ERRATUM: EULERIAN DYNAMICS WITH A COMMUTATOR
FORCING

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The publication [1] has a minor gap in the argument presented in Section 6.2 where the authors establish control over the first derivatives of density and momentum. Specifically, the bound on \( \Lambda \rho \) used in the momentum equation involves term \( \sqrt{D \rho'}(x) \), which propagates into formula (6.21). At that point the authors combined (6.21) with (6.19) to get rid of the \( D \)-term. The mistakes present in the fact that the point \( x \) at which the \( D \)-term is evaluated in 6.19 is different from the point \( x \) at which it is evaluated in 6.21. Hence the values may be different.

To avoid using combination of 6.19 and 6.21 we argue as follows. We produce a uniform bound on \( |\rho''|_2 \) on the time interval in question. This uniform bound, by Sobolev embedding, implies that \( \rho' \in C^{1/2} \) uniformly. Then the trivial bound

\[ |\Lambda \rho|_{\infty} \leq |\rho'|_{C^{1/2}}. \]

implies uniform control over \( \Lambda \rho \). Hence it is not necessary to resort to 6.19 to contain \( \Lambda \rho \), and the rest of the estimates on \( m' \) follow as documented in [1].

To achieve uniform bound on \( |\rho''|_2 \) we differentiate the density equation twice:

\[
\partial_t \rho'' + u \rho''' + u' \rho'' + e'' \rho + 3e' \rho' + 2e \rho'' = -2 \rho'' \Lambda \rho - 3 \rho' \Lambda \rho' - \rho \Lambda \rho''.
\]

Using that \( u' = e + \Lambda \rho \), we obtain

\[
\partial_t \rho'' + u \rho''' + e'' \rho + 3e' \rho' + 3e \rho'' = -3 \rho'' \Lambda \rho - 3 \rho' \Lambda \rho' - \rho \Lambda \rho''.
\]

At this point we know that \( |e^{(k)}| \lesssim \rho^{(k)} \), and we have uniform bounds on \( \rho, \rho' \). So, testing with \( \rho'' \), integrating by parts in \( u \rho''' \rho'' \) term, and using the \( e \) quantity again, we obtain

\[
\partial_t |\rho''|_2^2 \lesssim |\rho''|_2 + |\rho''|_2 + |\Lambda \rho|_{\infty} |\rho''|_2^2 + |\rho''|_2 |\Lambda \rho'|_2 - \int_T \rho \rho'' \Lambda \rho'' \, dx.
\]

Using that \( |\Lambda \rho'|_2 \lesssim |\rho''|_2 \), and log-Sobolev inequality

\[ |\Lambda \rho|_{\infty} \leq |\rho'|_{\infty} (1 + \log_+ |\rho''|_2) \lesssim 1 + \log_+ |\rho''|_2, \]

we further obtain

\[
\partial_t |\rho''|_2^2 \lesssim C + |\rho''|_2^2 (1 + \log_+ |\rho''|_2) - \int_T \rho \rho'' \Lambda \rho'' \, dx.
\]

Using symmetrization in the remaining dissipation term we have

\[
(0.1) \quad - \int_T \rho \rho'' \Lambda \rho'' \, dx = - \int_T D \rho'' \, dx + R,
\]

where

\[
R = \int_T \rho''(x) \int_T \frac{(\rho(x) - \rho(y))(\rho''(x) - \rho''(y))}{|x - y|^2} \, dy \, dx.
\]

\[ Date: October 21, 2018. \]
Using the bound $|\rho'| < C$ we further conclude

$$|R| \lesssim \int_T |\rho''(x)| \int_T \frac{|\rho''(x) - \rho''(y)|}{|x - y|} dydx \leq \int_T |\rho''(x)| \sqrt{D\rho''} dx \leq |\rho''|_2 \sqrt{\int T D\rho'' dx}. $$

By Young, the latter is bounded by

$$|R| \leq \varepsilon \int T D\rho'' dx + C_\varepsilon |\rho''|_2^2,$$

where $\varepsilon$ is smaller than the lower bound on the density on the given time interval. This gets the $D$-term absorbed into dissipation term in (0.1). We thus arrive at

$$\partial_t |\rho''|_2^2 \lesssim C + |\rho''|_2^2 (1 + \log_+ |\rho''|_2).$$

The result follows by integration.

Since in the estimates above we relied on second order a priori bound $|e''| \lesssim |\rho''|$ it is necessary to raise the regularity class from $H^3$ as in [1] to $H^4$ so that the local transport equation for $e''$ can be solved classically. The idea to avoid using higher order a priori bounds $|e^{(k)}| \lesssim |\rho^{(k)}|$ is to abandon the use of momentum equation for $m$, where $e$ quantity is explicitly present, and instead come back to the $u$-equation. This was performed in [1] up to the order 3 space $H^4$, and the argument is entirely similar going one more derivative up to $H^4$. We therefore state our final result as follows.

**Theorem 0.1.** Consider the system of equations (1.1), [1], with $1 \leq \alpha < 2$ subject to initial data $(u_0, \rho_0) \in H^4(T^1) \times H^{3+\alpha}(T^1)$. Then the system admits a global solution in the same class.

**References**


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