On the Approximation of Conservation Laws by Vanishing Viscosity

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Consider the scalar conservation law
\[ u_t + f(u)_x = 0, \quad x \in \mathbb{R}, \quad t \in [0, T], \quad (1) \]
with fixed, bounded initial data of the form
\[ u(0, x) = u_0(x), \quad (2) \]
and strictly convex flux \( f \) of class \( C^2 \). As shown by the analysis of Goodman and Xin, [GX], in the case the solution \( u \) contains finitely many non-interacting entropic shocks, its viscous approximations \( u^\varepsilon \), defined as the solutions of the family of Cauchy problems
\[ u^\varepsilon_t + f(u^\varepsilon)_x = \varepsilon u^\varepsilon_{xx}, \quad u^\varepsilon(x, 0) = u(x, 0), \quad (3) \]
admit a singular perturbation expansion. In particular, they admit expansions in terms of powers of \( \varepsilon \) both in the region where \( u \) is smooth and near the discontinuities. Our work, [BD2], provides a positive answer to the question whether a similar inner and outer expansion can still be performed for \( t > \tau \), in the case the solution \( u \) to the conservation law (1) contains arbitrarily many shock interactions, until at a certain time \( \tau \) an isolated shock emerges. An important point in our proof is the estimate on the time needed for the shock to appear, [BD2]. A brief discussion on this estimate, in the more general setting where the flux \( f \) is allowed to be non-convex, will be presented as well.

References

