Global Solutions to the Multi-Dimensional Compressible Magnetohydrodynamics

Dehua Wang
University of Pittsburgh, USA
dwang@math.pitt.edu

The equations for the three-dimensional viscous and compressible magnetohydrodynamic flows are considered. First an initial-boundary value problem is studied in a bounded domain with large data. In the isentropic case, the existence and large-time behavior of global weak solutions are established through a three-level approximation, energy estimates, and weak convergence for certain adiabatic exponents and constant viscosity coefficients. For the full heat conducting magnetohydrodynamic flows, a solution is constructed through an approximation scheme and a weak convergence method. The existence of a global variational weak solution with large data is established. The viscosity coefficients and heat conductivity can depend on the temperature. Then, the compactness of weak solutions to the magnetohydrodynamic equations for the viscous, compressible, heat conducting fluids is considered in both the whole three-dimensional space and the three-dimensional periodic domains. The viscosities, the heat conductivity as well as the magnetic coefficient are allowed to depend on the density, and may vanish on the vacuum. New ideas are developed to show the compactness of solutions of the full magnetohydrodynamic flows. A new entropy identity is derived. It is shown that the limit of a sequence of weak solutions is still a weak solution to the compressible magnetohydrodynamic equations. Finally, the relation between the compressible magnetohydrodynamic flows with low Mach number and the incompressible magnetohydrodynamic flows is investigated. The zero Mach limit is established for weak solutions, that is, it is proved that the weak solutions of the compressible isentropic magnetohydrodynamic equations converges to the weak solutions of the incompressible magnetohydrodynamic equations, as the Mach number approaches zero.