Test models for filtering of moisture-coupled tropical waves[1]

John Harlim[2]

Collaborator: Andrew J. Majda[3] (CIMS)

Department of Mathematics
North Carolina State University

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- When Gaussianity and linearity are assumed, one obtains the famous linear Kalman filter.
- For high dimensional nonlinear dynamics with multiscale structure, the computational burden is in the **forecast** step.

Why the tropical waves?

- A recent article in the bulletin of the World Meteorological Organization [Moncrieff et al 2007] reported that the difficulties in improving weather and climate predictions from days to years is essentially due to the limited representation of the tropical convection and its multiscale organization in the contemporary convection parameterization.

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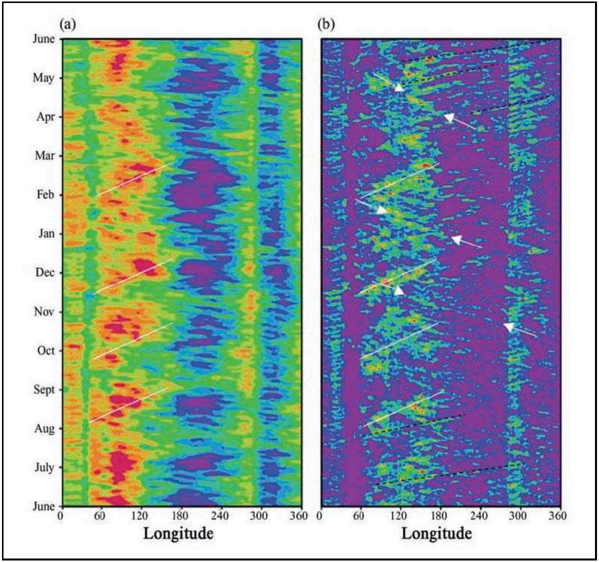
- We have a relatively accurate prediction for midlatitude weather dynamics, what so difficult about the tropics?

midlatitude	tropics
geostrophic balance	coriolis vanishes, convection
Rossby and inertio-gravity waves	Kelvin, MRG, ER, IG waves
denser measurement	sparse velocity measurements

Tropical waves multiscale structure

Period of 2000-2001 [Zhang 2005]



Longitude-time plots of daily (a) zonal wind (m/s) at roughly 1.5 km above the sea level from the NCEP/NCAR reanalysis and (b) precipitation (mm/day) from the GCPC combined data set.

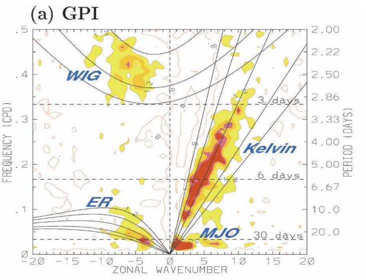
The MJO (Madden-Julian Oscillation) signal propagates eastward with phase speed of roughly 5m/s with periods of about 1 month (white solid line). The convectively coupled Kelvin waves (black dashes) which propagates eastward (15m/s) with shorter periods (5-10 days) and westward propagating (white arrows) waves (with periods less than 5 days) that are associated with Rossby or mixed-Rossby gravity waves.

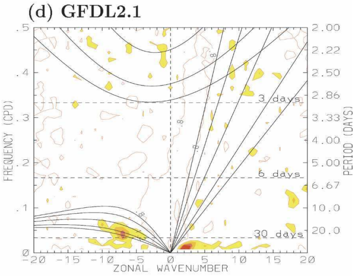
Wheeler-Kiladis space-time spectra

Multi-scale clouds and waves in the tropics

Observations

General Circulation Model (GCM)





from Lin et al. (2006)

MJO & CCEWs are **not** *currently part of GCM's intrinsic variability*

$\longrightarrow$ Implications for MJO prediction

Our contribution in term of data assimilation

- Instead of focusing on developing the analysis step (e.g., various EnKFs, variational based approaches, etc), we study the effect of model errors in the prior forecasting step in the filtering scheme.

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- In particular, we ask whether we can obtain accurate filtering skill by committing judicious model errors with a surrogate prior statistics $\tilde{P}$. We approximate the following filtering problem

$$P(u|v) \propto P(u)P(v|u)$$

with

$$P(u, \lambda|v) \propto \tilde{P}(u, \lambda)P(v|u, \lambda).$$

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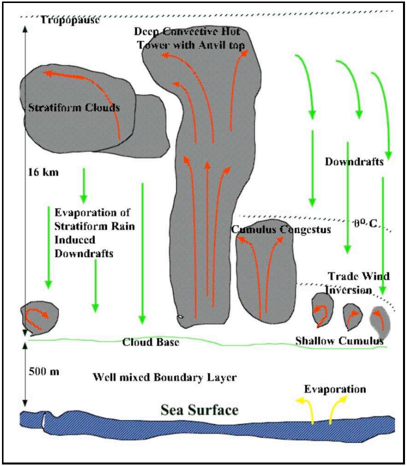
$$P(u,\lambda|v) \propto \tilde{P}(u,\lambda)P(v|u,\lambda).$$

- The main questions are: How to choose $\tilde{P}$? How to parameterize λ? How do you justify your choice of $\tilde{P}$ are optimal? We need a reasonable test model for the tropical atmospheric dynamics, need to choose $\tilde{P}$ that is data-driven.

Three-cloud model [Khouider and Majda 2007, 2008]



Three-cloud mechamism: (1) lower troposphere moistening through cumulus clouds triggers the (2) deep convection heating, and finally (3) trailing decks of stratiform precipitation.

The three-cloud mechanism has been observed on the eastward propagating convectively coupled Kelvin waves [Wheeler and Kiladis 2005], on the westward two-days waves [Haertl and Kiladis 2004], and on the MJO [Kiladis 2005, Dunkerton and Crum 1995].

Basic equations [KM 2007, 2008]

Two SWE obtained by a Galerkin projection of hydrostatic primitive eqn with constrant buoyancy frequency onto the first two baroclinic modes:

$$
\begin{aligned}
\frac{\partial \mathbf{v}_j}{\partial t} + \bar{\mathbf{U}} \cdot \nabla \mathbf{v}_j + \beta y \mathbf{v}_j^{\perp} - \theta_j &= -C_d u_0 \mathbf{v}_j - \frac{1}{\tau_w} \mathbf{v}_j, \quad j = 1, 2 \\
\frac{\partial \theta_1}{\partial t} + \bar{\mathbf{U}} \cdot \nabla \theta_1 - \mathrm{div}\ \mathbf{v}_1 &= P + S_1, \\
\frac{\partial \theta_2}{\partial t} + \bar{\mathbf{U}} \cdot \nabla \theta_2 - \frac{1}{4} \mathrm{div}\ \mathbf{v}_2 &= -H_s + H_c + S_2.
\end{aligned}
$$

Here, the total velocity and potential temperature is given by

$$
\begin{aligned}
\mathbf{V} &\approx \bar{\mathbf{U}} + G(z)\mathbf{v}_1 + G(2z)\mathbf{v}_2, \\
w &\approx -\frac{H_T}{\pi}\Big[G'(z)\mathrm{div}\ \mathbf{v}_1 + \frac{1}{2} G'(2z)\mathrm{div}\ \mathbf{v}_2\Big], \\
\Theta &\approx z + G'(z)\theta_1 + 2G'(2z)\theta_2
\end{aligned}
$$

where $G(z) = \sqrt{2}\cos(\pi z/H_T)$ and $G'(z) = \sqrt{2}\sin(\pi z/H_T)$.

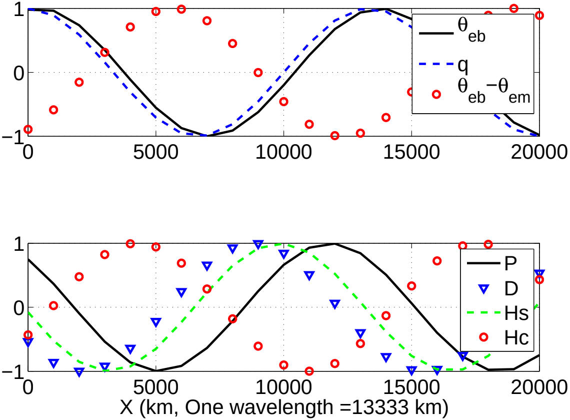
Convective parameterization [KM 2007, 2008]

The 2-layer SWE is coupled with

$$
\begin{aligned}
\frac{\partial \theta_{eb}}{\partial t} &= \frac{1}{h_b}(E-D) = \frac{1}{\tau_e}(\theta_{eb}^* - \theta_{eb}) - \frac{1}{h_b} D, \\
\frac{\partial q}{\partial t} &+ \bar{\mathbf{U}} \cdot \nabla q + \mathrm{div}\,(\mathbf{v}_1 q + \tilde{\alpha} \mathbf{v}_2 q) + \tilde{Q} \mathrm{div}\,(\mathbf{v}_1 + \tilde{\lambda} \mathbf{v}_2) = -\frac{2\sqrt{2}}{\pi} P + \frac{1}{H_T} D. \\
\frac{\partial H_s}{\partial t} &= \frac{1}{\tau_s}(\alpha_s P - H_s) \\
\frac{\partial H_c}{\partial t} &= \frac{1}{\tau_c}\left(\alpha_c \frac{\Lambda - \Lambda^*}{1-\Lambda^*} \frac{D}{H_T} - H_c\right), \\
P &= \frac{1-\Lambda}{1-\Lambda^*} P_0 = \frac{1-\Lambda}{1-\Lambda^*} \frac{1}{\tau_{conv}} \left[a_1 \theta_{eb} + a_2 (q - \hat{q}) - a_0(\theta_1 + \gamma_2 \theta_2)\right]^+ \\
D &= \Lambda D_0 = \Lambda \frac{m_0}{\bar{P}} \left[\bar{P} + \mu_2 (H_s - H_c)\right]^+ (\theta_{eb} - \theta_{em}) \\
\Lambda &= \begin{cases} 1 & \text{if } \theta_{eb} - \theta_{em} > \theta^+ \\ A(\theta_{eb} - \theta_{em}) + B & \text{if } \theta^- \le \theta_{eb} - \theta_{em} \le \theta^+ \\ \Lambda^* & \text{if } \theta_{eb} - \theta_{em} < \theta^- . \end{cases} \\
\theta_{em} &\approx q + \frac{2\sqrt{2}}{\pi}(\theta_1 + \alpha_2 \theta_2).
\end{aligned}
$$

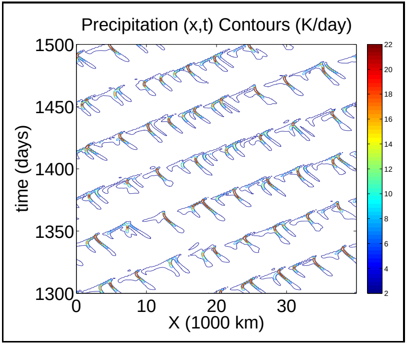
Key point: there is a parameter Λ that nonlinearly switches the dynamics to moistening the lower troposphere and inhibit the deep convection heating episodes.

Simple multicloud model with MJO-like behavior:



An MJO-analogue parameter regime

[Majda-Stechmann-Khouider 2007]

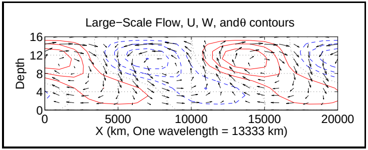


5m/s phase speed westerly low frequency planetary scale envelope with intermittent mesoscales westward propagating deep convection event

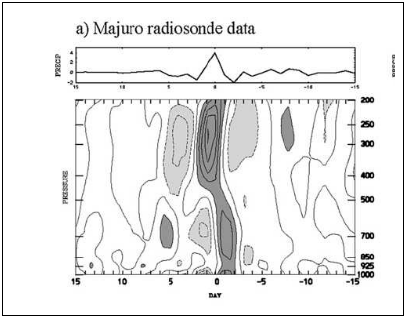
An MJO-analogue parameter regime

[Majda-Stechmann-Khouider 2007] Velocity

fields and heating contour.



Vertical tilting profile from [Straub et al. 2010]



How to choose $\tilde{P}(u, \lambda)$ and how to parameterize λ?

Given a dynamical system,

$$\frac{du}{dt} = Fu + \mathcal{N}(u),$$

on a periodic domain (for simplicity). Following turbulent closure approaches [DelSole 2004, Majda and Timofeyev 2004, etc], we consider the following approximation:

$$\mathcal{N}(u) \to \sum_{k=0}^{J-1} \left(-\lambda_k u_k + \sigma_k \dot{W}_k \right) e^{2\pi i k j / J},$$

then parameterizes λ_k, σ_k by fitting to the exact equilibrium statistical solutions of the linear reduced stochastic models, such as to the energy spectrum and correlation time [see MH2012, Ch 12].

Mean Stochastic Model for the three-cloud model

In our case, $\mathbf{\Psi} = (u_1, u_2, \theta_1, \theta_2, \theta_{eb}, q, H_s, H_c)^T \in \mathbb{R}^8$. Consider the linearized multicloud model about the RCE [KM2007]:

$$\frac{\partial \mathbf{\Psi}'}{\partial t} = \mathcal{P}(\partial_x)\mathbf{\Psi}',$$

where $\mathbf{\Psi}'$ denotes the perturbation field about the RCE and $\mathcal{P}$ denotes the linearized differential operator of the multicloud model at RCE.

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Consider a spatial discretization about $\Delta x = 2000$km such that

$$\frac{d\hat{\boldsymbol{\Psi}}_k}{dt} = \mathrm{i}\omega(k)\hat{\boldsymbol{\Psi}}_k, \quad |k| \leq M/2 = 10.$$

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Consider eigenvalue decomposition $\mathrm{i}\omega(k)\mathbf{Z}_k = \mathbf{Z}_k\mathbf{\Lambda}_k$ such that the MSM is given by the following diagonal 8×8 system of SDEs:

$$d\hat{\mathbf{\Phi}}_k = \Big[(-\mathbf{\Gamma}_k + \mathrm{i}\mathbf{\Omega}_k)\hat{\mathbf{\Phi}}_k + \mathbf{f}_k\Big]dt + \mathbf{\Sigma}_k dW_k,$$

where $\hat{\mathbf{\Phi}}_k = \mathbf{Z}_k^{-1}\hat{\mathbf{\Psi}}_k$.

Parameterization for the MSM

We'd like to parameterize

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Dry and Cold model: To mimic the approach in [Zagar 2004], we use only fitting the model to $(u_1, u_2, \theta_1, \theta_2, \theta_{eb})$. Mathematically, we replace the transformation in

$$\hat{\mathbf{\Phi}}_k^{dc} = \mathbf{Z}_k^{-1}\begin{bmatrix}\mathbf{I}_{5\times 5} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\hat{\mathbf{\Psi}}_k.$$

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Moist and Cold model: We also consider fitting the model to $(u_1, u_2, \theta_1, \theta_2, \theta_{eb}, q)$.

Observation networks

We define our observation model as follows:

$$\mathbf{G\Psi}^{o}_{j,m} = \mathbf{G\Psi}_{j,m} + \mathbf{Gs}_{j,m}, \quad \sigma_{j,m} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}^{o}),$$

at every $x_j = 2000$km and consider various observation networks:

- Surface Observations (SO): wind, potential temperature at surface height $z_s = 100m$ and θ_{eb}.
- Surface Observations + Middle Troposphere Temperature (SO+MT): add temperature at 8km.
- Surface Observations + Middle Troposphere Temperature & Velocity (SO+MTV): add velocity at 8km.
- Complete Observations (CO).

Fourier Domain Kalman Filter

Our discrete-time Kalman filtering problem with MSM as the prior model is defined on each horizontal wavenumber k:

$$
\begin{aligned}
\hat{\mathbf{\Psi}}_{k,m} &= \mathcal{F}_k(\Delta t)\hat{\mathbf{\Psi}}_{k,m-1} + \mathbf{g}_{k,m} + \eta_{k,m}, \\
\mathbf{G}\hat{\mathbf{\Psi}}^o_{k,m} &= \mathbf{G}\hat{\mathbf{\Psi}}_{k,m} + \mathbf{G}\hat{\sigma}_{k,m},
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{\sigma}_{k,m} &\sim \mathcal{N}(\mathbf{0}, \mathbf{R}^o/M) \\
\mathcal{F}_k(t) &= \mathbf{Z}_k \exp\Big((-\mathbf{\Gamma}_k + \mathrm{i}\mathbf{\Omega}_k)t\Big)\mathbf{Z}_k^{-1}, \\
\mathbf{g}_{k,m} &= -(\mathbf{I} - \mathcal{F}_k(t_m))(-\mathbf{\Gamma}_k + \mathrm{i}\mathbf{\Omega}_k)^{-1}\mathbf{f}_k, \\
\mathbf{Q}_k &= \frac{1}{2}\mathbf{Z}_k\mathbf{\Sigma}_k^2\mathbf{\Gamma}_k^{-1}(\mathbf{I} - |\mathcal{F}_k(\Delta t)|^2)\mathbf{Z}_k^*.
\end{aligned}
$$

Fourier Domain Kalman Filter

Applying the standard Kalman filter formula, we obtain recursive formula for the the prior statistics

$$\text{Predictor:} \quad \begin{cases} \hat{\mathbf{\Psi}}^{b}_{k,m} = \mathcal{F}_k(\Delta t)\hat{\mathbf{\Psi}}^{a}_{k,m-1} + \mathbf{g}_{k,m} \\ \mathbf{R}^{b}_{k,m} = \mathcal{F}_k(\Delta t)\mathbf{R}^{a}_{k,m-1}\mathcal{F}_k(\Delta t)^{*} + \mathbf{Q}_k, \end{cases}$$

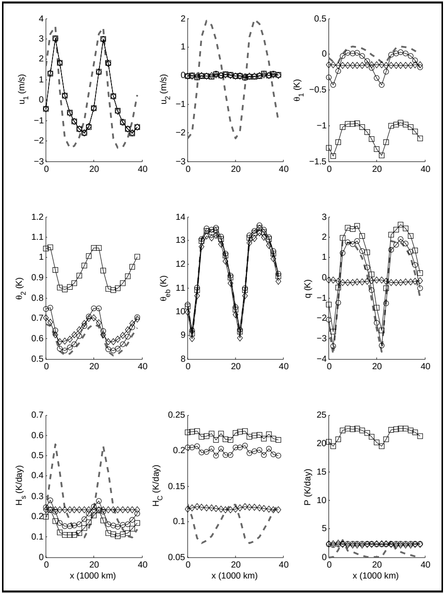
and the posterior statistics

$$\text{Corrector:} \quad \begin{cases} \hat{\mathbf{\Psi}}^{a}_{k,m} = \hat{\mathbf{\Psi}}^{b}_{k,m} + \mathbf{K}_{k.m}(\mathbf{G}\hat{\mathbf{\Psi}}^{o}_{k,m} - \mathbf{G}\hat{\mathbf{\Psi}}^{b}_{k,m}) \\ \mathbf{R}^{a}_{k,m} = (\mathbf{I} - \mathbf{K}_{k.m}\mathbf{G})\mathbf{R}^{b}_{k,m}, \\ \mathbf{K}_{k.m} = \mathbf{R}^{b}_{k,m}\mathbf{G}^{*}(\mathbf{G}(\mathbf{R}^{b}_{k,m} + \mathbf{R}^{o}/M)\mathbf{G}^{*})^{-1}. \end{cases}$$

We also consider a version of 3D-VAR by simply setting the prior error covariance statistics to be independent of time

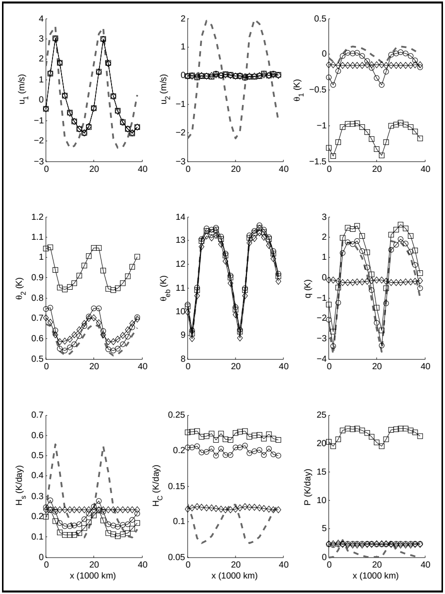
$$\mathbf{B}_k \equiv \lim_{\Delta t \to \infty} \mathbf{R}^{b}_{k,m} = \frac{1}{2}\mathbf{Z}_k\mathbf{\Sigma}^{2}_{k}\mathbf{\Gamma}^{-1}_{k}\mathbf{Z}^{*}_{k}.$$

Observation networks: Surface Observations



Unrealistic estimates for precipitation with MSM-filter (squares), the complete 3D-VAR (circles), "dry and cold" 3D-VAR (diamonds)!

Observation networks: Surface Observations



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Precipitation budget: $P_0 = \frac{1}{\tau_{conv}}\left[a_1\theta_{eb} + a_2(q - \hat{q}) - a_0(\theta_1 + \gamma_2\theta_2)\right]^+$

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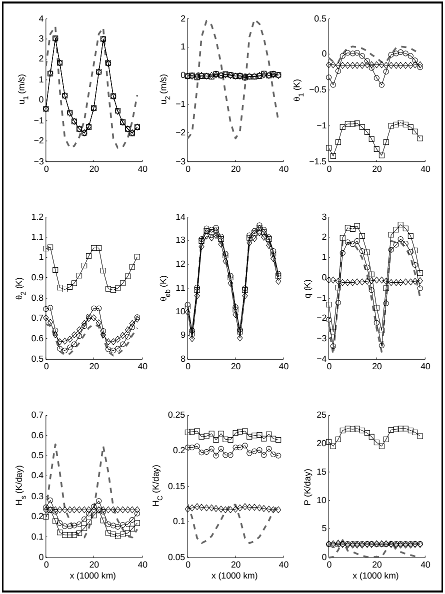

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$a_1 = 0.1, a_2 = .5, a_0, \gamma_2 = 1.2$, but $a_0 = 12$!

Observation networks: Surface Observations + Midtroposphere Temperature & Velocity

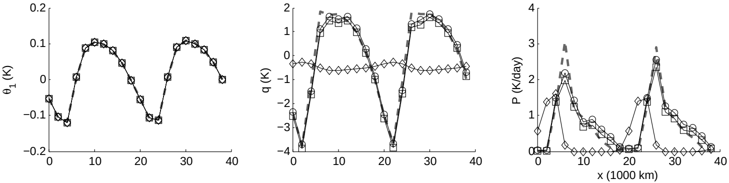
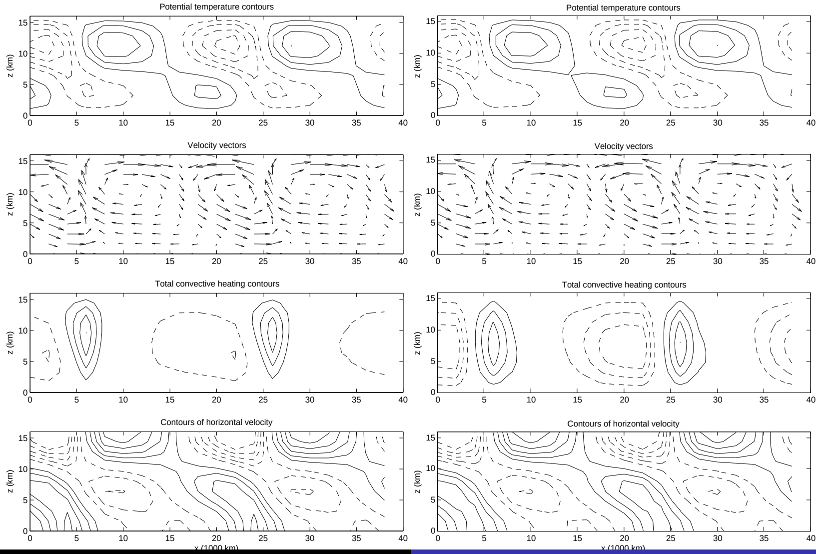


Figure: Moving average of the multi-cloud model variables with their vertical structure reconstructed from observing only wind and temperature spatially at every 2000km and 24h. True (grey dashes), posterior mean estimates from the 3D-VAR with moist background covariance matrix (circles), the MSM-Filter (squares), and the 3D-VAR with dry background covariance matrix (diamonds).

True (left) vs Observation networks: Surface Observations + Midtroposphere Temperature & Velocity (right)



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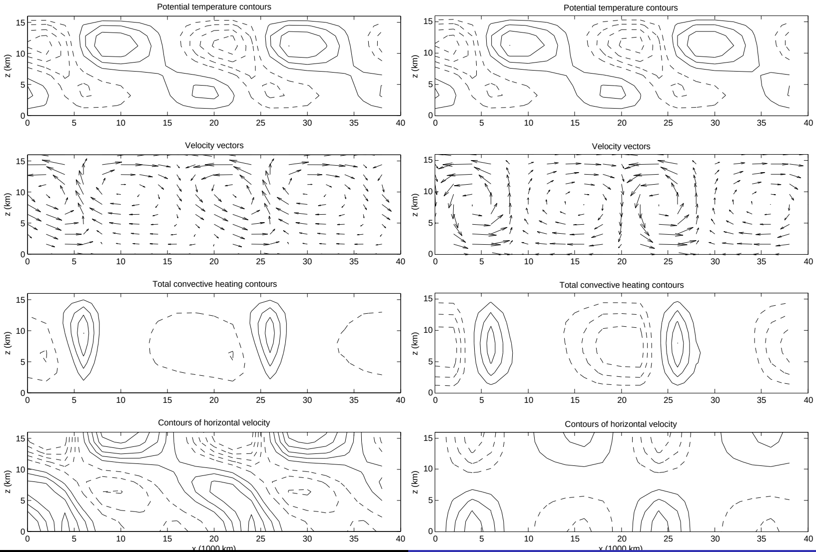
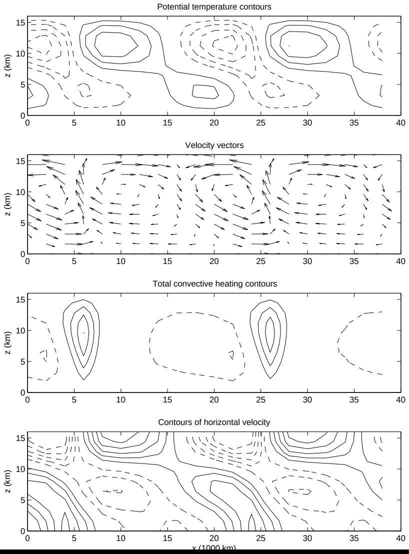
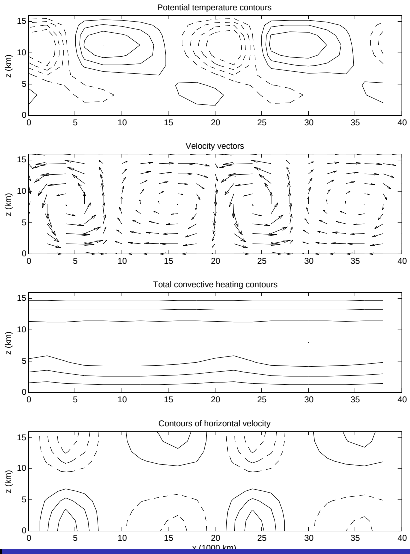
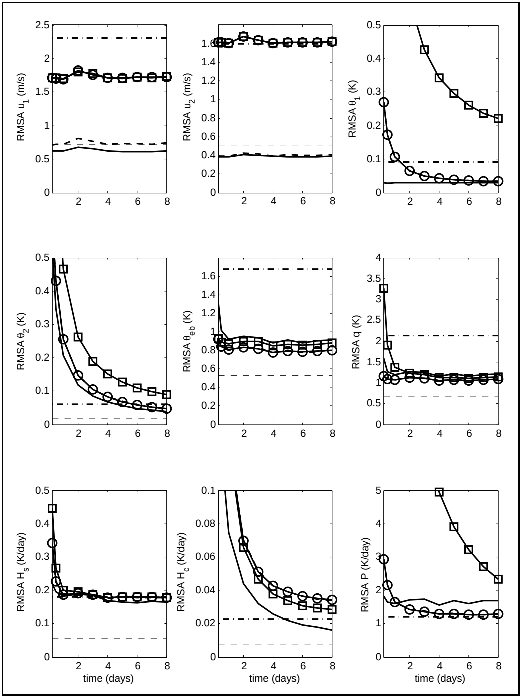


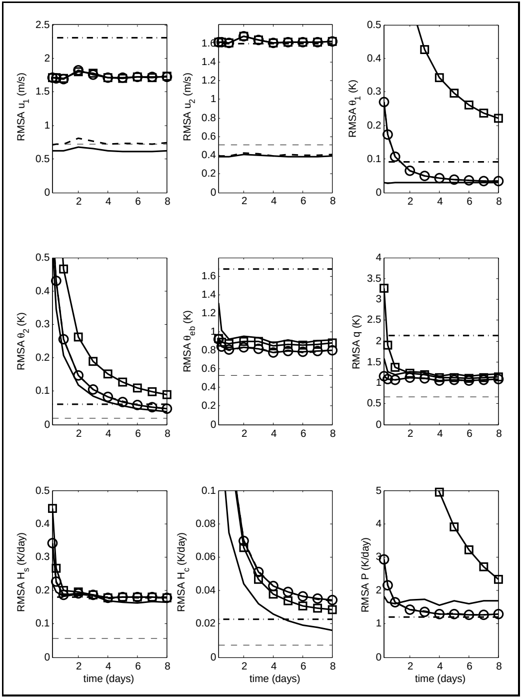
True (left) vs Observation networks: Surface Observations (right)







Observation error (thin dashes) about 10% of the climatological errors (dash-dotted line): CO (thick solid line), SO+MTV (thick dashes), SO+MT (circles) and SO(squares).



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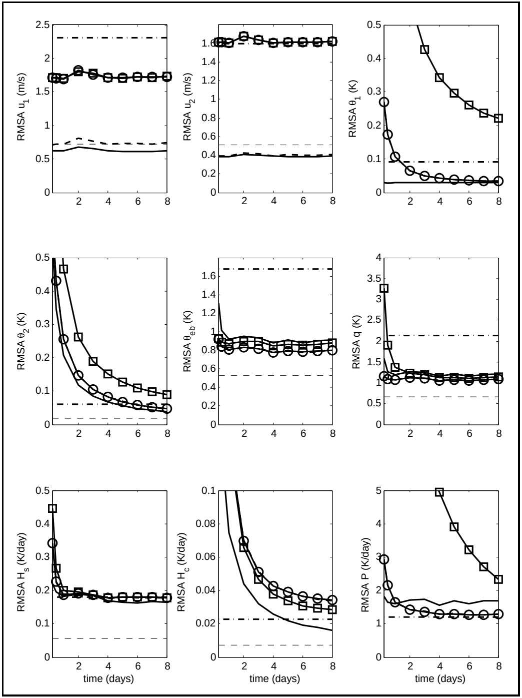
The filter except for SO is skillful (better than climatological errors) for $u_1, \theta_1, \theta_{eb}, q$



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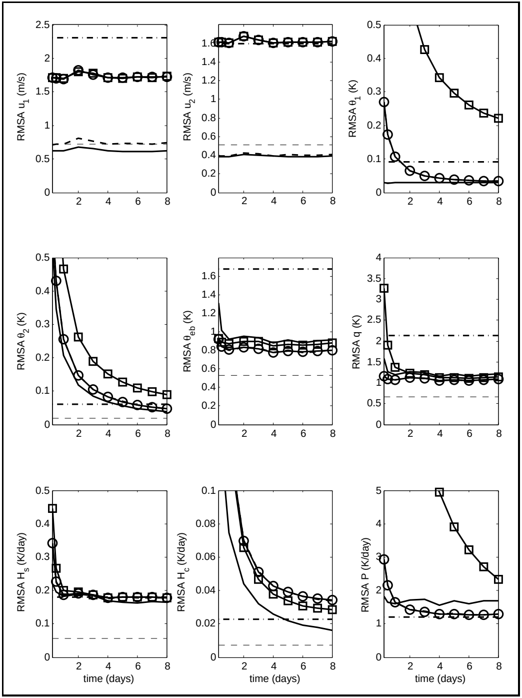

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For SO+MT: When $\Delta t = 6$h, $\lambda_1(\mathcal{F}_k) = 0.9899$ and when $\Delta t = 72$h, $\lambda_1(\mathcal{F}_k) = 0.8836$. The shorter time is marginally stable! In this case, the observability condition that is necessary for filter stability [Anderson and Moore 1979, MH 2012] is practically violated here! In this case, the observability matrix is ill-conditioned, $\det\left([\mathbf{G}^T\,(\mathbf{G}\mathcal{F}_k)^T]\right) \approx 10^{-20}$.

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- A better estimate for the tropical convection wave patterns requires more than surface wind and potential temperature observations.
- The skill of the reduced filtering methods with horizontally and vertically sparse observations suggests that more accurate filtered solutions are achieved with less frequent observation times. Such a counterintuitive finding is justified through an analysis of the classical observability and controllability conditions which are necessary for optimal filtering especially when the observation timescale is too short relative to the timescale of the true signal.