An Engquist-Osher-type scheme for conservation laws with discontinuous flux adapted to flux connections

Raimund Bürger∗
Departamento de Ingeniería Matemática, Universidad de Concepción, Concepción, Chile
e-mail rburger@ing-mat.udec.cl
Kenneth H. Karlsen
Centre of Mathematics for Applications, University of Oslo, Oslo, Norway
e-mail kennethk@math.uio.no
John D. Towers
MiraCosta College, Cardiff-by-the-Sea, CA, USA
e-mail john.towers@cox.net

We consider scalar conservation laws
\[ u_t + F(x, u)_x = 0, \quad x \in \mathbb{R}, \quad t > 0; \quad u(x, 0) = u_0(x), \quad x \in \mathbb{R}, \]
with the spatially varying flux
\[ F(x, u) = H(x)f(u) + (1 - H(x))g(u), \]
where \( H(x) \) is the Heaviside function and \( f \) and \( g \) are smooth nonlinear functions. Adimurthi, Mishra, and Veerappa Gowda [1] pointed out that such a conservation law admits many \( L^1 \) contraction semigroups, one for each so-called connection \((A, B)\). Here we define entropy solutions of type \((A, B)\) involving Kružkov-type entropy inequalities that can be adapted to any fixed connection \((A, B)\). It is proved that these entropy inequalities imply the \( L^1 \) contraction property for \( L^\infty \) solutions, in contrast to the “piecewise smooth” setting of Adimurthi et al. For a fixed connection, these entropy inequalities include a single adapted entropy of the type used by Audusse and Perthame [2]. We prove convergence of a new difference scheme that approximates entropy solutions of type \((A, B)\) for any connection \((A, B)\) if a few parameters are varied. The scheme relies on a modification of the standard Engquist-Osher flux, is simple as no \( 2 \times 2 \) Riemann solver is involved, and is designed such that the steady-state solution connecting \( A \) to \( B \) is preserved. In contrast to most analyses of similar problems, our convergence proof is not based on the singular mapping or compensated compactness methods, but on standard \( BV \) estimates away from the flux discontinuity. Some numerical examples are presented.

This contribution is based on the paper [3].

References
