Computing the Effective Hamiltonian for a Time-Dependent Hamiltonian

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Over the last years, homogenization has become more and more important. An important ingredient in the homogenization of Hamilton-Jacobi equations is the effective Hamiltonian, i.e., the unique constant $\bar{H}(p)$ such that the equation

$$\partial_t u + H(x, t, p + \nabla u) = \bar{H}(p)$$

has a viscosity solution $u \in C(T^n \times T)$. Here, $H \in C^2(T^n \times T \times \mathbb{R}^n)$ denotes a strictly convex Hamiltonian and $T$ denotes the flat torus.

Over the last decade, several numerical methods have been developed to approximate $\bar{H}$ for convex Hamiltonians $H$ that do not depend on time. Some of these methods can be ported to the time-dependent case by setting

$$\tilde{H}(x, p) = p_0 + H((x_1, \ldots, x_n), x_0, (p_1, \ldots, p_n)).$$

Unfortunately, this results in an $(n + 1)$-dimensional problem.

We propose a method to compute the effective Hamiltonian from the large time behaviour of an $n$-dimensional initial value problem Hamilton-Jacobi equation:

$$\partial_t w + \lambda w + H(x, t, p + \nabla w) = 0.$$ 

Under suitable assumptions, convergence of this method can be proved and we provide numerical evidence for this convergence.