Entropy-fix upwind schemes for the ideal magnetohydrodynamic equations

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The ideal magnetohydrodynamic (MHD) equations form a non-strictly hyperbolic system of conservation laws. This system of equations has been widely studied from different analytical and numerical methodologies ([1, 2, 3, 6, 9, 13]). The standard approach is based on the study of the wave structure of the equations ([5, 3]) and its numerical treatment ([3, 6, 9, 13]).

The complexity of the ideal MHD wave dynamics compared to the Euler equations of gas dynamics lies on the presence of the magnetic field that generates new magnetic and magnetoacoustic waves. The rotation of the magnetic field induces the non-strictly hyperbolicity of the MHD system and non-genuinely nonlinearity (non-convexity) of some of the local wave fields ([7, 3, 12]).

Numerical schemes for the approximation of the solution of ideal MHD equations demand dissipation mechanisms in order to reproduce the complex wave structure appearing in the dynamics. Most numerical schemes for ideal MHD equations are designed to prescribe global dissipation ([1, 2, 6]) ensuring stability and computational efficiency. On the other hand the excessive dissipation implies loss of accuracy and smearing of fine structure.

Upwind schemes are known for their low local dissipation behavior [4]. These schemes are defined according to the sign of the local wave velocities and propagate waves with correct speed. The main drawback of upwind schemes is the lack of dissipation around points where new wave structure is generated (sonic points in convex dynamics) and an entropy correction is required to determine the appropriate dissipation around them to ensure correct formation of new waves (transonic rarefaction waves in convex dynamics), ([10]). Non-convex dynamics shares sonic points with convex dynamics and adds the so-called non-convexity points as points where new structure is formed. Thus, an additional entropy correction in the neighborhood of points where nonlinear characteristic fields are non-convex is necessary to ensure correct formation of magnetoacoustic compound waves.

In this research work we present an analytical study of the wave structure of the ideal MHD equations based on the local decomposition in characteristic wavefields. We establish an explicit criteria to detect non-convexity points and propose a consistent entropy correction for MHD upwind schemes. We formulate a general purpose shock capturing scheme based on the local characteristic field decomposition that allows to prescribe analytic viscosities local in space and different in each wavefield. High order accuracy is obtained by means of a third order compact hyperbolic reconstruction procedure ([11]) for the approximation in space and the third order Shu-Osher Runge-Kutta method [10] for the integration in time. We test our model through a set of one- and two-dimensional MHD numerical examples including Brio-Wu MHD Riemann problems ([3]) and the Orszag-Tang vortex system ([8, 13]).

Numerical results indicate that the proposed high order accurate entropy-fix upwind scheme behaves accurate, low dissipative and stable under a high CFL number.
References


