Self-force driven inspiral of a scalar point particle into a Schwarzschild black hole: a progress report

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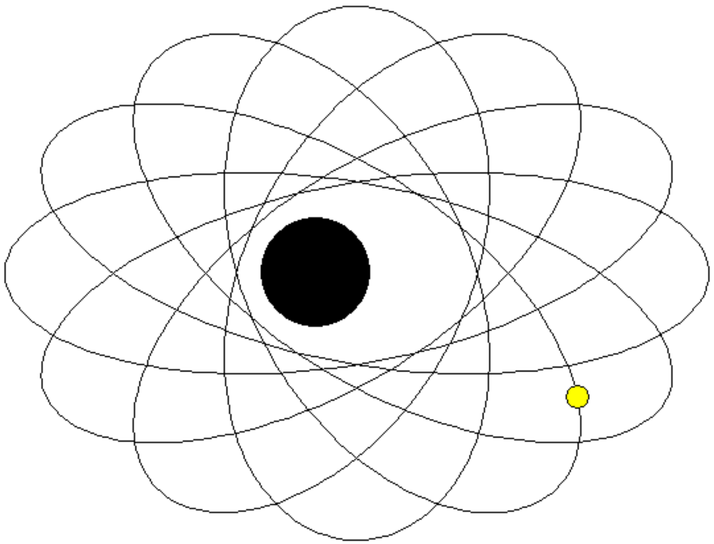
Extreme Mass Ratio Inspirals



- Solar-mass, compact object moving around a supermassive black hole
  - energy lost due to radiation leads to orbital decay
  - possible source for space based GW antenna
- 10^5 wave cycles / year
  - below noise level
  - accurate modeling to detect signal
  - encode geometry around central object
- employ small mass ratio approach
  - first order expansion in μ/M
  - no weak-field or slow motion assumption

Scalar self force in Schwarzschild

field sourced by particle

$$\Box \Phi = -4\pi\rho$$

$$m(\tau)\ddot{x}^{\alpha} = q(g^{\alpha\beta} + u^{\alpha}u^{\beta})\nabla_{\beta}\Phi$$

$$\dot{m} = -qu^{\alpha}\nabla_{\alpha}\Phi$$

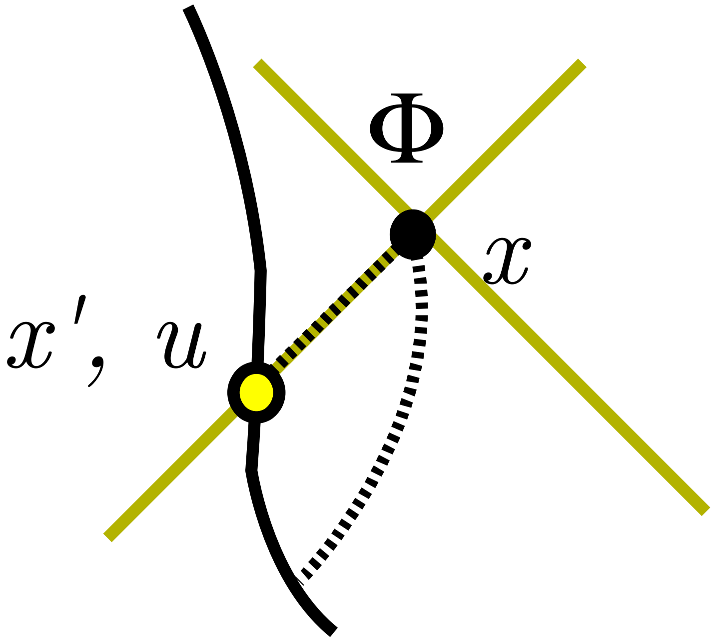
loss of rest mass to monopole radiation

particle moves under influence of field

- toy model for gravity
  - classical, massless field
  - “particle” is stellar sized object in orbit around black hole
- self-consistent evolution must evolve field and particle simultaneously

- Motivation
  - current mode-sum approach calculates the self-force on a geodesic
  - one inspiral calculation requires data from a large number of geodesics ⇒ is calculated as a postprocessing step
  - inefficient, since a large bank of self-force templates has to be computed in advance
- Idea
  - calculate the self-force self-consistently along an accelerated world line and use it to evolve forward in time
  - calculation of self-force and evolution of orbit occur at the same time

Detweiler-Whiting decomposition



- waves travel both
  - directly along the light cone
  - by scattering off curvature in the tail
- particle interacts with its own radiation

- direct piece is singular and must be removed
- tail piece is regular and solely responsible for the force

$$\nabla_\alpha \Phi^R = \nabla_\alpha \Phi - \nabla_\alpha \Phi^S$$

controls motion

compute numerically

known analytically

Singular field removal

- Mode sum regularization
  - regularizes after computing Φ
  - efficient in Schwarzschild, hard to extent to Kerr
  - first method to be used successfully [Barack, Mino, Nakano, Ori, Sasaki 2002]
- Effective source method
  - regularizes source term in wave equation
  - full 3D simulation for field, extension to Kerr is simpler
  - first method to compute self-consistent motion [Diener, Vega, Wardell, Detweiler 2011]
- m-mode regularization
  - combines aspects of effective source and mode sum regularization
  - designed to work in Kerr [Barack, Golbourn, Sago 2007]
- Green function methods

Mode sum regularization

- Spacetime is spherically symmetric
  - decompose field into spherical harmonic modes
  - modes decouple
  - each mode is finite at location of particle

$$\nabla_\alpha \Phi^S = \sum_\ell \left(\nabla_\alpha \Phi^S\right)_\ell P_\ell$$

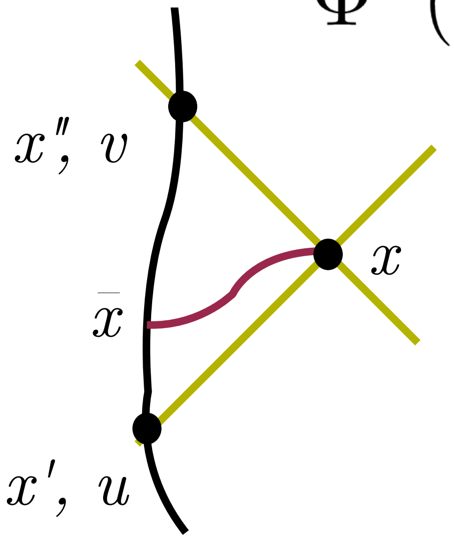
vanishes

$$\left(\nabla_\alpha \Phi^S\right)_\ell = (\ell + 1/2) A_\alpha + B_\alpha + \frac{C_\alpha}{\ell + 1/2} + \frac{D_\alpha}{(2\ell - 1)(2\ell - 3)} + O(\ell^{-3})$$

independent of ℓ

(numerical $\nabla_\alpha \Phi$ must match this structure)

Singular field near the world line



$$\Phi^S(x) = \frac{q}{2r}U(x,x') + \frac{q}{2r_{\rm adv}}U(x,x'') - \frac{1}{2}q\int_u^v V(x,z)d\tau$$

x', x'' retarded/advanced point

- Expand (bi-)tensors in terms of $\sigma^{\bar{\alpha}} \equiv \nabla^{\bar{\alpha}}\sigma(x,\bar{x})$ (covariant)

$$A(x,\bar{x}) = A(\bar{x}) + A_{\bar{\alpha}}(\bar{x})\sigma^{\bar{\alpha}} + \ldots$$

- Expand σ^{α} in terms of $(x-\bar{x})^{\bar{\alpha}}$ (not covariant)

$$\sigma^{\bar{\alpha}} = (x-\bar{x})^{\bar{\alpha}} + B^{\bar{\alpha}}_{\bar{\beta}\bar{\gamma}}(x-\bar{x})^{\bar{\beta}}(x-\bar{x})^{\bar{\gamma}} + \ldots$$

Changes due to acceleration

- Covariant local expansion of the singular field
  - Synge's world function σ links points on the world line $\Rightarrow$ acceleration and higher derivatives appear in its expansion along the world line
  - Retarded and advanced times depend on σ $\Rightarrow$ acceleration appears
- Coordinate expansion of bitensors
  - Unchanged as the point x on the world line is arbitrary

$$A_{(\mu)} = \hat{A}_{(\mu)}(x^\alpha, u^\alpha)\,\mathrm{sign}(\Delta)$$

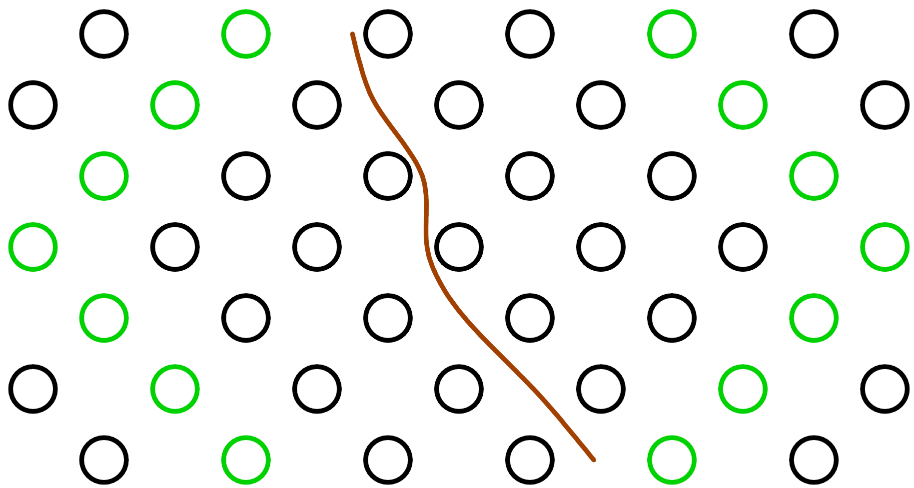
$$B_{(\mu)} = \hat{B}_{(\mu)}(x^\alpha, u^\alpha, a^\alpha)$$

$$C_{(\mu)} = 0$$

$$D_{(\mu)} = \hat{D}_{(\mu)}(x^\alpha, u^\alpha, a^\alpha, \dot{a}^\alpha)$$

Retarded field

$$\Box^{1+1}_{\text{flat}}\,(r\Phi_{\ell m}) - V\,\Phi_{\ell m} = S_{\ell m}\,\delta(r - r(\tau))$$

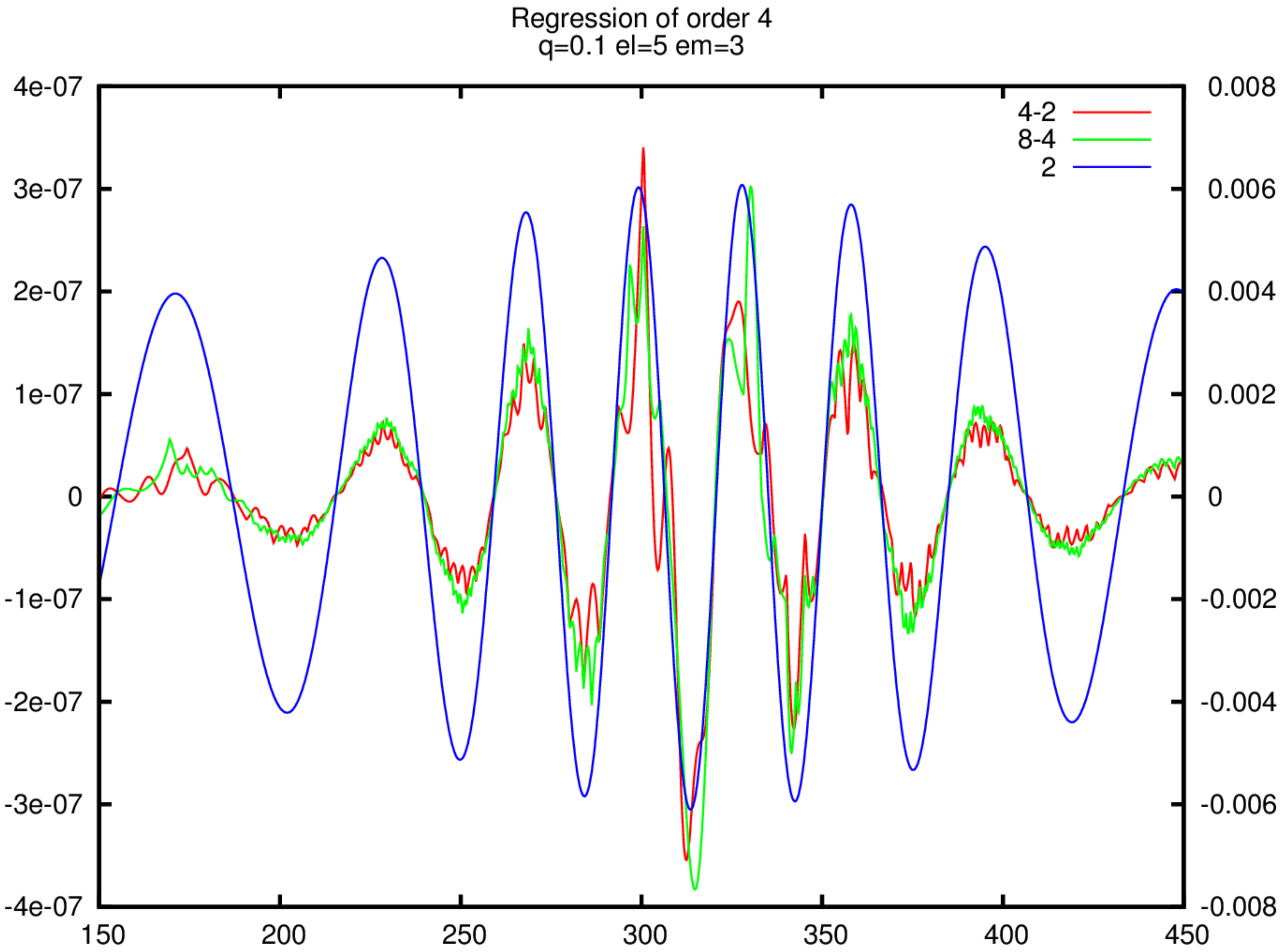


- Fourth-order accurate algorithm
- No boundary conditions are enforced,instead the grid matches the domain of dependence in each timestep
- No physical initial data is specified, we wait until the initial radiation contents has propagated away
- Both the field and the source term are evolved concurrently using a predictor-corrector scheme

Changes to handle acceleration

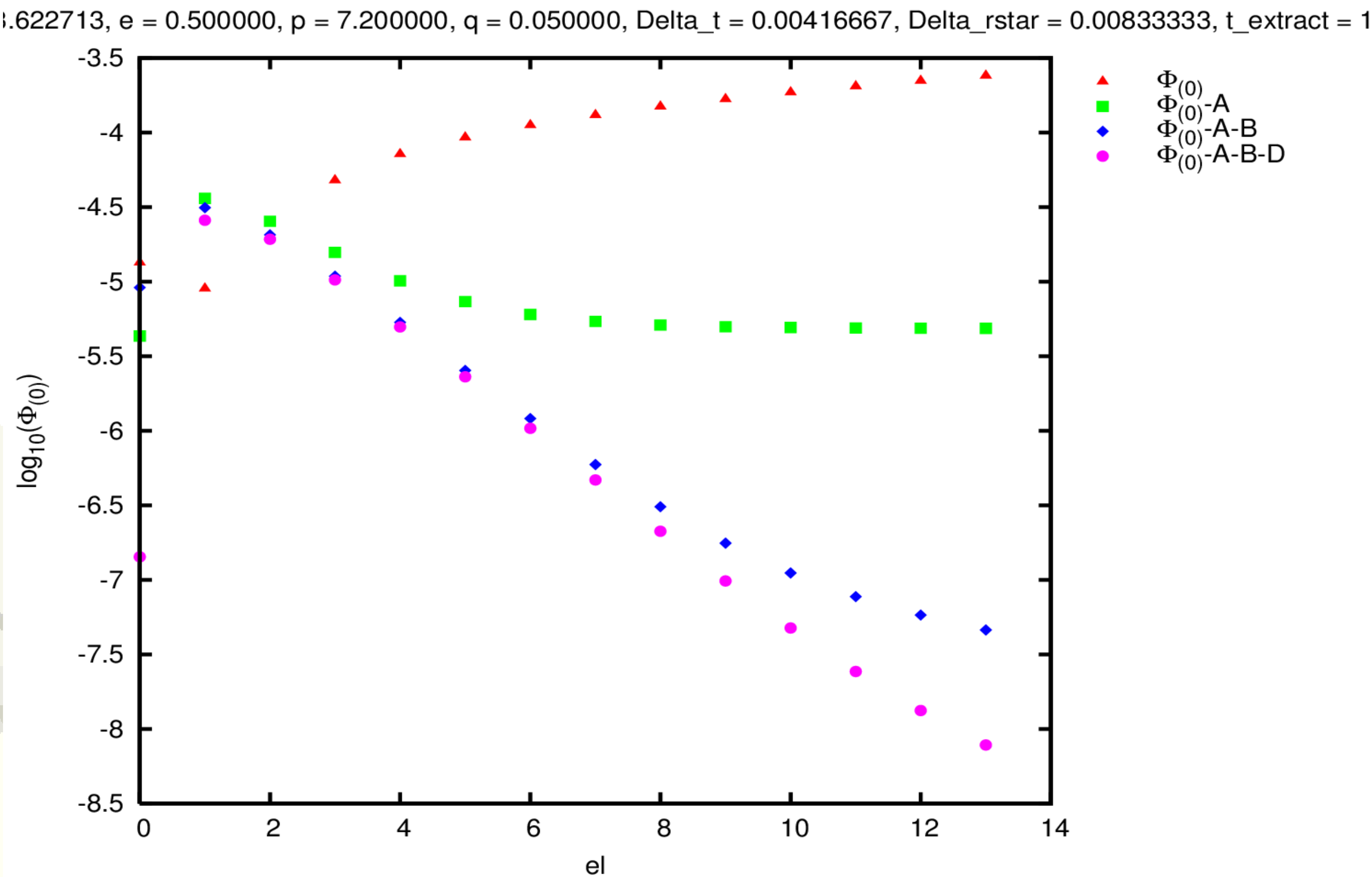
- field evolved using the original fourth-order accurate algorithm of [Lousto 2009]
  - time update requires only two time slices
  - straightforward to adapt to accelerated motion
  - fast
- particle evolved in step with field using an Adams-Bashforth-Moulton multistep timestepper
  - only uses time steps for which field values are available
  - 4th order accurate in time
- extraction of field value at particle location uses partial information on jumps. Only jumps independent of acceleration are used

Code convergence



Mode falloff for geodesic orbits

$$\left(\nabla_\alpha \Phi^S\right)_\ell = (\ell + 1/2)A_\alpha + B_\alpha + \frac{C_\alpha}{\ell + 1/2} + \frac{D_\alpha}{(2\ell - 1)(2\ell - 3)}$$



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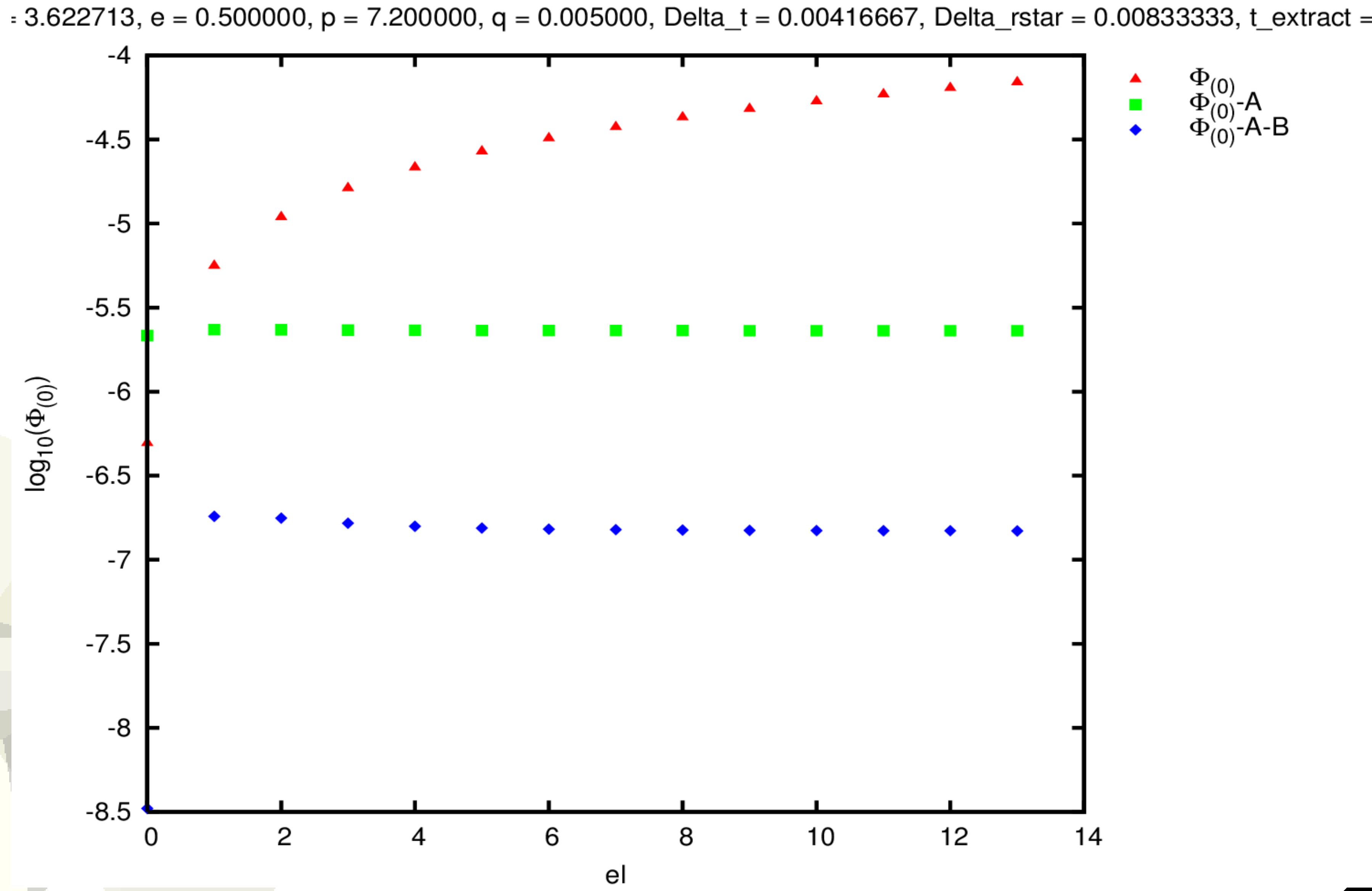
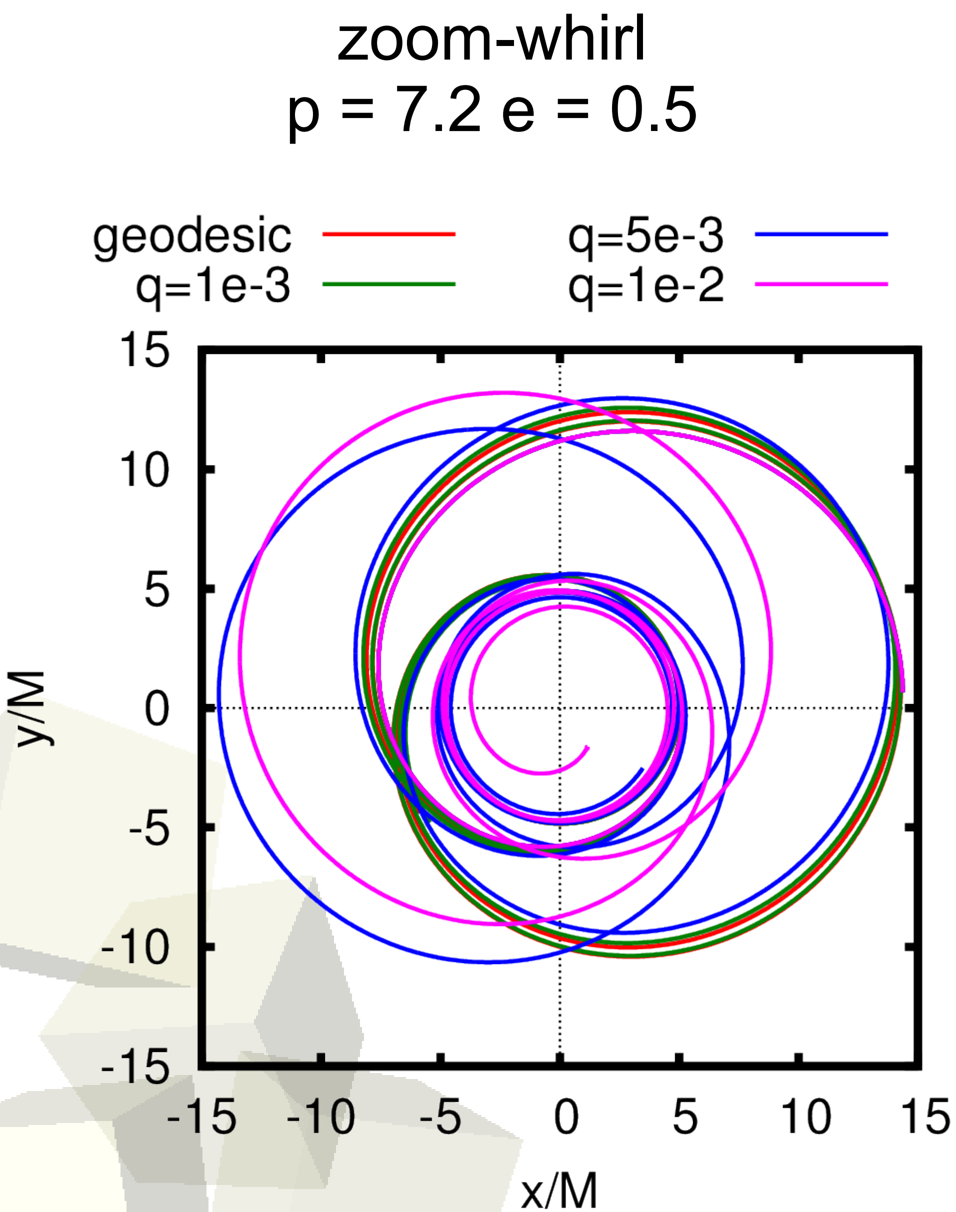
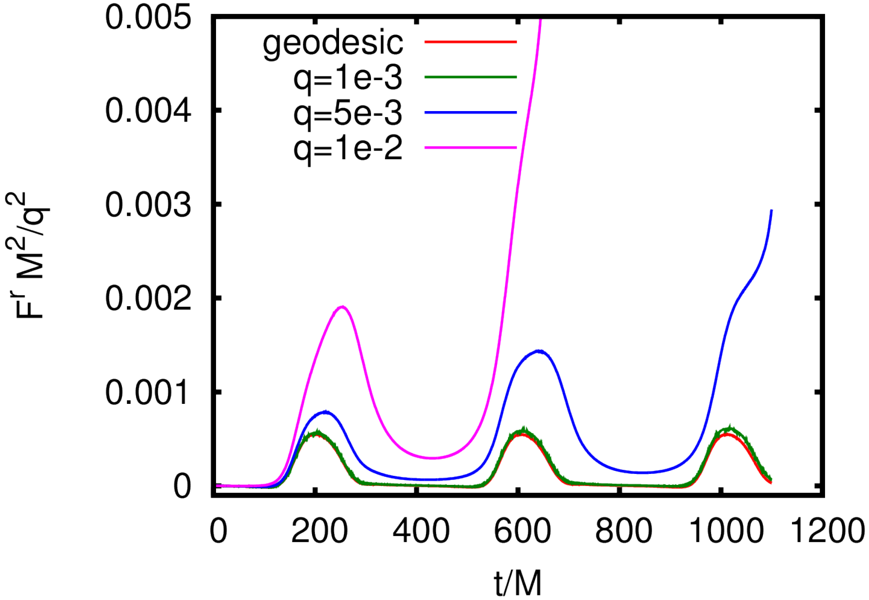


Trajectories

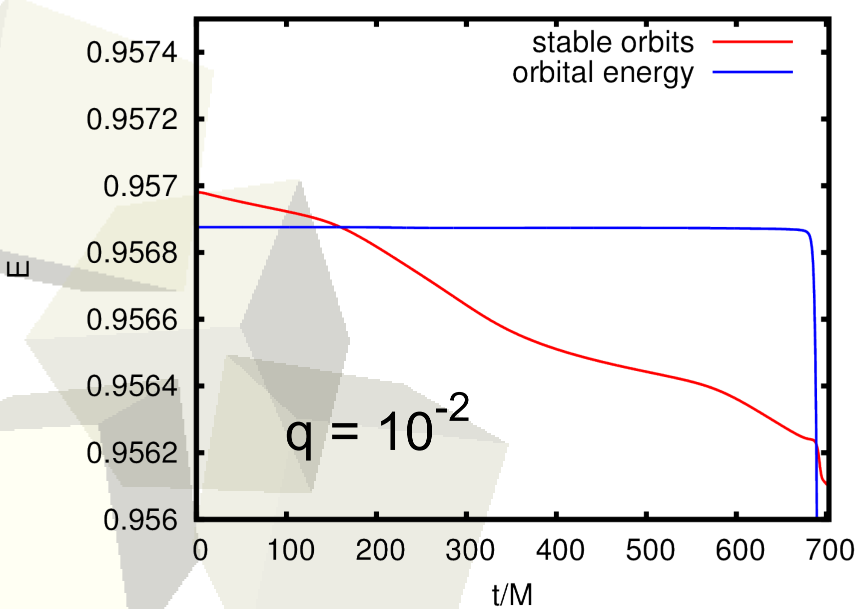


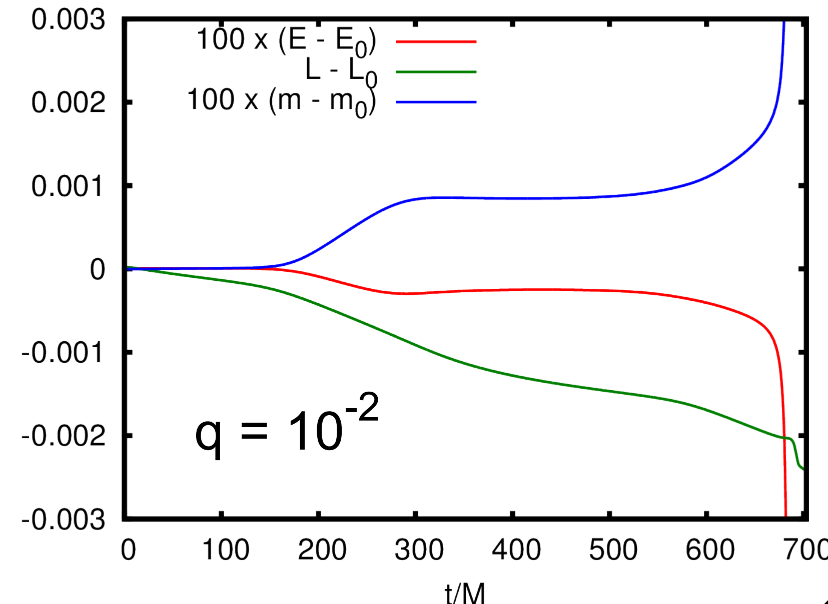
- zoom whirl orbit
  - copious radiation emitted during whirl phase
  - penetrates deep within the strong field region
- self force computed locally
- increased perihelion advance due to self force
- sudden transition from inspiral to plunge

Variable constants of motion



- charge controls number of orbits before plunge
- mass loss due to scalar radiation
- E, L oscillate





Summary

- self-consistent evolution of charge under the influence seems possible but there are still bugs in the code
- self-force likely increases perihelion advance
- onset of plunge once
  - angular momentum is sufficiently low
  - particle on inbound trajectory