Analysis and Computation for Bose-Einstein Condensate Ground States

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In this talk, we study asymptotically and numerically the nonlinear Schrodinger equation arising from the Bose-Einstein condensation (BEC). We begin with the time-independent Gross-Pitaevskii equation (GPE) and reformulate it into a singularly perturbed nonlinear eigenvalue problem under constraint. Asymptotic approximations for the eigenfunctions and eigenvalues as well as the corresponding energies are presented in strongly interacting regimes. Boundary and/or interior layers and their widths are presented. An efficient and accurate numerical scheme, the backward Euler sine-pseudospectral (BESP) method, based on the imaginary time method and the normalized gradient flow, is proposed to compute the ground state and/or other higher energy excited states of the condensate.

For spinor condensate with internal degree of freedom, difficulties arise if the normalized gradient flow method is applied directly due to insufficient normalization conditions. For spin-1 condensate described by a three-component coupled GPEs, we introduce a normalization condition, in addition to the two existing conditions: the conservation of total particle number and the conservation of magnetization. The third normalization condition is derived from the relationship between the chemical potentials of each spinor component together with a time splitting scheme applied to the continuous normalized gradient flow. The BESP scheme is then extended to compute the ground state of spin-1 condensate.

References

