A possible definition of strong/symmetric hyperbolicity for a second-order system of evolution equations is that it admits a reduction to first order which is strongly/symmetric hyperbolic. We investigate the general system that admits a reduction to first order and give necessary and sufficient criteria for strong/symmetric hyperbolicity of the reduction in terms of the principal part of the original second-order system. An alternative definition of strong hyperbolicity is based on the existence of a complete set of characteristic variables, and an alternative definition of symmetric hyperbolicity is based on the existence of a conserved (up to lower order terms) energy. Both these definitions are made without any explicit reduction. Finally, strong hyperbolicity can be defined through a pseudo-differential reduction to first order. We prove that both definitions of symmetric hyperbolicity are equivalent and that all three definitions of strong hyperbolicity are equivalent (in three space dimensions). We show how to impose maximally dissipative boundary conditions on any symmetric hyperbolic second order system. We prove that if the second-order system is strongly hyperbolic, any closed constraint evolution system associated with it is also strongly hyperbolic, and that the characteristic variables of the constraint system are derivatives of a subset of the characteristic variables of the main system, with the same speeds.