Hybrid Scheme for the Baer-Nunziato Two-Phase Flow Model

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The Baer-Nunziato two-phase flow model describes flame propagation in gas-permeable reactive granular materials. In this work, we neglect terms due to combustion processes, drag and heat transfer between the phases and we consider the flow of compressible gas in a porous particle bed,

\[
\begin{align*}
\rho_g \phi_g \frac{\partial g}{\partial t} + \rho_g \phi_g u_g \frac{\partial g}{\partial x} &= 0 \\
\rho_g \phi_g \frac{\partial g}{\partial t} + \rho_s \phi_s \frac{\partial g^2}{\partial x} + p_g \phi_g &= p_g \phi_g \\
E_g \phi_g \frac{\partial g}{\partial t} + (u_g (\phi_g E_g + \phi_g p_g)) \frac{\partial g}{\partial x} &= p_g u_g (\phi_g) \\
\rho_s \phi_s \frac{\partial s}{\partial t} + \rho_s \phi_s u_s \frac{\partial s}{\partial x} &= 0 \\
\rho_s \phi_s u_s \frac{\partial s}{\partial t} + \rho_s \phi_s u_s^2 + p_s \phi_s &= p_s \phi_s \\
E_s \phi_s \frac{\partial s}{\partial t} + (u_s (\phi_s E_s + \phi_s p_s)) \frac{\partial s}{\partial x} &= p_s u_s (\phi_s) \\
\phi_s \frac{\partial s}{\partial t} + u_s (\phi_s) \frac{\partial s}{\partial x} &= 0
\end{align*}
\]

Here \( \rho, u, p \) and \( E \) denote the density, velocity, pressure and energy of the gas and solid phases respectively. Both phases are assumed ideal and obey the Equation of State

\[
E = \frac{1}{2} \rho u^2 + \frac{p}{\gamma - 1}
\]

and \( \phi \) is the porosity, satisfying

\[
\phi_g + \phi_s = 1.
\]

This is an averaged flow model, expressing conservation of mass, and momentum and energy balance of the gas and solid phases. The last equation is a compaction equation describing the evolution of the porosity. The system is only conditionally hyperbolic, and may fail to have a complete set of eigenvectors if

\[
(u_s - u_g)^2 = c_s^2
\]

It is also in non-conservation form due to momentum and energy exchange between the phases. The presence of the non-conservative terms has major consequences both theoretically and computationally.
We are concerned with flows in which the porosity changes discontinuously, but is otherwise constant in different flow regions. In this case, the phases 'talk' to each other only across the porosity jump, and otherwise obey the single fluid Euler system respectively. The jump conditions across the porosity jump are obtained using the Riemann Invariants.

\[
\begin{align*}
  u_s, \quad \eta_g, \quad \eta_s, \quad \phi_g \rho_g v_g, \quad \phi_g p_g + \phi_s p_s + \phi_g \rho_g v_g^2, \quad \frac{1}{2} v_g^2 + \frac{c_g^2}{\gamma - 1}
\end{align*}
\]

where \( \eta \) denotes the entropy, \( v_g = u_g - u_s \) denotes the speed of the gas relative to the speed of the compaction wave, and \( c = \sqrt{\gamma p/\rho} \) the speed of sound.

In the special case where the particle bed does not move, \( u_s = 0 \), and the solid phase is assumed incompressible, the system reduces to the Euler equations with area variation, where the porosity may be identified with the cross sectional area. In this case, the Riemann Invariants across the porosity jump are the mass flux \( \phi p u \), the entropy, \( \eta = p/\rho^\gamma \) and the specific enthalpy \( h = \frac{1}{2} u^2 + c^2/(\gamma - 1) \).

In either case, exact solutions to the Riemann problem may be obtained, and for certain wave configuration they are not unique [?, ?, ?]. In [?], a study has shown that even state-of-the-art numerical methods may fail miserably to compute solutions to the Riemann problem. The methods were based on the conservative variables and the failure of the scheme was attributed to inability of conservative formulation to keep the entropy constant across the porosity jump. We note that the evolution equation for the entropy is given by

\[
\eta_t + u \eta_x = 0
\]

and trivially recognizes and respects constant entropy across the interface. With this observation at hand, we have formulated the Baer-Nunziato system in terms of the Riemann Invariants, and propose a hybrid algorithm that uses the Riemann Invariants formulation across the compaction wave, and the conservative formulation away from the compaction wave. Within this framework, one's favorite discretization scheme may be used. A Roe-type wave based upwind scheme enables a seamless switch between the formulations.

The talk will describe the hybrid scheme and present numerical results.

References


