Macroscopic limits of kinetic models revisited

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1 Introduction

A fluid, a plasma or a solid-state device consists of a large number of particles which interact among themselves and with their environment. The large scale (or macroscopic) behaviour of the system is, to a large extent, determined by the nature or the elementary particle interactions at the microscopic level. The study of the interplay between the microscopic and macroscopic worlds has been the subject of an intense research for more than a century (starting from the works of Boltzmann and Maxwell).

Recently, this field of research has received an increased interest driven by the quest for new models accompanying the development of new technologies. In this talk, after an introduction to the field, we shall present some new developments in this area and illustrate them by examples pertaining with space and plasma devices technologies.

2 Summary of the talk

We shall start with a basic introduction to kinetic models, which will constitute our microscopic level of physical description. In a kinetic model, the state of the ensemble of particles is described by a phase-space particle density $f(x, v, t)$, which describes the number of particles at a given location x, with a given velocity v at time t. The equation for f models how particles evolve, subject to force fields and interactions among themselves or with their surrounding (obstacles, boundaries, other species of molecules, radiation, etc). This leads to the so-called Boltzmann equation. Here, we shall mainly deal with particle interactions with their surroundings.

The kinetic distribution function f is related with more conventional continuum (or macroscopic) variables, like the number density $n(x, t)$ or the temperature $T(x, t)$ through an integration of f with respect to the velocity variable (velocity moments). One central question is to try to deduce evolution equations for these variables from that of the distribution function. This question has been solved, at least formally, for conventional models (like Euler, Navier-Stokes equations or Drift-Diffusion equations). However, many mathematical questions remain unanswered and are the subject of an intense research activity.

Nevertheless, in this talk, we shall take another direction, and discuss how to extend these techniques in order to derive new models. Indeed, new technologies using advanced fluid mechanics, solid-state or plasmas technologies require the design of new physical models, the standard ones lacking of physical accuracy. The main reason for this discrepancy is a question of scale. The continuum models are derived on the assumption that the scale of the system is much larger than the scale of microscopic interactions. Many high technology devices have a smaller size, which is intermediate between what can be considered as a macroscopic scale, and the microscopic scale of elementary particle interactions. This is the problem of mesoscale modeling.

We shall present an approach to derive mesoscale models. In most physical cases, the collision operator (the mathematical object which describes the particle interactions) exhibits a multiscale behaviour: it forces the evolution of certain quantities to be faster than other ones. A typical example is the interaction between electrons and ions in a plasma. Because of the mass ratio (ions are typically 10^3 to 10^5 times heavier than electrons), electron momentum is modified much faster than electron energy. At large scales, one can consider that both momentum and energy will have relaxed towards those of the ions. However, on shorter time or length scales, it is very likely that mean electron momentum will be close to that of the ions, but not the mean energy.

When full relaxation towards thermodynamical equilibrium has occured (under the influence of collisions), the system is amenable to a description by a reduced set of variables, namely the macroscopic ones (like density, or temperature). This leads to the conventional continuum models. However, when incomplete relaxation has occured, which often happens at the mesoscale, then modeling of the system requires to keep track of some of the microscopic variables, but not necessarily all of them. For instance, in the above example, it is enough to keep track of the energy distribution function of the electrons, but not of their angular velocity distribution since this will be isotropic as a result of collisions against the ions. In this way, intermediate descriptions between the microscopic kinetic model and the macroscopic continuum models are obtained.

The aim of this talk will be to develop a mathematical apparatus which describes how such intermediate models can be obtained. More specifically, we shall focus on the derivation of the *SHE* and *Energy-Transport* models (SHE is an acronym coming from the physics literature for 'Spherical Harmonics Expansion, but no such expansion is needed in current derivations of this model). Both are systems of diffusion equations which can describe various physical situations. Various examples and numerical applications will be borrowed from plasma device technology. As far as possible, the status of the mathematical theory of these models will be precised.

This talk will present an overview of work done in collaboration with various authors: N. Ben Abdallah, J. P. Boeuf, F. Deluzet, L. Garrigues, S. Génieys, A. Jüngel, V. Latocha, D. Levermore, B. Lucquin-Desreux, S. Mancini, A. Mellet, F. Poupaud, C. Schmeiser, R. Talaalout, M. H. Vignal.