A Relaxation Method for Modeling Two-Phase Shallow Granular Flows

Marica Pelanti
École Normale Supérieure, Paris, France
Marica.Pelanti@ens.fr

François Bouchut
CNRS and École Normale Supérieure, Paris, France
Francois.Bouchut@ens.fr

We consider a depth-averaged two-phase model for gravity-driven flows made of solid grains and fluid. The model system consists of mass and momentum balance equations for the solid and fluid components, coupled together by both conservative and non-conservative terms involving the derivatives of the unknowns. The system can be shown to be hyperbolic at least for phase velocity differences sufficiently small compared to the characteristic speeds of flow in kinematic equilibrium.

Assuming hyperbolicity holds, we numerically solve the model equations by a Godunov-type finite volume scheme built via a relaxation approach.

In the spirit of a relaxation technique recently proposed by Berthon and Marche [1], a relaxation model is formulated by introducing a set of new variables that replace the unknowns in the spatial gradients of the original system. The auxiliary variables are governed by linear equations with coefficients that determine the eigenvalues of the relaxation model, and whose definition is subject to stability constraints. All the characteristic fields of the relaxation system are linearly degenerate, and this property makes exact Riemann solutions of the relaxation model easily obtained. The proposed relaxation strategy results in the definition of a particular approximate Riemann solver for the original model equations, which has the advantage of a certain degree of freedom in the specification of the wave speeds through the choice of the relaxation parameters. This flexibility may be exploited for deriving a solver able to preserve positivity of flow depth and phase volume fractions at the discrete level, which is the main interest and motivation of our studies on this relaxation approach.

Several numerical experiments are presented to show the efficiency and the stability property of the proposed relaxation method.

References