Spectral Methods for Time dependent Problems

Tuesday & Thursday 2–3:30

IPAM Building Room #1180

Instructor: Professor Eitan Tadmor
e-mail: tadmor@math.ucla.edu, Office: MS7945

During the last three decades, spectral methods have emerged as successful, and often superior, alternatives to the better known finite difference and finite element methods. Applications include several key areas of computational fluid dynamics, modeling different type of wave motion, weather forecasting and more. In these lectures we will study how and why spectral methods work for the approximate solutions of time dependent problems encountered in such applications, what are their advantages and when and why they fail.

We begin with a self contained overview of the main ingredients of spectral approximations – Fourier and Chebyshev interpolants, their spectral accuracy, the role of aliasing, differentiation matrices ... For time dependent problems, we make reference to four prototype model problems: the energy balance for the linear wave and heat equations and the phenomena of dissipation and dispersion for nonlinear KDV and Navier-Stokes equations are studied and modeled by the spectral methods. In particular, the questions of stability and convergence of spectral methods, and the interplay between aliasing, smoothing and high resolution are addressed.

We also discuss specific issues related to non-periodic problems. in this context we study the intricate issue of the CFL stability restriction on the permitted time-step for the stable Chebyshev method. Finally, we will discuss spectral approximation of piecewise smooth data with various applications from shock formation to image processing.

References:
B. Fornberg, A Practical Guide to Pseudospectral Methods
Lloyd N. Trefethen, Spectral Methods in MATLAB

1Or other mutually agreeable time agreed at our first meeting