Mass Conservation and Positivity Preservation with Ensemble-type Kalman Filter Algorithms

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Maintaining conservative physical laws numerically has long been recognized as being important in the development of numerical weather prediction (NWP) models. Numerical conservation of the total mass of air, water in its different phases, chemical species, etc., including sign (positivity) preservation in each grid volume, is sometimes taken to be the first of a number of fundamental design principles for NWP models. In the broader context of data assimilation, concerted efforts to maintain conservation laws numerically and to understand the significance of doing so have begun only recently.

In this talk we consider the use of equality and inequality constraints in ensemble-based Kalman filter algorithms to enforce mass conservation and positivity preservation. We show that the analysis steps of ensemble transform Kalman filter (ETKF) algorithm and ensemble Kalman filter algorithm (EnKF) can conserve the mass integral, but do not preserve positivity. Further, if localization is applied or if negative values are simply set to zero, then the total mass is not conserved either. In order to ensure mass conservation, a projection matrix that corrects for localization effects is constructed. In order to maintain both mass conservation and positivity preservation through the analysis step, we construct two data assimilation algorithms based on quadratic programming and ensemble Kalman filtering.

We examine the extent to which imposing mass conservation and positivity preservation in data assimilation changes the assimilation results and we demonstrate the benefits of imposing these constraints in two simple experimental setups: a solid body rotation experiment and idealized problems of environmental pollution. The results show clear improvements in both analyses and forecasts, particularly in the presence of localized features.

References

[1] Janjic, T., D. B. McLaughlin, S. E. Cohn: "Preservation of physical properties with ensemble based Kalman filter algorithms", ***Mathematical and Algorithmic Aspects of Atmosphere-Ocean Data Assimilation, Mathematisches Forschungsinstitut Oberwolfach*****, Report No. 58/2012, 17–20, doi:10.4171/OWR/2012/58.**

[2] T. Janjic, D. B. McLaughlin, S. E. Cohn: "Conservation of mass and preservation of positivity with ensemble type Kalman filter algorithms", submitted to ***Monthly Weather Review*****.**