Stalking: Aggressive Shadowing of a Noisy Trajectory

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δ_x-Shadowing Definition

Given a system $y_{n+1} = f(y_n)$, a tolerance $\delta_x$, and a sequence $\{p_n\}_{n=a}^b$, a true orbit $\{x_n\}_{n=a}^b$, $x_{n+1} = f(x_n)$, $\delta_x$-shadows $\{p_n\}_{n=a}^b$ on $a \leq n \leq b$ if $|x_n - p_n| < \delta_x$
Noise may push the target solution outside of the ensemble ellipse. This is fine, so long as *some* ensemble member is still consistent with the target (red and blue overlap).
However, if a contracting direction begins to expand, or there is a near tangency of the stable and unstable manifolds, we may have a shadowing failure or *glitch*. 
However, if a contracting direction begins to expand, or there is a near tangency of the stable and unstable manifolds, we may have a shadowing failure or \textit{glitch}.
A few words about Breeding

\[ \text{svd}(B) = U\Sigma V' \]
Ensemble variance inflation

\[ \sigma_1 u_1 \]

\[ \sigma_2 u_2 \]

\[ (\sigma_2 + \epsilon) u_2 \]

\[ \delta u_1 \]

\[ \epsilon = \text{inflation} \]

\[ \delta_x \leq \delta \leq \sigma_1 \]
$\delta_x$-Stalking Definition

Given a system $y_{n+1} = f(y_n)$, a tolerance $\delta_x$, and a sequence $\{p_n\}_{n=a}^b$, an $\varepsilon$-pseudo orbit $\{x_n\}_{n=a}^b$, $|x_{n+1} - f(x_n)| < \varepsilon$, $\delta_x$-stalks $\{p_n\}_{n=a}^b$ on $a \leq n \leq b$ if $|x_n - p_n| < \delta_x$.
\[ y_{n+1} = f(y_n), \] a tolerance \( \delta_x \), and a sequence \( \{ p_n \}_{n=a}^b \), an \( \epsilon \)-pseudo orbit \( \{ x_n \}_{n=a}^b \), \[ |x_{n+1} - f(x_n)| < \epsilon, \] \( \delta_x \)-stalks \( \{ p_n \}_{n=a}^b \) on \( a \leq n \leq b \) if \( |x_n - p_n| < \delta_x \)
Iteration of the Henon map leaves the truth outside ellipse
Step 1: inflate contracting directions by $\varepsilon$
Step 2: rescale expanding directions
Goal

● Fixing the following parameters:
  ● variance inflation ($\epsilon$)
  ● rescale of expanding axes ($\delta_x \leq \delta \leq \sigma_i$)
  ● stalking distance ($\delta_x$)
  ● noise ($\delta_f$), where $|p_{n+1} - f(p_n)| < \delta_f$
● How long will $x_n$ stalk $p_n$?
Stalking time in Lorenz 40-D system for parameters $\varepsilon$ (inflation), $\delta_f$ (noise), $\delta$ (rescale). Failure occurs when target leaves ellipse.
Conclusion

- Agreement across $O(\delta_f)$ (noise) and $O(\varepsilon)$ (ensemble variance inflation) indicates a substantial relationship between stalking time and parameters.
- Stalking solutions to the Lorenz 40-D model require inflation $>>$ noise, what if we increase the stalking distance?