On the convergence rate of Glimm scheme
for non genuinely nonlinear hyperbolic systems

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Consider the Cauchy problem for an $N$-dimensional, strictly hyperbolic, quasilinear system

$$u_t + A(u)u_x = 0, \quad u(0, x) = \bar{u}(x),$$

(1)

where $u \mapsto A(u)$ is a smooth matrix-valued map, and the initial data $\bar{u}$ is assumed to have small total variation. We investigate the rate of convergence of approximate solutions of (1) constructed by the Glimm scheme, under the assumption that, for each $k$-th characteristic family, the linearly degenerate manifold

$$M_k = \{ u \in \Omega : D\lambda_k(u) \cdot r_k(u) = 0 \}$$

($\lambda_k(u), r_k(u)$ being the $k$-th eigenvalue and a corresponding right eigenvector of $A(u)$) is either the whole space or a connected $N-1$-dimensional manifold, transversal to the characteristic vector field $r_k$. Relying on an adapted wave tracing method and on a suitable quadratic interaction potential, we obtain the same type of error estimate valid for hyperbolic systems satisfying the classical assumptions of genuine nonlinearity or linear degeneracy of the characteristic families (see [2]), i.e., a rate of convergence $\approx o(1) \sqrt{\Delta x} \log \Delta x$, thus improving the rate $o(1) \sqrt{\Delta x} \log \Delta x$ obtained in [3], under the assumption that $M_k$ consists of a finite number of connected $N-1$-dimensional manifolds.

References

