On Hyperbolic Equations Arising in the Theory of Longitudinal Vibration of Thick Bars

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Longitudinal vibration of bars are normally considered in mathematical physics in terms of classical model described by the wave equation under assumptions that the bar is thin and relatively long structure. More general theories were formulated taking in consideration the effect of lateral motion of relatively thick bar. Mathematical formulation of these models includes fourth order derivatives in the equation of their motion. Rayleigh did the simplest generalization of the classical model in 1894, by including the effects of lateral motion and neglecting the shear stress. Bishop obtained the next generalization of the theory in 1952. The Rayleigh-Bishop model is described by a forth order partial differential equation not containing the forth time derivative. He taken into accounts the effects of shear stress. Both Rayleighs and Bishops theories consider lateral displacement being proportional to the longitudinal strain [ref1]. The Bishops model was generalized by Mindlin and Herman. They considered the lateral displacement proportional to an independent function of time and longitudinal coordinate. This result is formulated as a system of two differential equations of second order, which could be replaced by a single equation of forth order resolved with respect to the highest order time derivative. We consider all above mentioned equations in frame of general theory of hyperbolic equations [ref2],[ref3]. The Greens functions for all these models can be constructed [ref4].

Generalizations of the linear Rayleigh and Bishop theories to the nonlinear cases are discussed and corresponding nonlinear partial differential equations and boundary conditions are derived from the Hamilton variational principle.

References

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