Linearization Techniques and L^1 Continuous Dependence for Nonlinear Hyperbolic Systems

Philippe G. LeFloch
University of Paris 6 and CNRS
LeFloch@ann.jussieu.fr

We discuss several questions of existence and uniqueness of discontinuous solutions to adjoint problems associated with nonlinear hyperbolic systems of conservation laws. By generalizing the Holmgren-Haar arguments and relying on Glimm-type or front-traching approximations, we then establish that entropy solutions to systems of conservation laws depend continuously upon their initial data in the natural L^1 norm.

The main difficulty is coping with linear hyperbolic systems with discontinuous coefficients, possibly in a non-conservative form. Our analysis begins with the following key observation for systems with general flux that need not be genuinely nonlinear. While entropy solutions, by definition, contain compressive shocks only, the averaged matrix $\overline{A}(u, v) = \int_0^1 Df(u + \theta(v - u))\, d\theta$ associated with two entropy solutions u, v has compressive or undercompressive shocks, but *no rarefaction-shocks.*

On the other hand, rarefaction shocks are recognized as a possible source for non-uniqueness and instability. The proposed method rests on geometric properties of the averaged matrix and also takes into account wave cancellation effects along generalized characteristics. In the special case of genuinely nonlinear systems, this strategy was first carried out in [5, 6].

References

[1] P.G. LeFloch, *Entropy weak solutions to nonlinear hyperbolic systems in nonconservative form,* Comm. Part. Diff. Equa. 13 (1988), 669–727.

[2] P.G. LeFloch, *An existence and uniqueness result for two nonstrictly hyperbolic systems,* IMA Volumes in Math. and its Appl.,“Nonlinear evolution equations that change type”, ed. B.L. Keyfitz and M. Shearer, Springer Verlag, Vol. 27, 1990, pp. 126–138.

[3] P.G. LeFloch and Z.-P. Xin, *Uniqueness via the adjoint problems for systems of conservation laws,* Comm. Pure Appl. Math. 46 (1993), 1499–1533.

[4] P.G. LeFloch and T.-P. Liu, *Existence theory for nonlinear hyperbolic systems in nonconservative form,* Forum Math. 5 (1993), 261–280.

[5] J. Hu and P.G. LeFloch, *L^1 continuous dependence property for systems of conservation laws,* Arch. Rational Mech. Anal. 151 (2000), 45–93.

[6] P.G. LeFloch, *Hyperbolic Systems of Conservation Laws,* Lect. in Math., ETH Zürich, Birkhäuser, 2002.

[7] P.G. LeFloch, *Haar method, averaged matrix, wave cancellation, and L^1 stability for hyperbolic systems,* Jou. Hyper. Diff. Equa. 3 (2006), 701–739.