A Dielectric Invisibility Carpet

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Theory developed with Sir John Pendry,
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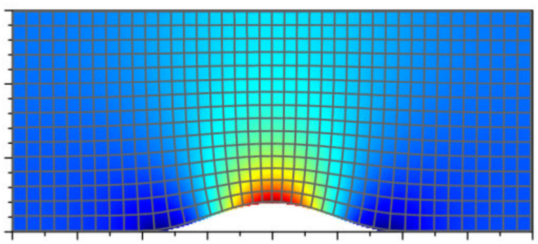
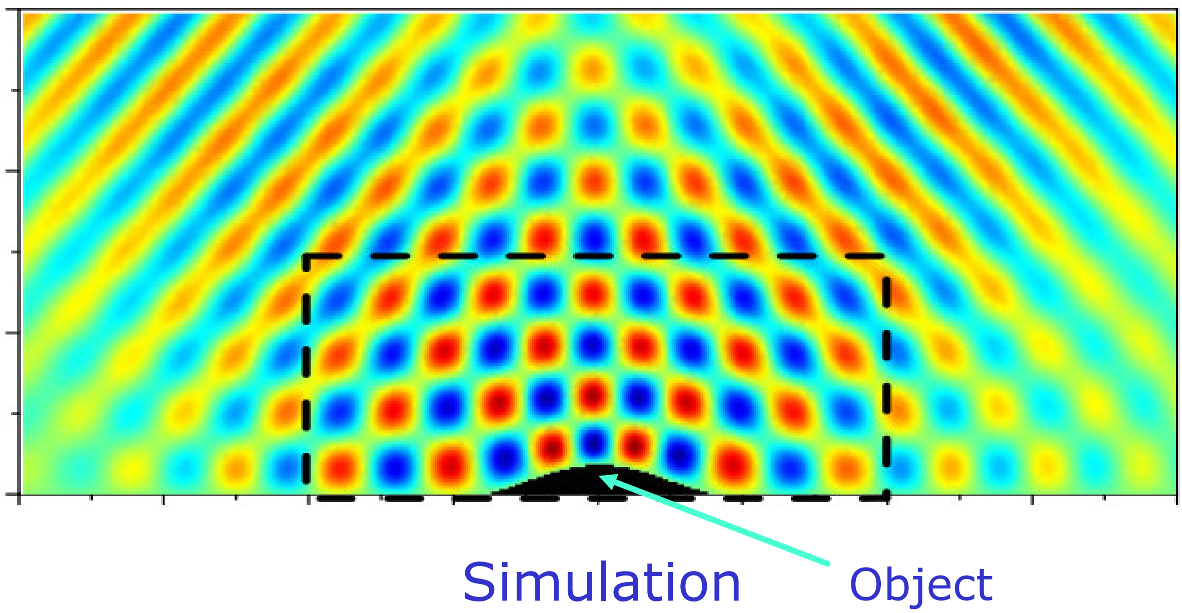
and Mathematical Modeling,
University of Maryland, College Park

Invisibility Carpet

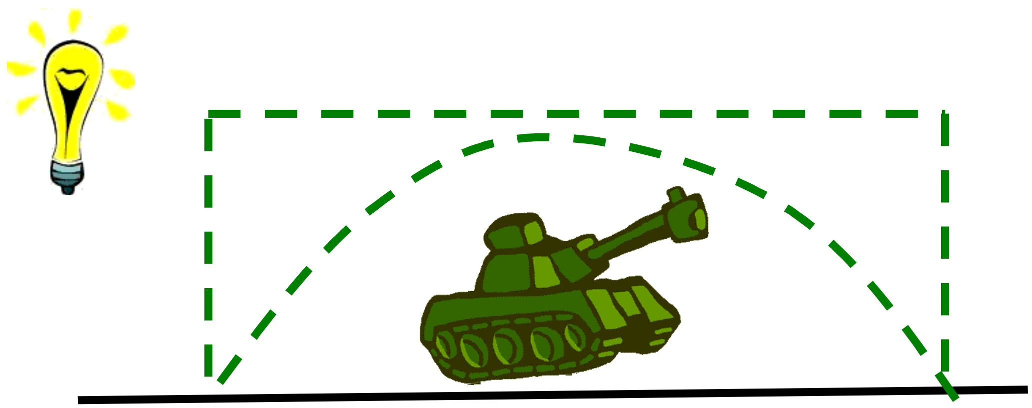
Conceal an object on the ground
Work for optical frequencies
Can be made from dielectrics – practical for experiments



Profile of cloak



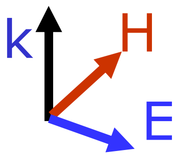
Simulation

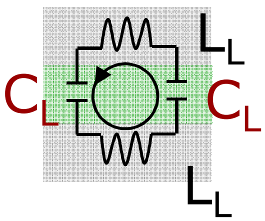


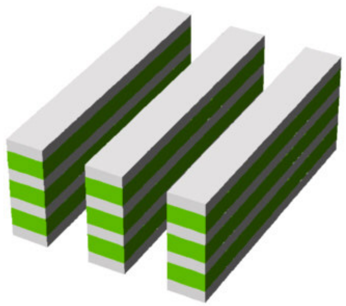
Metamaterials fabricated in Xlab …

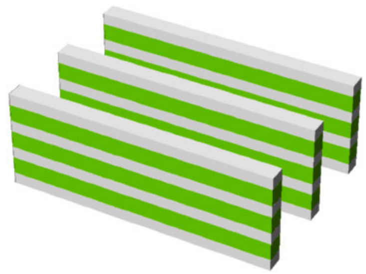
A Negative Index Material by Fishnet

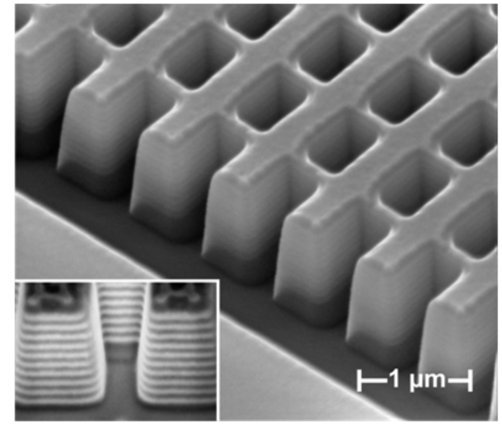
Thick Metal Strips + Thin Metal Wires = Fishnet







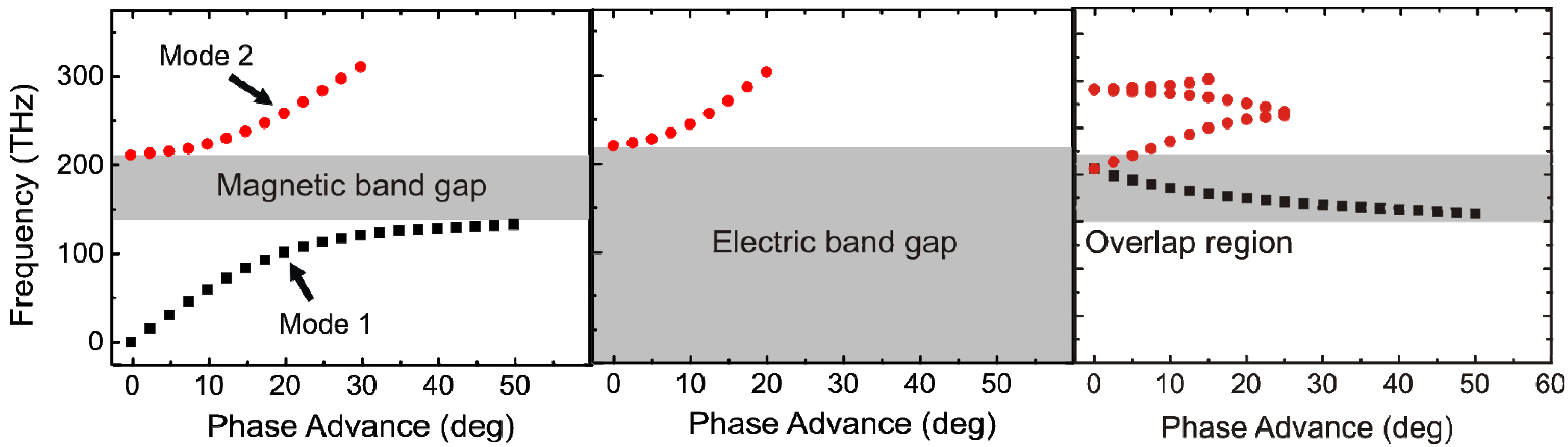




Magnetic Dispersion

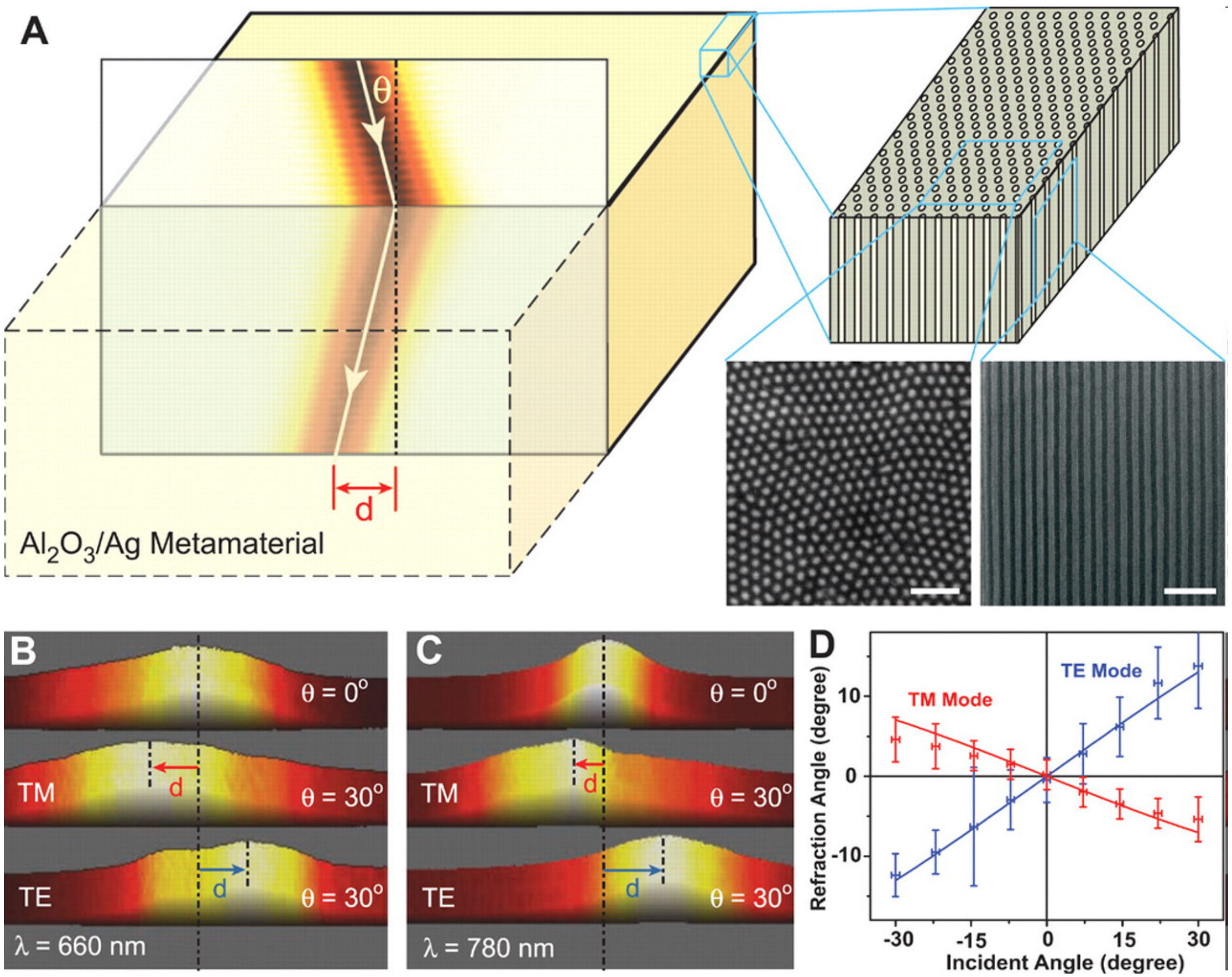
Electric Dispersion

Combined Dispersion



J. Valentine, et. al., Nature **455**, 376 (2008).

Optical Negative Refraction in Bulk Metamaterials Made of Metallic Nanowires



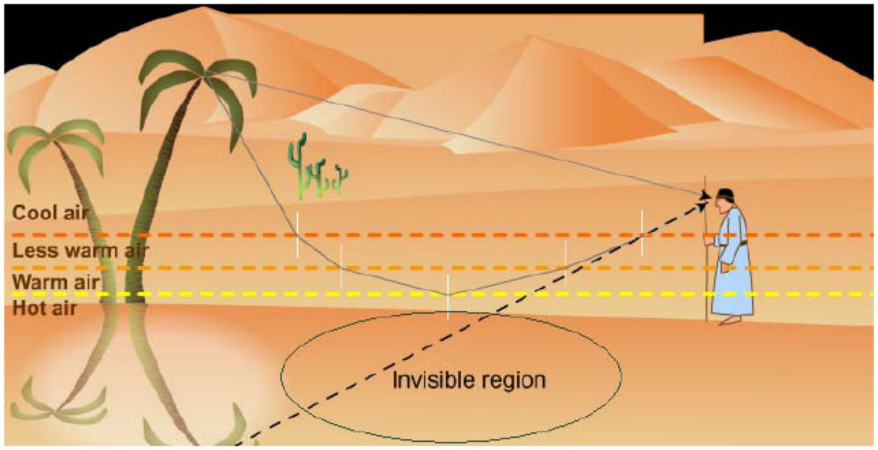
J. Yao, et. al., Science **321**, 930 (2008).

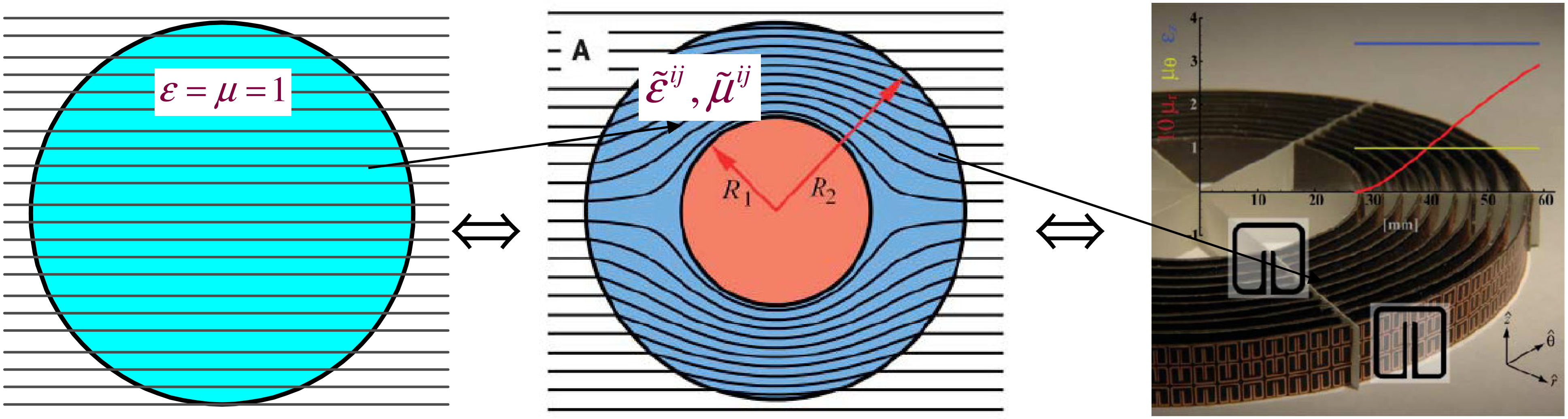
Outline

- Cloaking and Transformation Optics
- Limitation of Metamaterials with metals at Optical Frequencies
- Invisibility Carpet
  - Compress object to flat conducting sheet
  - Design using quasi-conformal map
  - A profile of $\varepsilon>0$, $\mu>0$ without extreme values, easier to fabricate and broadband
  - Full wave simulations

A controlled mirage

- Cloaking
  - guide light as desired, principle like mirage
- Coordinate Transform
  - Maxwell Equation invariant
  - Only material parameters (ε and μ) changed





Virtual system

Physical system

Realization

D Schurig, et. al., Science **314**, 977 (2006).

Race to lower loss near optical frequency

Gold nanorods pair

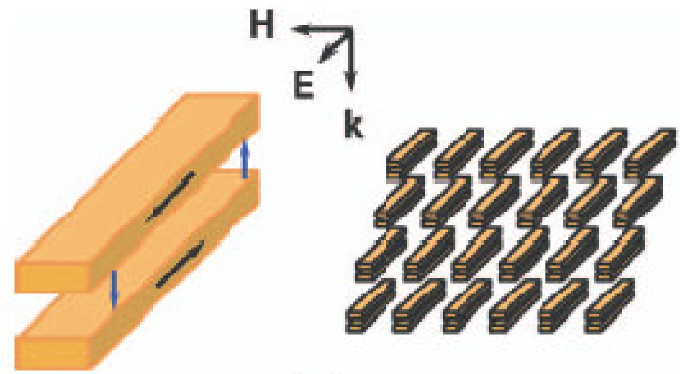
- Resonating element giving rise to both electric magnetic response
- $n' = -0.3$, $F = |n'|/n'' = 0.1$ at $1.5\mu m$

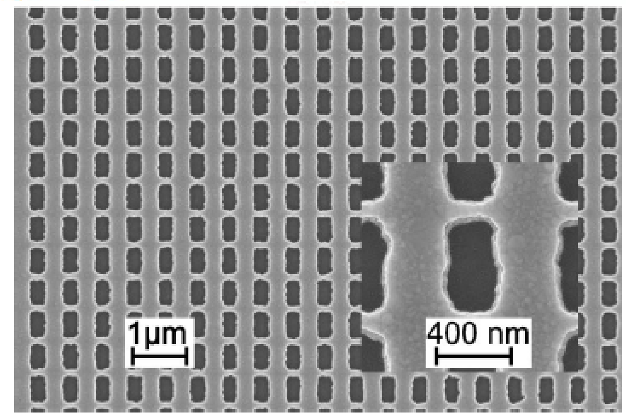
Fishnet

- $n' = -1$, $F = |n'|/n'' = 3$ at $1.5\ \mu m$
- $n' = -0.6$, $F = |n'|/n'' = 0.5$ at $780nm$

3D Fishnet

- $n' = -1.23$, $F = |n'|/n'' = 3.5$ at $1.8\ \mu m$





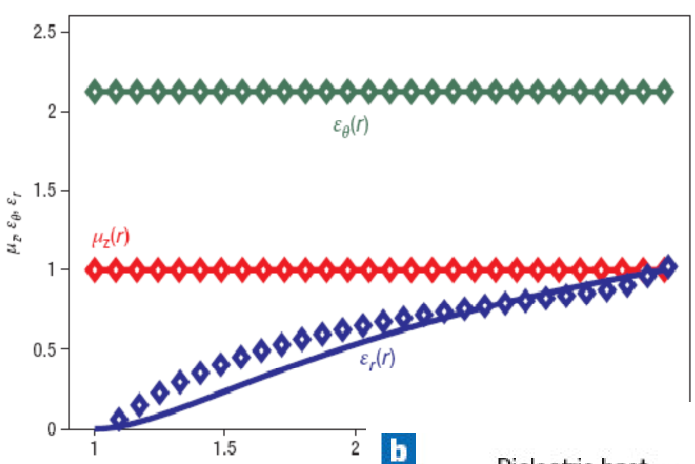
Reasons of high loss

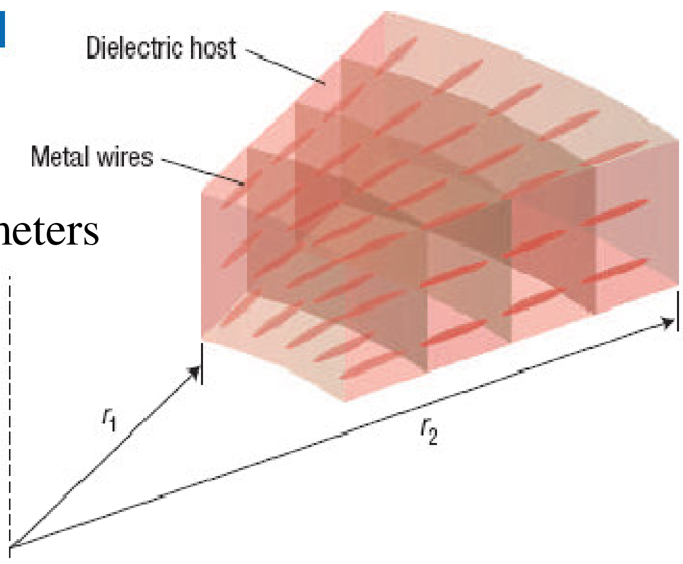
- Resonant nature of the structure
- Very near to magnetic resonance

V M Shalaev, et. al., Opt. Lett. **30**, 3356 (2005).
G Dolling, et. al., Opt. Lett. **32**, 53 (2007).
J. Valentine, et. al., Nature **455**, 376 (2008).

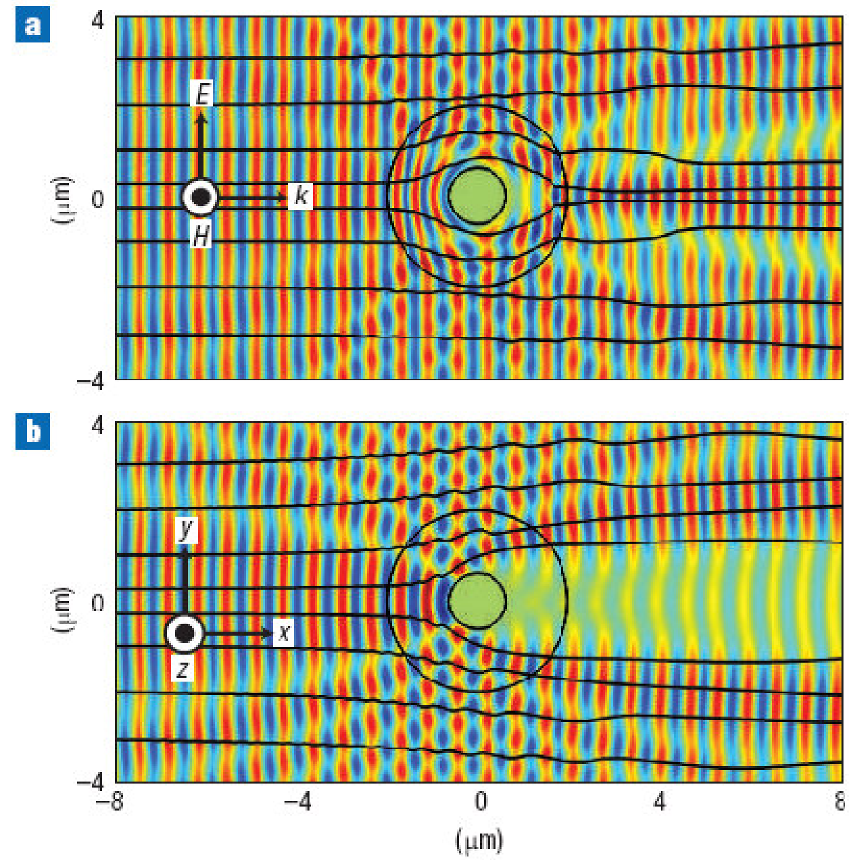
Avoid Magnetic resonance for cloaking at optical

- Ag nanowires in Silica
  - Reduced parameter approximation: $(\varepsilon_\theta, \varepsilon_r, \mu) \to (\varepsilon_\theta \mu, \varepsilon_r \mu, 1)$
  - Size of unit cell ~ 100 nm, H-polarization @ 632.8nm
  - non-magnetic metamaterials in reduced material parameters





TM, reduced parameters



W Cai, et. al., Nature Photonics **1**, 224 (2007).

At optical frequencies …

Metamaterials (resonating elements)

- Advantage: large range of material parameters
- Disadvantage: Elements subwavelength, absorption

Dielectrics (far away from resonance)

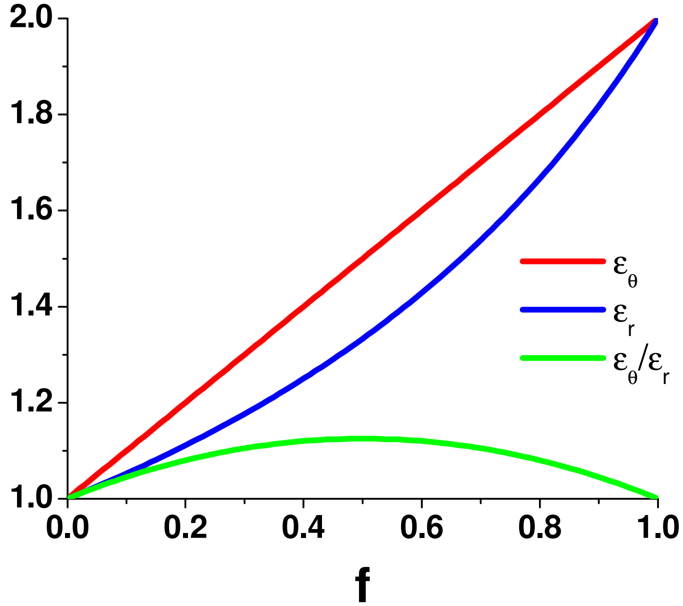
- Low absorption, easy to fabricate
- Broadband: Frequency independent

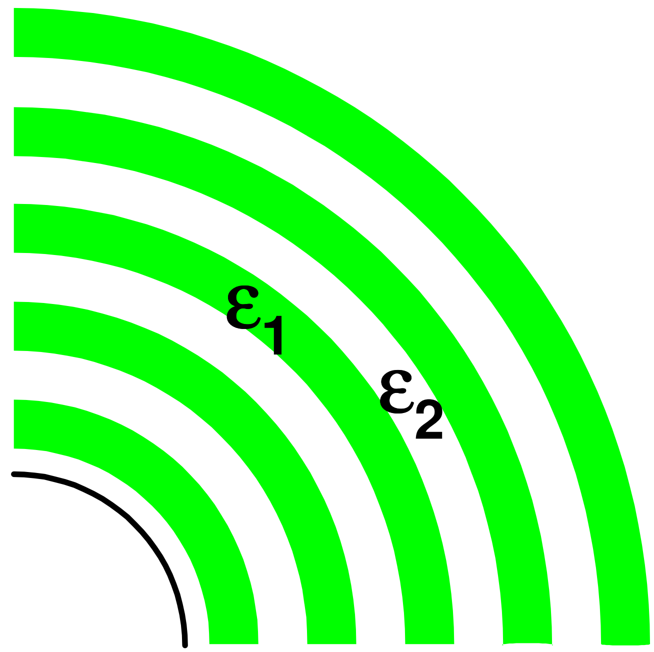
Limitation of Dielectrics

Limited anisotropy for fixed permittivities

$$\varepsilon_\theta = f\varepsilon_1 + (1-f)\varepsilon_2$$

$$1/\varepsilon_r = f/\varepsilon_1 + (1-f)/\varepsilon_2$$





Extreme permittivities needed for large anisotropy

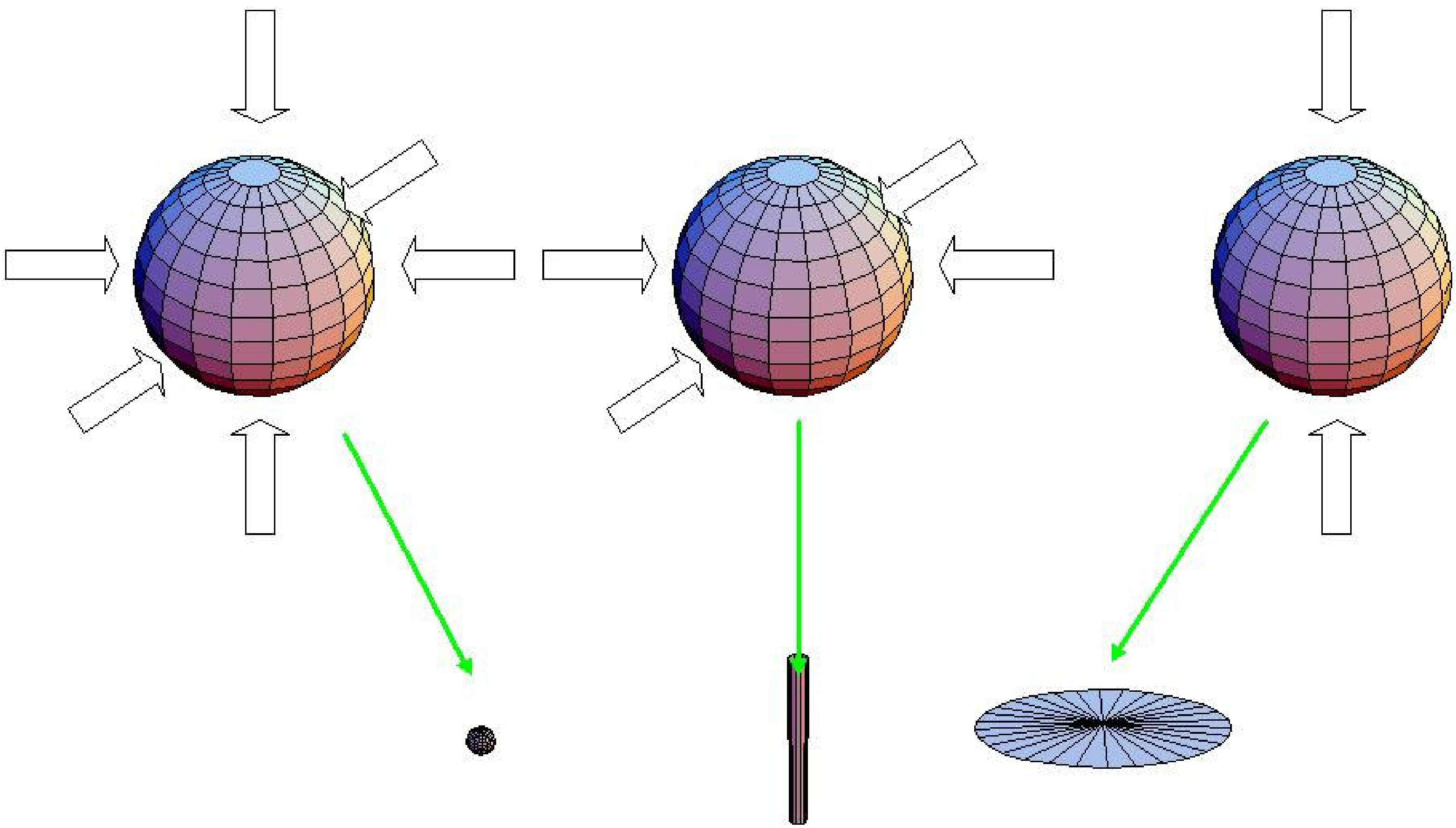
$\varepsilon_1 = 0.05, \varepsilon_2 = 8$ from Y. Huang, et.al., Opt. Exp. 2007

Extreme values related to the topology of cloak

- how we crush an object

Three ways to crush an object

- Crushing an object into a point, a line or a plane



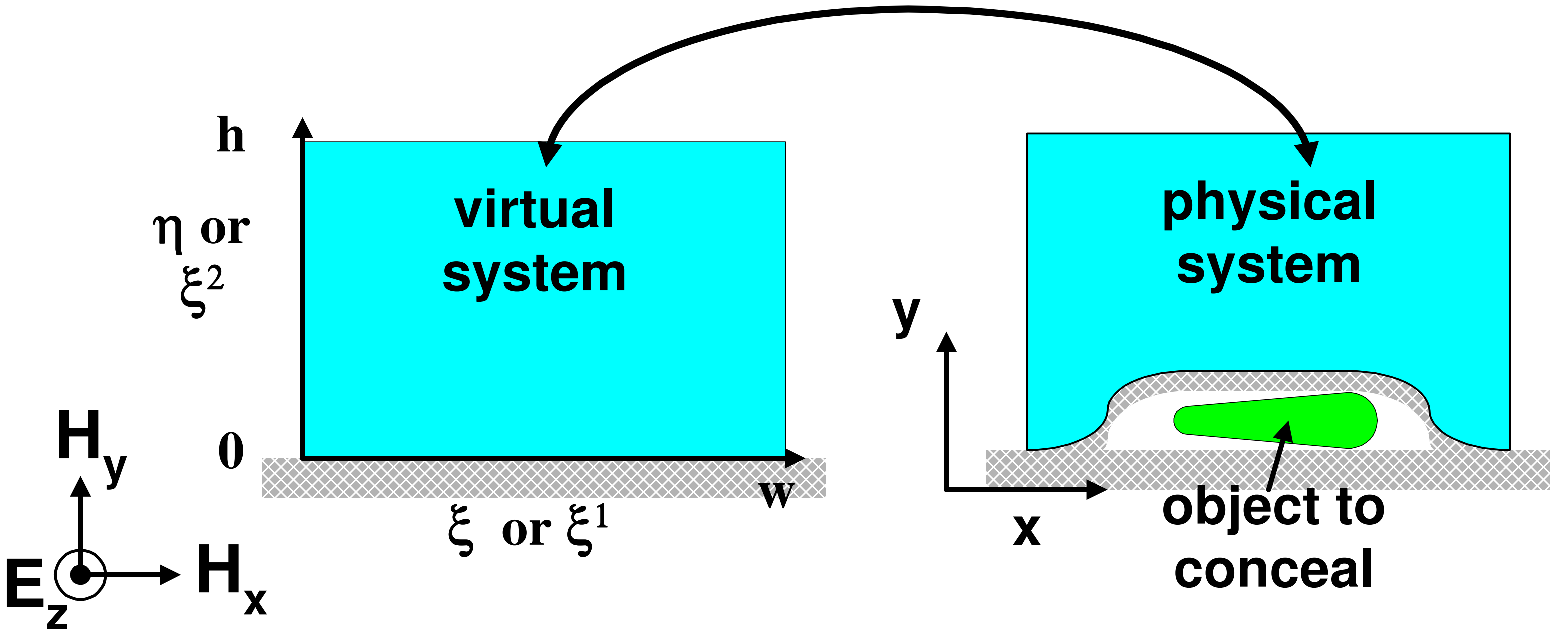
3D Spherical Cloak
Singular

2D Cylind. Cloak
Singular

Carpet Cloak
Non-Singular

Hiding under a carpet

- Perceived as a flat ground plane
- Avoid singular/extreme values for ε and μ



Material parameters: $\vec{\mu}, \varepsilon_z$

Minimize the anisotropy by an appropriate coordinate transformation

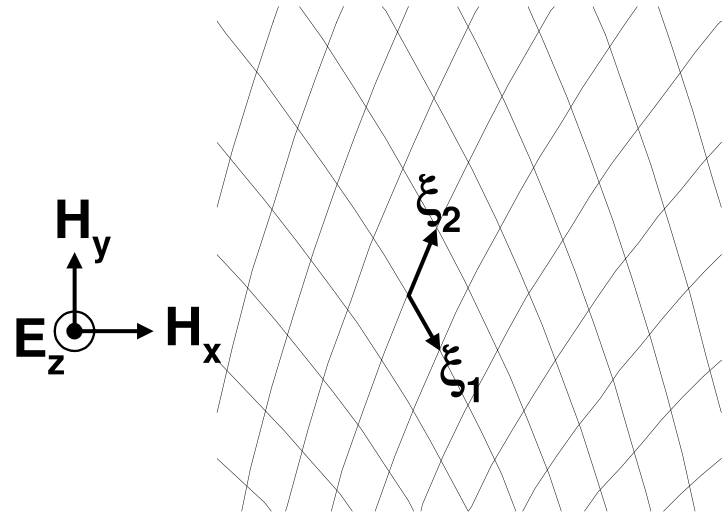
Transformation Optics

- E-polarization for 2D

Material parameters:

$$\vec{\mu} = \begin{pmatrix} \mu_{xx} & \mu_{xy} \\ \mu_{xy} & \mu_{yy} \end{pmatrix}, \varepsilon$$

$$\nabla \cdot \frac{\vec{\mu}}{\det \vec{\mu}} \cdot \nabla E = -\left(\frac{\omega}{c}\right)^2 \varepsilon E$$



In general coordinates:

$$\frac{\partial}{\partial q^i}\left(\frac{\Omega\mu^{ij}}{\det\left(\Omega\mu^{ij}\right)}\frac{\partial E}{\partial \xi^j}\right) = -\left(\frac{\omega}{c}\right)^2 \Omega\varepsilon E$$

$$\tilde{\mu}^{ij}\left(\xi\right) = \delta^{ij} \qquad \tilde{\varepsilon}\left(\xi\right) = 1$$

ε and μ in ξ as if it is Cartesian

$$\left[\mu^{ij}\left(x\right)\right] = SS^T / \Omega$$

$$\varepsilon\left(x\right) = 1/\Omega$$

General Coordinate: $\left(\xi^1, \xi^2\right)$

$$\boldsymbol{\xi}_i = \frac{\partial \mathbf{r}}{\partial \xi^i}$$

$$g_{ij} = \boldsymbol{\xi}_i \cdot \boldsymbol{\xi}_j \qquad S^i{}_j = \frac{\partial x^i}{\partial \xi^j}$$

$$\Omega = \left|\boldsymbol{\xi}_1 \times \boldsymbol{\xi}_2\right| = \det S = \sqrt{\det g}$$

$$\mu^{ij}\left(\xi\right) = \boldsymbol{\xi}^i \cdot \vec{\mu} \cdot \boldsymbol{\xi}^j$$

Local Dispersion Surface

- The contravariant tensor

$$\frac{\partial}{\partial \xi^i}\left(\frac{\Omega\mu^{ij}}{\det\left(\Omega\mu^{ij}\right)}\frac{\partial E}{\partial \xi^j}\right) = -\left(\frac{\omega}{c}\right)^2 \Omega\varepsilon E \qquad \text{with } \frac{\partial}{\partial \xi^i} \rightarrow ik_i$$

Local dispersion surface: $\gamma^{ij}(\xi)k_i(\xi)k_j(\xi) = \left(\frac{\omega}{c}\right)^2$

Contravariant

I

$$\Rightarrow \qquad \left[\gamma^{ij}(x)\right] = S\left[\gamma^{ij}(\xi)\right]S^T = SS^T$$

Eigenvalues of $\left[\gamma^{ij}(x)\right]: \frac{1}{n_L^2}, \frac{1}{n_T^2}$

General Coordinate: $\left(\xi^1, \xi^2\right)$

$$\boldsymbol{\xi}_i = \frac{\partial \mathbf{r}}{\partial \xi^i}$$

$$g_{ij} = \boldsymbol{\xi}_i \cdot \boldsymbol{\xi}_j \qquad S^i{}_j = \frac{\partial x^i}{\partial \xi^j}$$

$$\Omega = \left|\boldsymbol{\xi}_1 \times \boldsymbol{\xi}_2\right| = \det S = \sqrt{\det g}$$

$$\mu^{ij}(\xi) = \boldsymbol{\xi}^i \cdot \vec{\mu} \cdot \boldsymbol{\xi}^j$$

Anisotropy at a single point

Dispersion Surface: $\alpha = n_T/n_L$ measures the anisotropy

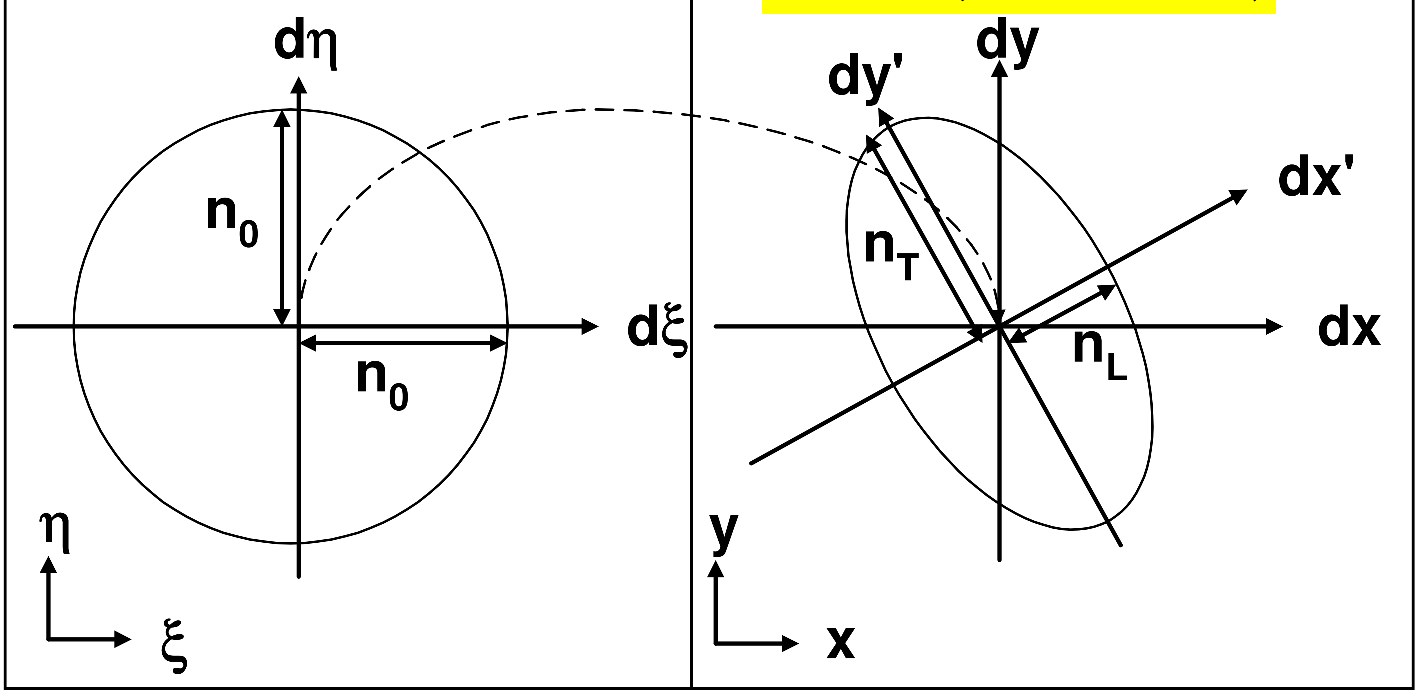
$n_T{*}n_L$ measures the size

$$\left(\frac{k_\xi}{n_0}\right)^2 + \left(\frac{k_\eta}{n_0}\right)^2 = \left(\frac{\omega}{c}\right)^2$$

$$\left(\frac{k_{x'}}{n_L}\right)^2 + \left(\frac{k_{y'}}{n_T}\right)^2 = \left(\frac{\omega}{c}\right)^2$$

Anisotropy factor:

$$\alpha = \max\left(n_T / n_L, n_L / n_T\right)$$



Quasiconformal map minimizes anisotropy

- Relationship between anisotropy factor and metric

$$\alpha = \max\left(n_T / n_L, n_L / n_T\right).$$

$$\alpha + \frac{1}{\alpha} = \frac{Tr(g)}{\sqrt{\det g}}$$

$$\left[g_{ij}\right] = S^T S$$

metric   Jacobian

- Minimizing anisotropy
  - Modified Liao generator

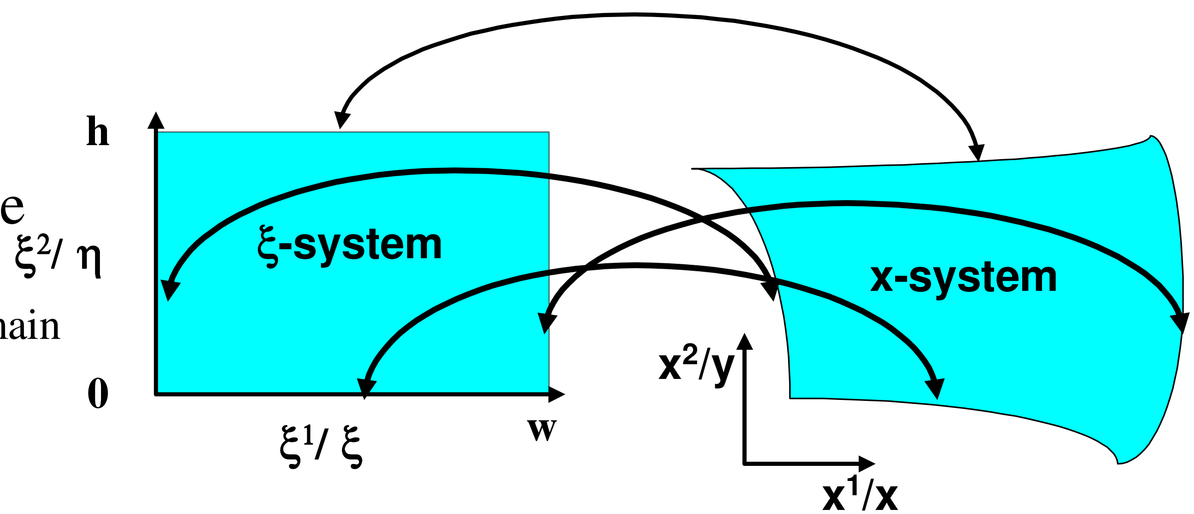
$$\Phi = \frac{1}{hw}\int_0^w d\xi \int_0^h d\eta \left(\frac{Tr(g)}{\sqrt{\det g}}\right)^2.$$

  - upon slipping boundaries generates the quasi-conformal map

$$\frac{Tr(g)}{\sqrt{\det g}} = \text{constant}$$

which can minimize

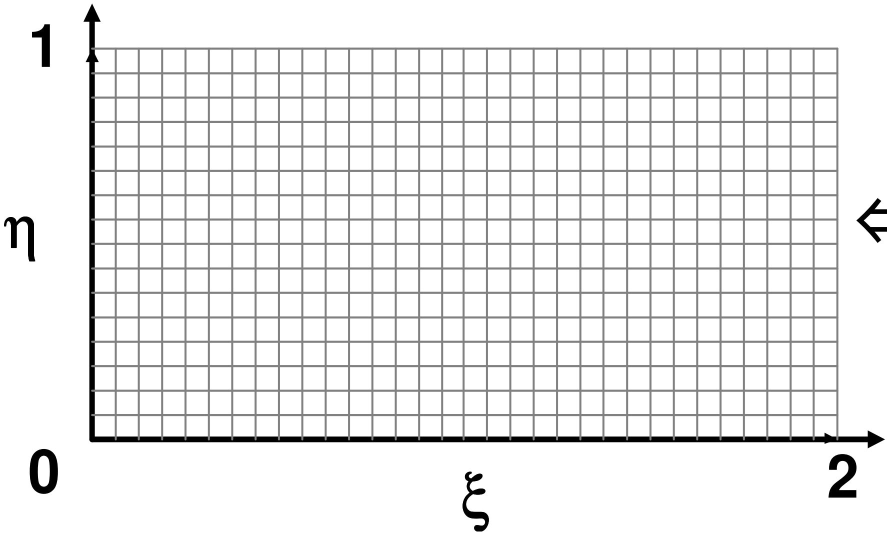
$$\left(\frac{Tr(g)}{\sqrt{\det g}}\right)_{\max}$$ in the whole domain



J. Thompson, Handbook of Grid Generation

Simple grid before minimization

Linear compression



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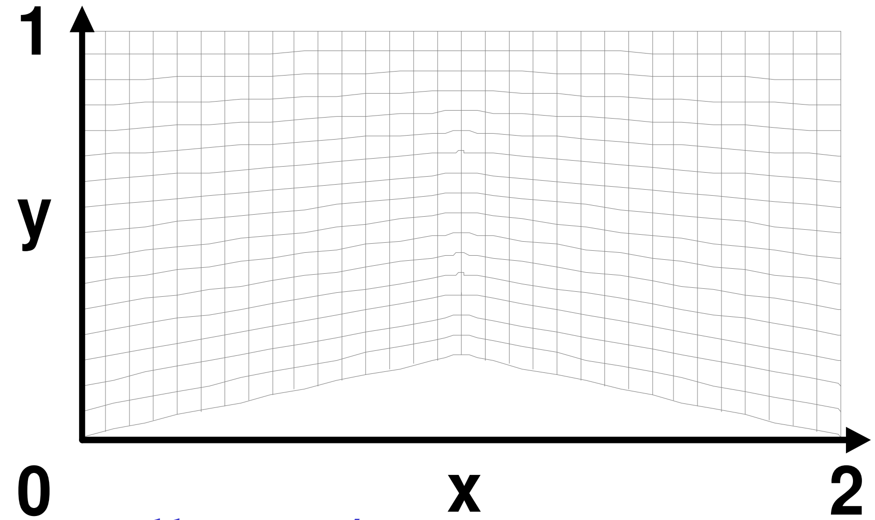
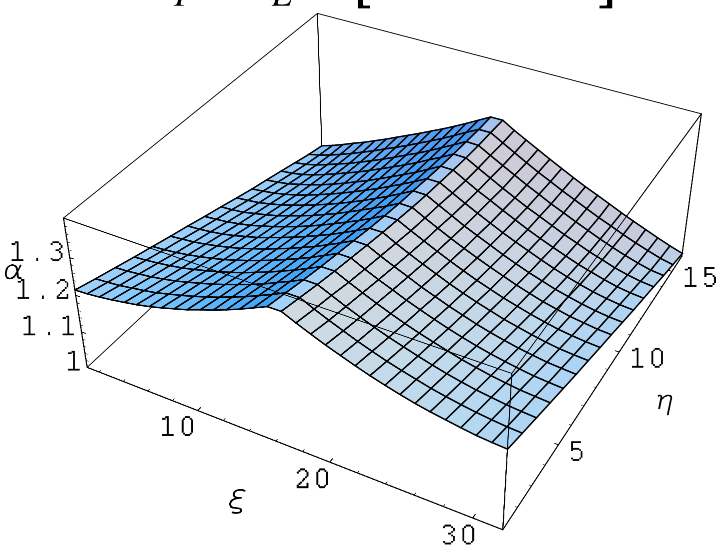
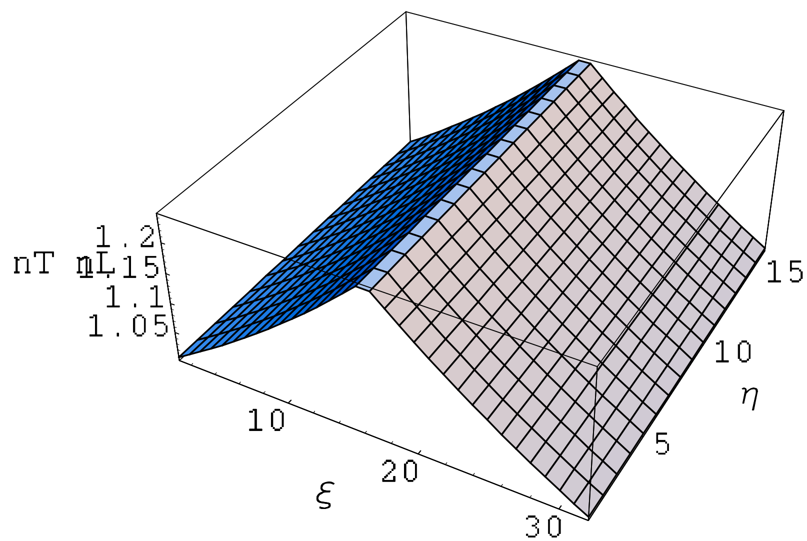


- No singular values but still want smaller n_T/n_L

$\alpha = n_T / n_L \in [1\ ,\ 1.355]$



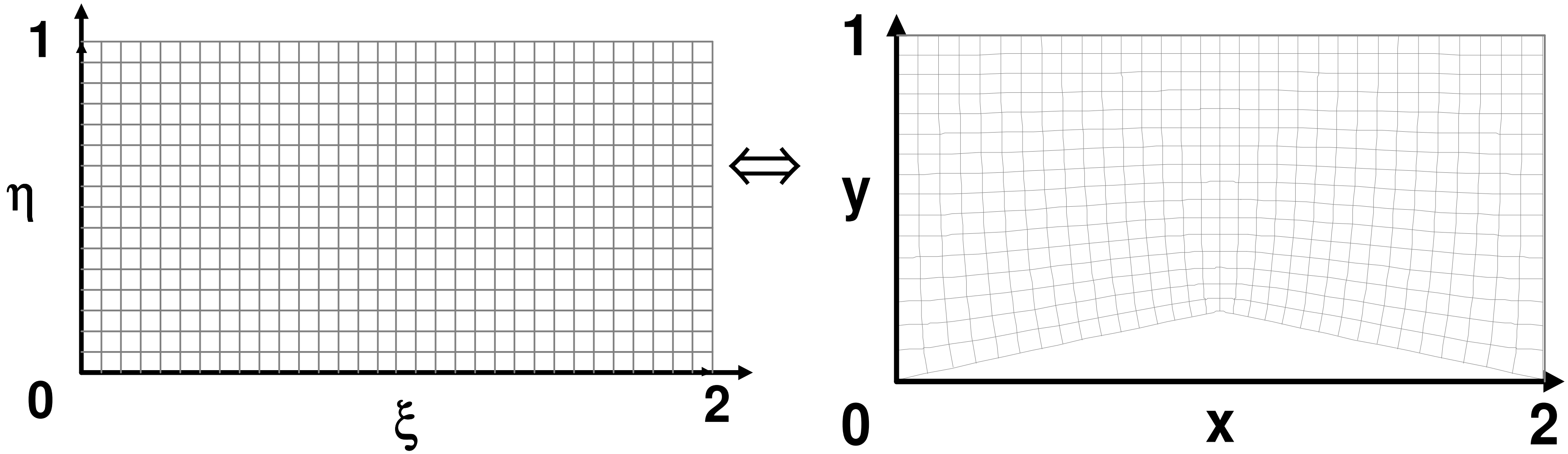
$n_T \times n_L \in [1\ ,\ 1.24]$



Quasi-conformal map

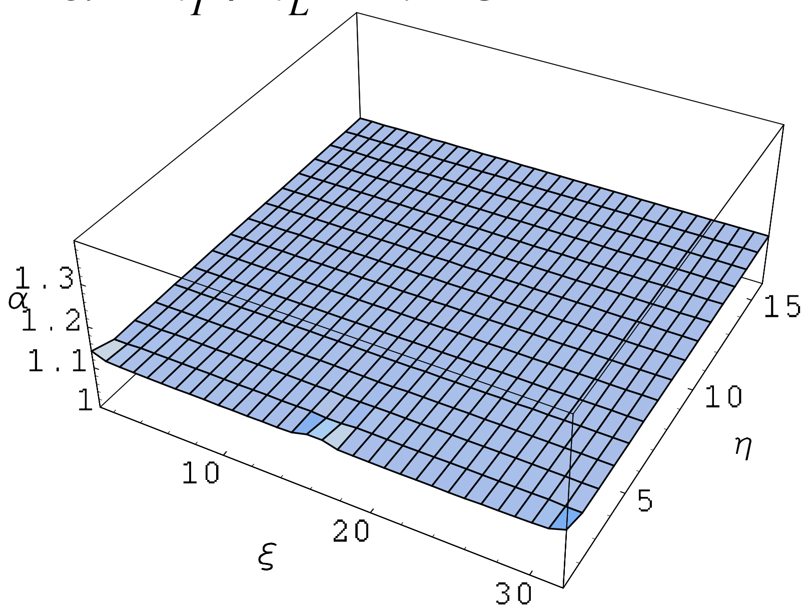
Orthogonal

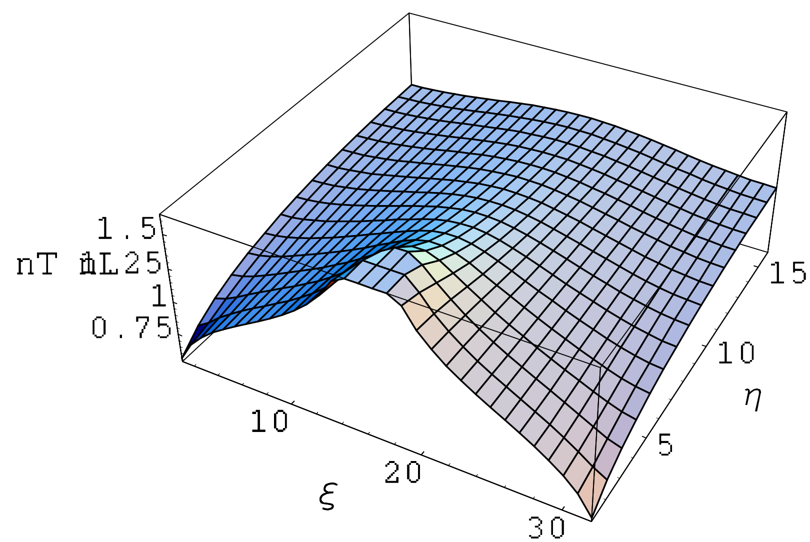
Anisotropy factor = constant cell aspect ratio



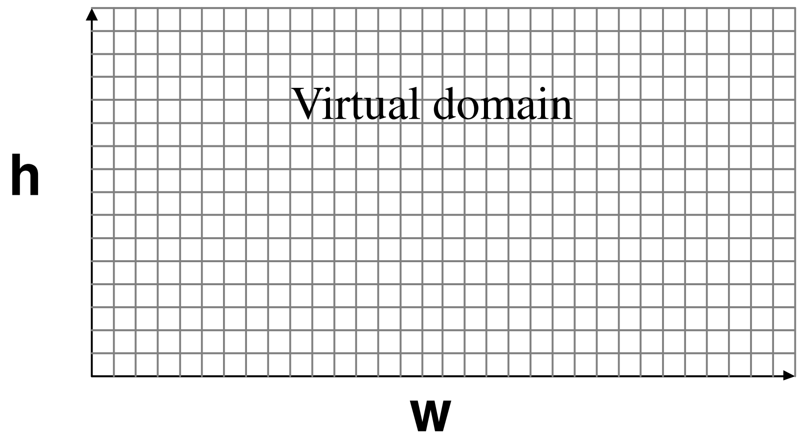
$\alpha = n_T / n_L \equiv 1.125$

$n_T \times n_L \in [0.56 \ , \ 2.26]$

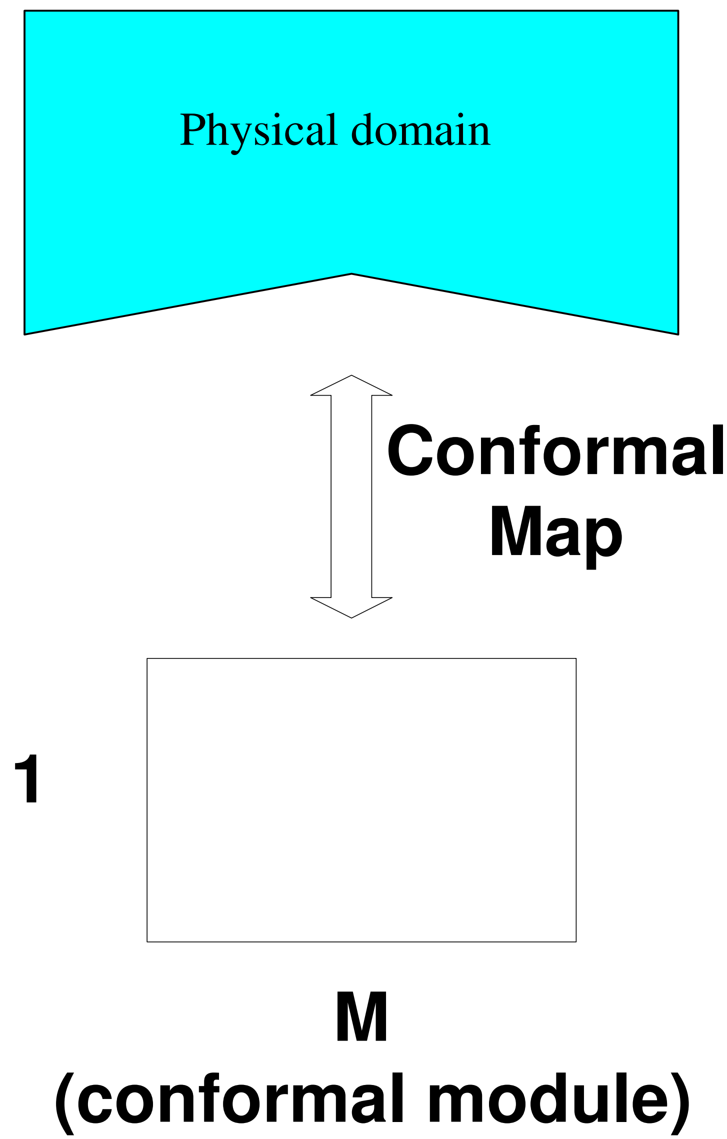




Properties of quasi-conformal map



conformal module m = w/h



- Properties
  - Orthogonal
  - Rectangular cell with constant aspect ratio M:m
  - Anisotropy factor/ cell aspect ratio: $\alpha = \frac{M}{m}$

A carpet cloak using quasi-conformal map

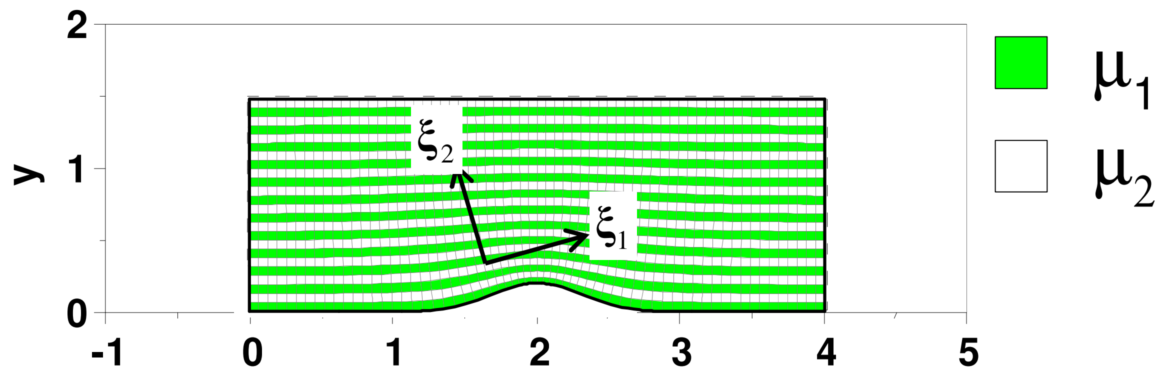
- Principal axes always align to the grid lines
- Anisotropy generated by stack of two isotropic materials

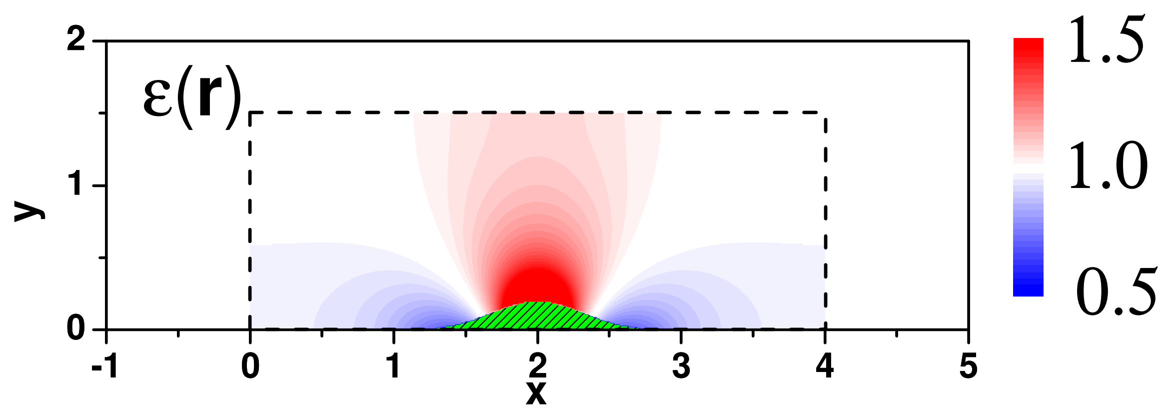
$$\vec{\mu} = \mu_L \vec{\xi}_1 \vec{\xi}_1 / \left|\vec{\xi}_1\right|^2 + \mu_T \vec{\xi}_2 \vec{\xi}_2 / \left|\vec{\xi}_2\right|^2$$

$$\mu_L = \alpha$$

$$\mu_T = 1/\alpha$$

$$\varepsilon = \frac{1}{\sqrt{\det g}} = \frac{1}{|\xi_1||\xi_2|}$$





Isotropic Approximation

$$(\mu_T, \mu_L, \varepsilon)$$

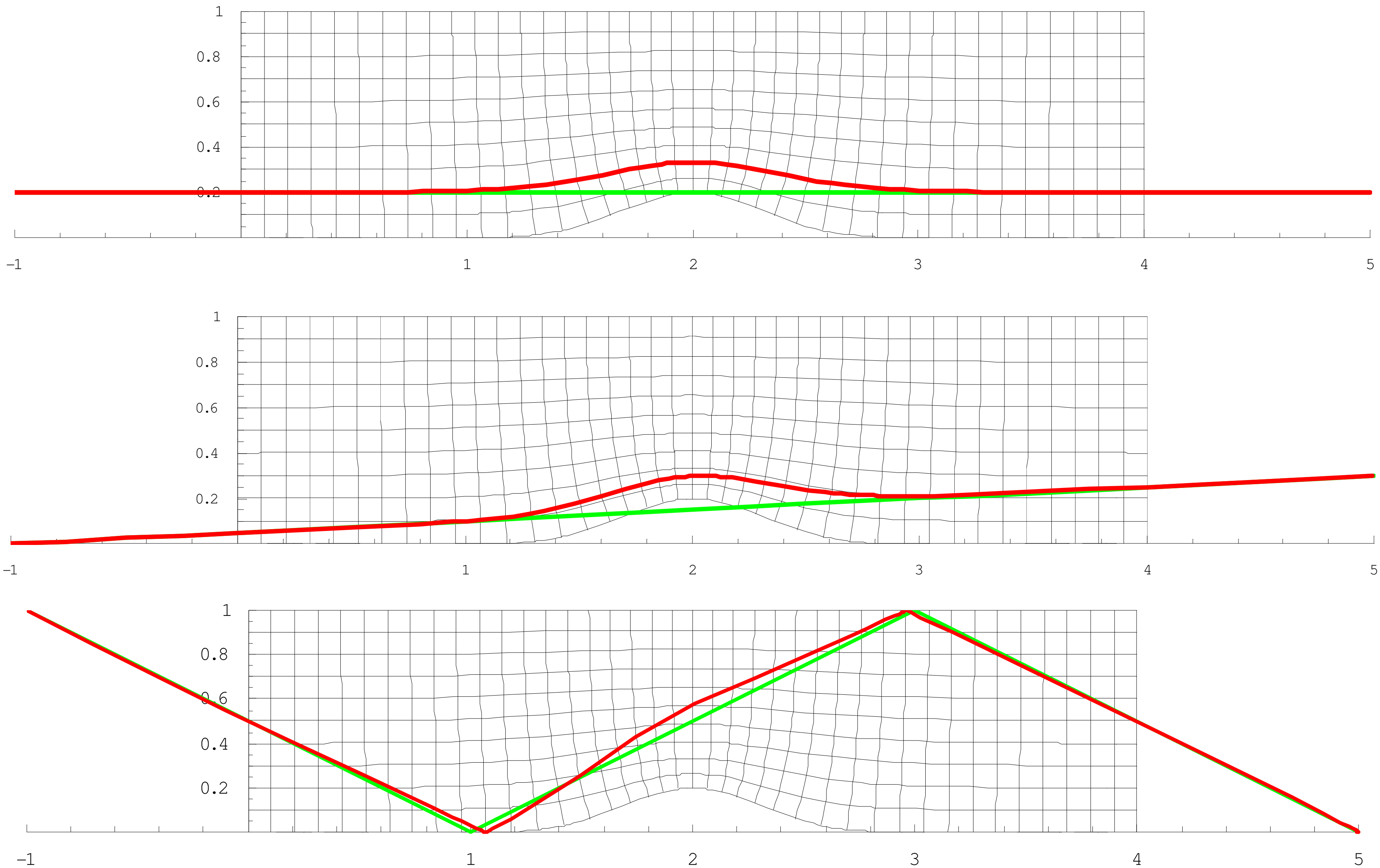
$$\downarrow$$

$$\left(\sqrt{\mu_T \mu_L} = 1, \sqrt{\mu_T \mu_L} = 1, \varepsilon\right)$$

To be applied later
for simplication

Ray tracing

- Geometrical optics limit



Further approximation to ease fabrication

- Reduced Parameter Approximation (Previous approaches)
  - Need impedance match at outer boundary of cloak

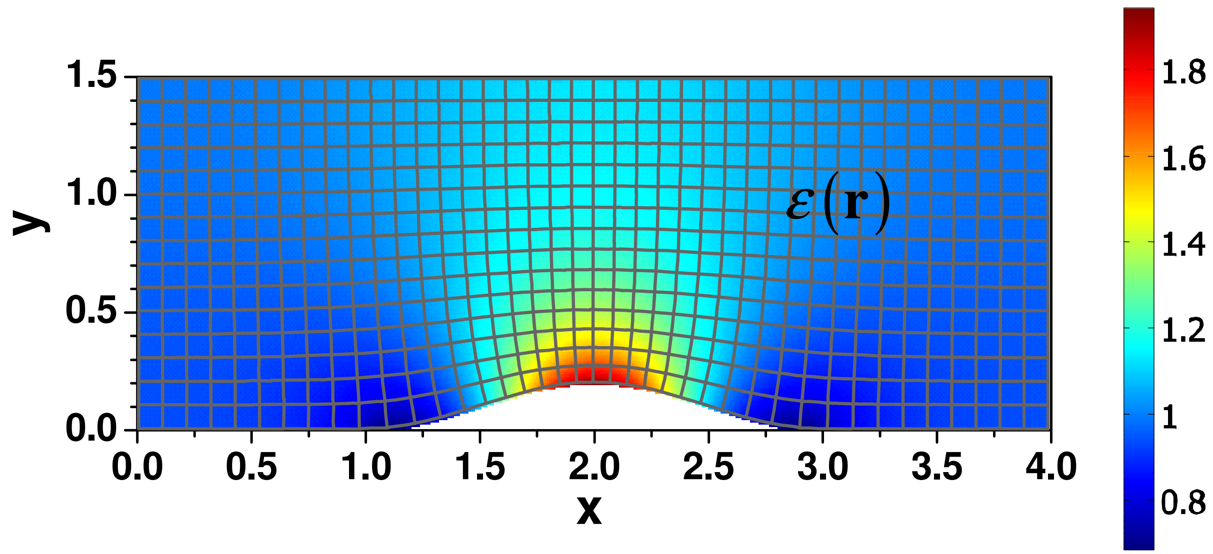
$$(\mu_T, \mu_L, \varepsilon) \rightarrow (\mu_T \varepsilon, \mu_L \varepsilon, 1)$$

- Isotropic Approximation (Our approach)
  - Need to have a thicker coating

$$(\mu_T, \mu_L, \varepsilon) \rightarrow \left(\sqrt{\mu_T \mu_L} = 1, \sqrt{\mu_T \mu_L} = 1, \varepsilon\right)$$

Dielectric Invisibility Carpet

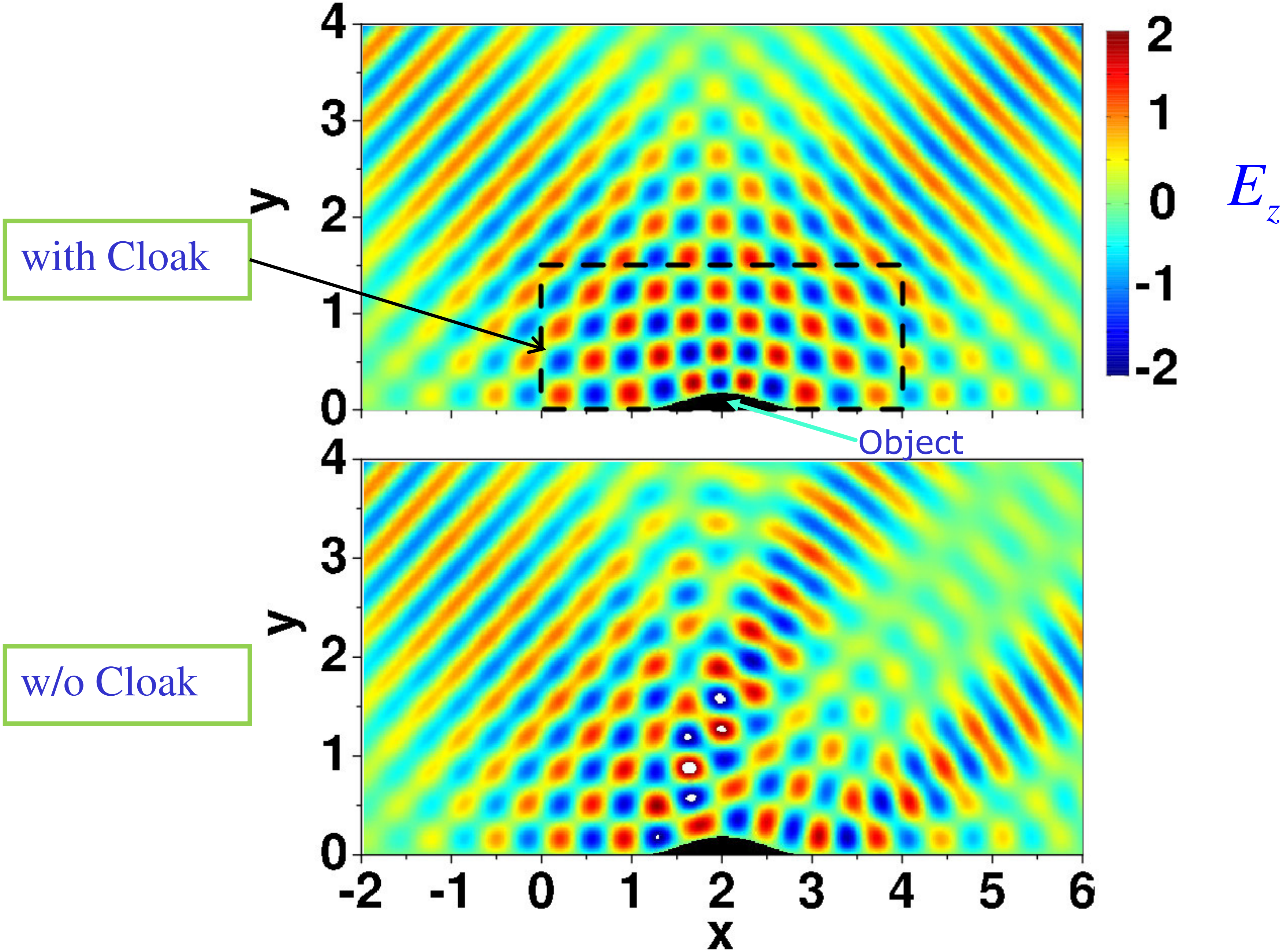
- $n_T/n_L = \alpha = 1.042$, regarded as 1, i.e. $\mu = 1$
- $\varepsilon = n_T\, n_L = 0.7$ to 2.0, relative to background
- $\varepsilon =$ 1.5 to 4.4 if SiO_2 is the background
  - Cloak can be obtained by drilling holes in Si



$$y_b(x) = \begin{cases} 0.2\cos(\pi x/2)^2 & 1 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

Cloak at oblique incidence angle

- Split into two separate beams without cloaking



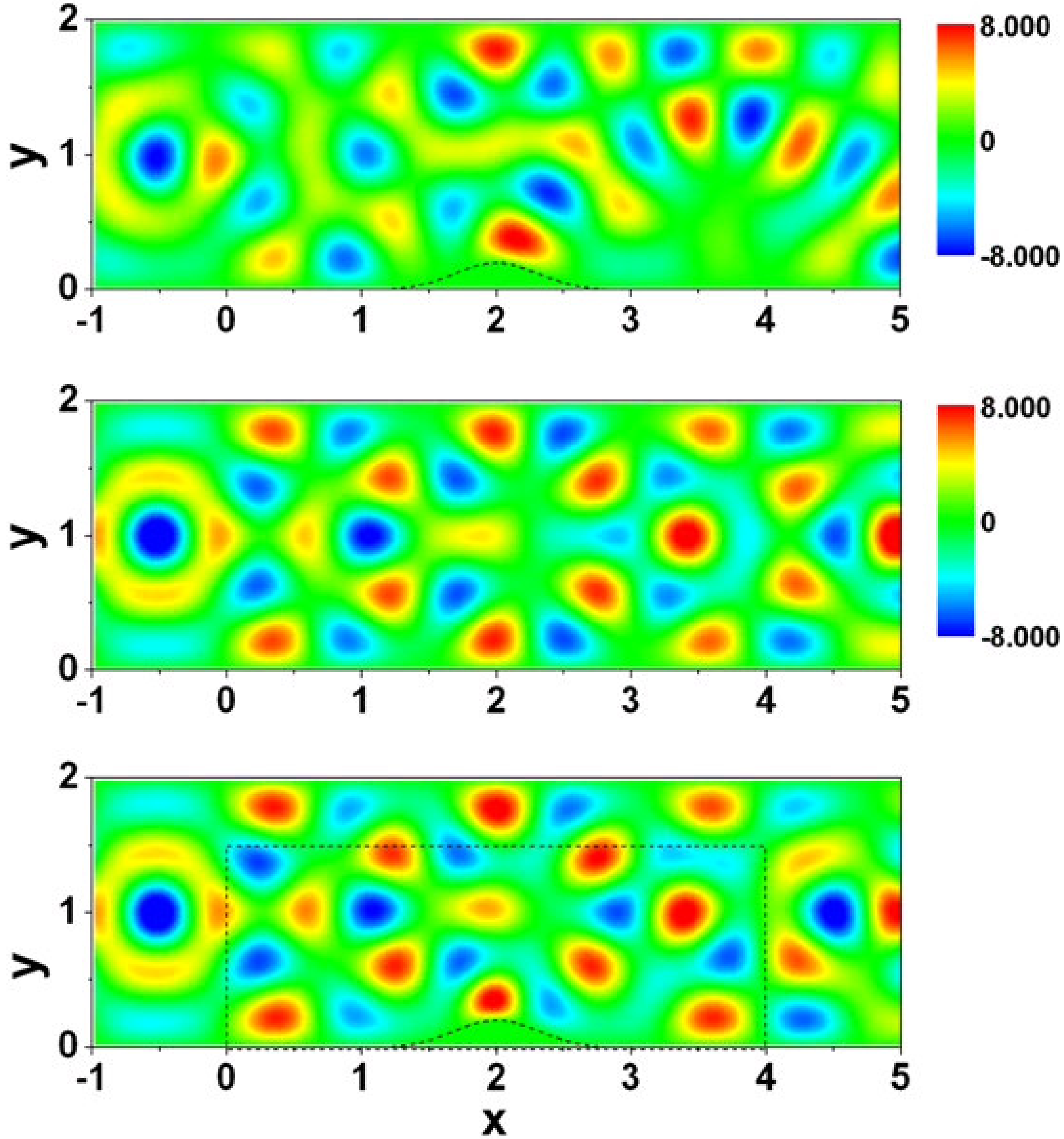
An extreme test on the cloak

- Within a hollow waveguide ($\lambda = 0.7$)
  - Near field pattern recovered (point source on the left)
  - Squeezed upwards and without reflection

Bump only

Flat space

Bump with Cloak

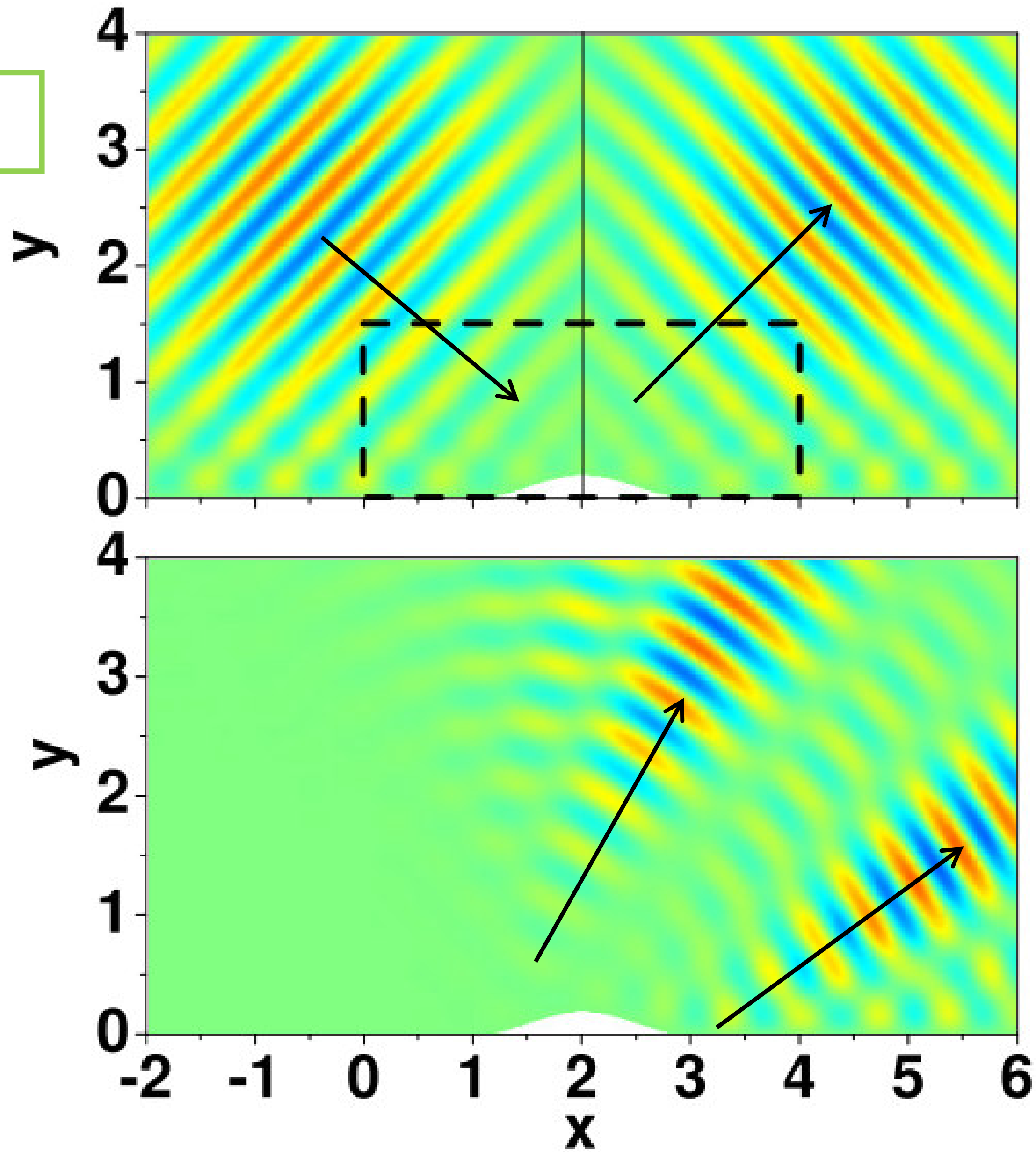


E field pattern

Broadband Cloaking

- Small range of dielectric constant & little frequency dispersion
- Reflected Gaussian wave packet undistorted

initial packet



after some time

with cloak

w/o cloak

Take Home Messages

- An invisibility carpet compresses object to a thin plate
- Quasi-conformal map to minimize anisotropy
- Dielectric fabrication is possible thru isotropic approximation
- Broadband Cloaking is possible

Manuscript available at arXiv:0806.4396