We study the classical problem of supersonic uniform inviscid gas flow onto a planar infinite wedge. As is well known [1, 2], theoretically this problem has two possible solutions, a strong shock corresponding to a subsonic gas flow behind the shock (i.e., $u_0^2 + v_0^2 < c_0^2$ where $u_0$ and $v_0$ are components of the velocity field, and $c_0$ is the sound speed in the gas at the rest) and a weak shock corresponding to a supersonic flow behind the shock (i.e., generally speaking, $u_0^2 + v_0^2 > c_0^2$).

However, as follows from numerous physical and numerical experiments, the case of a weak shock is realized in practice. Till recently, in spite of a large number of qualitative studies there was no correct mathematical justification why it is so. Therefore, it was impossible to predict a result in any concrete experiment. Actually, the powerful tool to resolve the above problem is the hypothesis by R. Courant and K.D. Friedrichs [1, 2] stating that the strong shock is unstable whereas the weak one is stable against small perturbations of the steady gas flow. That is, we should study the asymptotic behavior of the solution to the corresponding linearized problem as $t \to \infty$.

The first part of the hypothesis was confirmed in our works [3, 4, 5]. The main result is that the perturbations at the wedge’s vertex increase in time even for the case of compactly supported initial data unless some rigid conditions are required for them.

Now we study the case of a weak shock. We obtain the following results.

1) We find an exact solution with a weak shock of the linearized problem for the case of compactly supported initial data.

2) We find an asymptotic expansion of the exact solution in a neighborhood of the wedge’s vertex.

3) We prove the asymptotic Lyapunov’s stability of the weak shock and show that the solution of the linearized problem behaves as a certain power of $1/t$ as $t \to \infty$.

In other words, we strictly confirm the second part of Courant-Friedrich’s hypothesis, i.e., we prove that the weak shock is asymptotically stable (by Lyapunov) against small perturbations.

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References


