

$$LHS \text{ of (3.5)} = (e(t_1) - d(t_1))' \leq 0,$$

since $e(t) - d(t) > 0$ for all $t < t_1$; but this contradicts the positivity of the expression on the right of (3.5), for by (3.4)

$$RHS \text{ of (3.5)} = 0 - [\zeta(t_1) - \rho(t_1)] > 0.$$

□

In the next section, we employ phase plane analysis on the modified system (3.2). When translated in terms of the original system (3.1), however, such analysis can only yield blow-up results and is insufficient for global existence results. In other words, estimate (3.3) is only useful for proving $d \searrow -\infty$, the key mechanism for blow-up of C^1 solutions.

4. STABILITY ANALYSIS OF THE MAJORANT SYSTEM

We shall prove the blow-up of the majorant system (3.2), $e(t) \rightarrow -\infty$ as $t \uparrow t_c$, which in turn, by lemma 3.1 implies $d(t) \rightarrow -\infty$. Abusing notations, we express the majorant system in terms of the original variables $(e, \zeta) \mapsto (d, \rho)$:

$$(4.1a) \quad d' = -\frac{1}{n}d^2 - (\rho - 1),$$

$$(4.1b) \quad \rho' = -d\rho.$$

The (in-)stability analysis of (4.1) hinges on the path invariants of this system. To this end, we use the same q -transformation employed in [11, 10]: setting $q := d^2$ and differentiate along the path $\{(t, X(a, t)) \mid X_t(a, t) = u(t, X(a, t)), X(a, 0) = a\}$, we find

$$\frac{dq}{d\rho} = 2d\frac{d'}{\rho'} = \frac{2}{n\rho}q + 2\left(1 - \frac{1}{\rho}\right),$$

which yields

$$\frac{d}{d\rho}\left(q\rho^{-\frac{2}{n}}\right) = 2(1 - \rho^{-1})\rho^{-\frac{2}{n}}.$$

Upon integration, we arrive at the following key observation.

Lemma 4.1. *The majorant system (4.1) is equipped with the path invariant,*

$$I(d(t), \rho(t)) = I(d_0, \rho_0),$$

along each path $(t, x(t))$ initiated with a non-vacuum state $(d_0, \rho_0 > 0)$. Here,

$$(4.2) \quad I(d, \rho) := d^2\rho^{-\frac{2}{n}} - 2\int_1^\rho (1 - r^{-1})r^{-\frac{2}{n}} dr = \rho^{-\frac{2}{n}}(d^2 - nF(\rho)),$$

where $F(\cdot)$ is specified in (1.3b).

It is simple calculation to show that the majorant system (4.1) admits three distinct critical points (see figure 4.1):

$$(d^*, \rho^*) := (0, 1), \quad (d^\pm, \rho^\pm) := (\pm\sqrt{n}, 0).$$

and that $(0, 1)$ is a saddle point, $(-\sqrt{n}, 0)$ a nodal source and $(\sqrt{n}, 0)$ a nodal sink. The separatrix is given by the zero level set $I(d, \rho) = 0$. Moreover, the right branch of the separatrix, $d = \sqrt{nF(\rho)}$ connects critical points $(0, 1)$ and $(\sqrt{n}, 0)$ while the left branch, $d = -\sqrt{nF(\rho)}$ connects $(0, 1)$ and $(-\sqrt{n}, 0)$.

By inspection of the phase plane in figure 4.1, we postulate the following invariant region of finite-time blow-up for the modified system (4.1),

$$(4.3a) \quad \Omega = \Omega_1 \cup \Omega_2 = \{(d, \rho) \mid d < \operatorname{sgn}(\rho - 1)\sqrt{nF(\rho)}\}$$

where

$$(4.3b) \quad \Omega_1 := \{(d, \rho) \mid I(d, \rho) > 0 \text{ and } d < 0 \text{ and } \rho > 0\},$$

$$(4.3c) \quad \Omega_2 := \{(d, \rho) \mid I(d, \rho) < 0 \text{ and } \rho > 1\}.$$

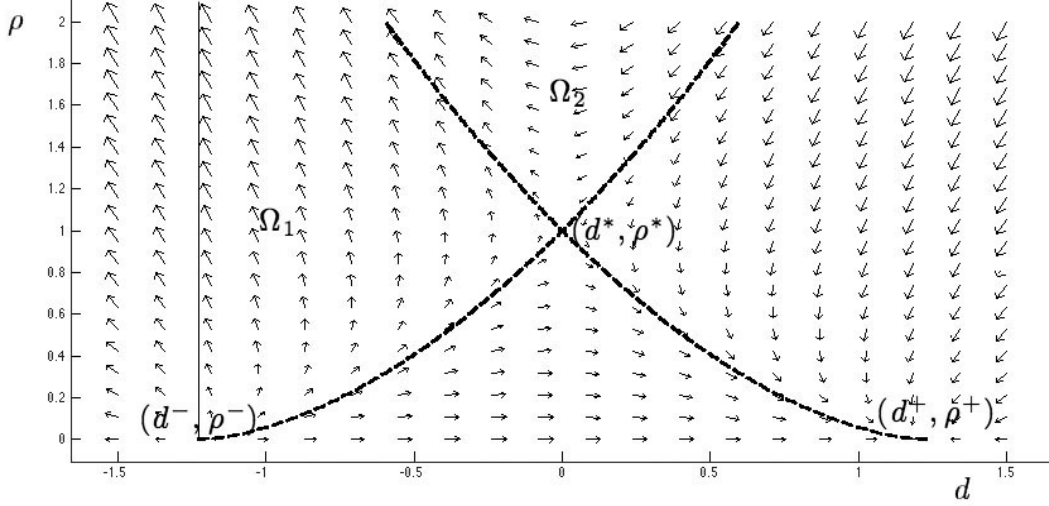


FIGURE 4.1. Phase plane of (4.1) with blow-up region $\Omega_1 \cup \Omega_2$ which extends the Chae-Tadmor region [2] $d \leq d^-$.

Lemma 4.2. *Consider the modified system (4.1), equipped with initial data (d_0, ρ_0) . If $(d_0, \rho_0) \in \Omega$, then $\text{div } \mathbf{u} \rightarrow -\infty$ and $\rho \rightarrow \infty$ at a finite time.*

Proof. We begin by recalling (1.3b), consult (4.2),

$$F(\rho) = \frac{2}{n} \rho^{\frac{2}{n}} \int_1^\rho (1 - r^{-1}) r^{-\frac{2}{n}} dr.$$

Clearly, $F(1) = F'(1) = 0$ and a simple calculation shows that $F''(\rho) = \frac{2}{n} \rho^{\frac{2}{n}-2}$, which implies that $F(\rho)$ is a strictly convex function of positive ρ and attains its only minimum at $\rho = 1$,

$$(4.4) \quad F(\rho) \geq F(1) = 0.$$

We shall also utilize the invariance of (4.2)

$$(4.5) \quad d^2 - nF(\rho) = \rho^{\frac{2}{n}} I_0, \quad I_0 = I(d_0, \rho_0).$$

We now turn to discuss the two possible blow-up scenarios, depending whether the initial data (d_0, ρ_0) belong to the blow-up regions Ω_1 or Ω_2 given in (4.3).

Case #1. Assume that $(d_0, \rho_0) \in \Omega_1$ so that the invariant I remains a *positive* constant

$$I > 0.$$

In this case, d remains negative, for otherwise, setting $d = 0$ in (4.5) would result in $F(\rho) = -\rho^{\frac{2}{n}}I/n < 0$, violating (4.4). Thus, (4.5) and (4.4) yield an upper bound,

$$d \leq -\rho^{\frac{1}{n}}\sqrt{I}.$$

Then, by (4.1b), we have a Riccati type of equation $\rho' \geq \sqrt{I}\rho^{1+\frac{1}{n}}$ for which the solution exhibits blow-up $\rho \rightarrow +\infty$ and the divergence $d = \operatorname{div} \mathbf{u}$ approaches $-\infty$ at a finite time due to (4.5).

Case #2. Assume that $(d_0, \rho_0) \in \Omega_2$ so that the invariant I remains a *negative* constant

$$I < 0.$$

In this case, $\rho - 1$ remains positive, for otherwise setting $\rho = 1$ in (4.5) would result in $F(1) = (d^2 - I)/n > 0$ in contradiction to (4.4). Now, for $\rho > 1$ we have

$$F(\rho) = \frac{2}{n}\rho^{2/n} \int_1^\rho \left(1 - \frac{1}{r}\right) \frac{1}{r^{2/n}} dr \leq \frac{2}{n}\rho^{2/n}(\rho - 1).$$

This together with (4.5) yield

$$\frac{2}{n}\rho^{2/n}(\rho - 1) \geq F(\rho) = \frac{1}{n}(d^2 - \rho^{2/n}I) \geq -\frac{1}{n}\rho^{2/n}I$$

and the lower bound, $\rho - 1 \geq -I/2$ follows. Thus, by (4.1a), we end up with a Riccati type of equation

$$d' \leq -\frac{d^2}{n} + \frac{I}{2}.$$

Since the invariant I remains a negative constant, the solution exhibits blow-up $d = \operatorname{div} \mathbf{u} \rightarrow -\infty$ at a finite time even if initially $d_0 > 0$. The density ρ also approaches ∞ in finite time due to (4.5). \square

The last step of proving the Main Theorem is just to combine the comparison principle in Lemma 3.1 with the above lemma. We notice that Ω is an open set and thus given any initial data $(d_0, \rho_0) \in \Omega$ for the original system, we can always find $\varepsilon > 0$ and initial data $(d_0 + \varepsilon, \rho_0 - \varepsilon) \in \Omega$ for the modified system. This latter initial data will lead to finite time blow-up of the modified system and therefore, by lemma 3.1, initial data $(d_0, \rho_0) \in \Omega$ will lead to finite time blow-up of the original system.

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