Unstaggered Central schemes
with Constrained Transport Treatment
for Shallow Water Magnetohydrodynamics

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In this work we propose one and two-dimensional unstaggered central finite volume methods for solving hyperbolic systems of conservation laws. Based on Nessyahu and Tadmor’s (NT) one-dimensional non-oscillatory central scheme and its multidimensional extensions on Cartesian and/or unstructured grids, we construct a new class of unstaggered, second-order, non-oscillatory, central finite volume schemes. In contrast with the original (NT) central scheme that evolves the numerical solution on an original grid (at even time steps) and a dual staggered grid (at odd time steps), the proposed unstaggered central scheme (UCS) evolves the numerical solution on a single grid and uses a "ghost" staggered grid to avoid the time consuming resolution of the Riemann problems arising at the cell interfaces. Like the original NT scheme, the proposed numerical method is second-order accurate both in space and time and shares the same stability conditions. We apply the UCS method and solve some classical shallow water magnetohydrodynamic problems. To satisfy the solenoidal property of the magnetic flux in the numerical solution, we apply Evans and Hawley’s constrained transport method (CT). The CT method applies easily with the UCS method and deals directly with the computed numerical solution (it does not require any additional staggering for the magnetic flux components), and preserves the second-order accuracy of the base scheme. Even without the application of the CT procedure, the numerical base scheme does not break down, and may even in some cases deliver reasonable results. However, to ensure an admissible physical solution the CT treatment must be applied. The numerical results show the efficiency and the potential of the proposed method and compare very well with their corresponding ones appearing in the recent literature.

References


