The $l^1$-error estimates for a Hamiltonian-preserving scheme to the Liouville equation with piecewise constant potentials

Xin Wen

Institute of Computational Mathematics, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, P.O.Box 2719, Beijing 100080, China.
wenxin@amss.ac.cn (Xin Wen)

Shi Jin
Department of Mathematics, University of Wisconsin, Madison, WI 53706, USA.
jin@math.wisc.edu (Shi Jin)

We present the $l^1$-error estimates for a Hamiltonian-preserving scheme, developed in [2], to the Liouville equation with a piecewise constant potential in one space dimension. This problem has important applications in computations of the semiclassical limit of the linear Schrödinger equation through barriers, and of the high frequency waves through interfaces. We use the $l^1$-error estimates established in [4, 3] for the immersed interface upwind scheme to the linear advection equations with piecewise constant coefficients. We prove that the scheme with the Dirichlet incoming boundary conditions is $l^1$-convergent for a class of bounded initial data, and derive the one-halfth order $l^1$-error bounds with explicit coefficients. The initial data conditions can be satisfied by applying the decomposition technique proposed in [1] for solving the Liouville equation with measure-valued initial data, which arises in the semiclassical limit of the linear Schrödinger equation.

References


