Inverse problem for scalar conservation laws with discontinuous coefficients

Holden H.
N.T.N.U., Norway
holden@math.ntnu.no

Priuli F. S.*
N.T.N.U., Norway
priuli@math.ntnu.no

Risebro N. H.
University of Oslo, Norway
nilshr@math.uio.no

Given a scalar conservation law with discontinuous coefficients of the form

\[ u_t + f(\gamma(x), u)_x = 0, \]  

(1)

for a smooth \( f \) and a suitable discontinuous function \( \gamma \), we are interested in reconstructing properties of the system from observations of the solution \( u(t, x) \) to the Cauchy problem for (1).

In its full generality, the problem cannot be dealt with. First of all, already in the case of a flux function \( f = f(u) \), which only depends on the conserved quantity \( u \), discontinuities in the solution correspond to a loss of information that normally makes the inverse problem ill-posed in uniqueness (see [1] and [2]). Moreover, the Cauchy problem for (1) is well posed only at price of additional assumptions on the flux function (see [3] and its rich bibliography)

Therefore, we will focus our attention on two special cases, arising in traffic flow and two phases flow in porous media, which are known to have well-posed Cauchy problems. For these models, we want to deduce additional informations on the structure of the flux by only observing either the solution at different times, or different solutions corresponding to different initial data.

Namely, our goal is to obtain a piecewise constant approximation of \( \gamma \). Indeed, finding a similar reconstruction procedure would help to address several engineering problems: one example is to identify flow conditions inside a tunnel by observing the flow at both its ends; another one is to deduce properties of the rock in water flooding of a core sample of a porous medium. In both examples, the approximation of \( \gamma \) is possible and the most delicate point is to find a good compromise between the hypotheses which are physically reasonable and the ones needed to obtain a well posed mathematical description.

References
