Applications of dispersive estimates to the acoustic pressure waves for incompressible fluid problems

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We show how in the case of the whole domain, the use of dispersive estimates, allows in fluidinamic to overcome the problem connected with the formation of acoustic waves. It is well known in the theory of the incompressible limit or the low Mach number limit that the presence of acoustic waves, which propagates with high speed in the space domain may causes the lost of strong convergence.

We show that similar problems arise also when we try to approximate the incompressible Navier Stokes equations by means of the artificial compressibility method, where we have to deal with acoustic waves for the pressure. In order to overcome such a difficulty we use the dispersive properties of these waves. We exploit the wave equation structure of the pressure which allows us to use $L^p - L^q$ Strichartz type estimate, leading to the strong convergence of the approximating sequences, see [DM1], [D1].

Similar phenomena appear also in modeling the Debye screening effect for semiconductor devices. Those kind of systems are obtained by coupling the compressible Navier Stokes equation with an external force given by the electrostatic potential provided by the Poisson equation for the electric field. While studying the quasineutral limit for this system, the incompressible limit regime yields to introduce a suitable time scaling, which introduces a singularity by the coupling term with the electric field and it leads to formation of acoustic waves. In order to handle these difficulties the previous model will be regarded as a semilinear wave equation and hence the dispersive analysis will lead to uniform estimates and then to the strong convergence, see [DM2].

Finally, we show same work in progress concerning the case of exterior domains (to a compact convex obstacle). In that case the analysis is harder due to the need of physical consistent boundary conditions.

References

[DM1] Donatelli D., Marcati P.,*A dispersive approach to the artificial compressibility approximations of the Navier Stokes equations in 3-D.*, Journal of Hyperbolic Differential Equations, 3, (2006), no. 3, 575-588.

[DM2] Donatelli D., Marcati P.,*A quasineutral type limit for the Navier Stokes Poisson system with large data*, Nonlinearity, 21, (2008), 135-148.

[D1] Donatelli D.,*On the artificial compressibility method for the Navier Stokes Fourier system*, Preprint 2008 (submitted).