Consider the two-dimensional isentropic compressible Euler system

\[
\begin{aligned}
\rho_t + (\rho u)_x + (\rho v)_y &= 0, \\
(\rho u)_t + (\rho u^2 + p)_x + (\rho uv)_y &= 0, \\
(\rho v)_t + (\rho uv)_x + (\rho v^2 + p)_y &= 0,
\end{aligned}
\] (1)

where \(\rho\) is the density, \((u, v)\) is the velocity and \(p\) is the pressure given by \(p(\rho) = K\rho^\gamma\) where \(K > 0\) and \(\gamma > 1\) is the gas constant. Cauchy problems for (1) are open. Riemann problems for (1) are a current research topic, as they are reducible to involve fewer independent variables while retain important features of general solutions.

Riemann problems are Cauchy problems with special initial data that are constant along each ray from the origin. The two-dimensional case was formulated, and the solution configurations conjectured, in [6] in 1990 by T. Zhang and Y.Zheng. The solution configurations are complicated, as further evidence shown afterward by several numerical simulations [1, 2, 3, 4, 5]. The difficulty to a rigorous proof lies in the shortage of effective methods of analysis.

In this talk, as a first success since the proposition [6], we construct a class of analytic solutions to a case of a configuration of the 2-D four-wave Riemann problems of (1), (i.e., Configuration B as known in the literature), using methods that we have developed in recent years. The construction is based on the analysis of (1) in three planes: The self–similar variables \((\xi, \eta) = (x/t, y/t)\), the inclination angles of characteristics \((\alpha, \beta)\), and the velocity \((u, v)\)-plane of the hodograph transformation. These forms enable us to do analysis effectively.

The results are from our new paper [7].

References


