In this talk, we present a scheme for the one dimensional two-layer shallow water equations aimed at high order of accuracy and explicit well-balancing for moving equilibria.

The mathematical model we consider neglects effects like friction between and mixture of the two layers, cf. [2]. It may be written in the general form:

$$\frac{\partial u}{\partial t} + \frac{\partial F}{\partial x}(u) = B(u) \frac{\partial u}{\partial x} + S(u) \frac{d\sigma}{dx}. \quad (1)$$

While the left hand side and the right hand term $S(u) \frac{d\sigma}{dx}$ resemble a conservation law with source, the so called coupling term $B(u) \frac{\partial u}{\partial x}$ depends on the unknown $u$ and derivatives thereof. Among other intricacies, this term affects the eigenvalues of the overall system.

In the development of our solver [3] we followed a framework developed by Manuel Castro, Carlos Parés and others, cf., e.g., [2]. By using a semi-discrete approach with WENO-reconstruction in space and Runge-Kutta methods in time, high order can be achieved. We succeeded in combining this work with techniques developed recently by Noelle, Xing and Shu [4]. In this paper, so called equilibrium variables are introduced which are constant in case of a steady state. These can be used to limit the high order point values found by a WENO reconstruction. With this combination of techniques both high order of accuracy and well-balancing are achieved.

We transferred these results to the two-layer case. We used an idea of Bale et al. [1] to write the scheme in terms of flux differences, simplifying the consideration of the correct characteristic speeds while working with the limited reconstructed point values. The result is a scheme with improved accuracy and an explicit well-balancing even for moving equilibria.

References

