Investigation of Model Covariance with Low-Order Dynamics

John M. Lewis [a] , S. Lakshmivarahan [b] , and William Bourke [c]

[a] National Severe Storms Laboratory, Norman, OK, 73072, and Desert Research Institute, Reno, NV, 89512, jlewis@dri.edu; [b] School of Computer Science, University of Oklahoma, Norman, OK, 73072; [c] Middle Park, Vic 3206, Australia.

Knowledge of a dynamical model's covariance is a critically important element of Bayesian data assimilation as practiced at operational weather prediction centers worldwide. This covariance is controlled by uncertainty in initial conditions and forcing as well as the effect of overall model formulation deficiencies. When these uncertainties are combined with covariance of observations, the optimal *a posteriori* state of the system can be determined. This estimated state improves the link between existing ensembles of data assimilation and assessment of alternative methods for creating initial conditions [1].

There are three ways to obtain a model's covariance: (1) stochastic – dynamic prediction (SDP) that delivers the moments of the probability density function (pdf), (2) Monte Carlo methods that generate the covariance through a series of forecasts stemming from a random body of initial states and random perturbations to the parameterization schemes, and (3) solution to Liouville's equation that yields the exact pdf and therefore the exact covariance through a "continuity of probability" principle.

The determination of a model's covariance is explored through use of Platzman's low-order spectral model of nonlinear advection [2] where uncertainty in initial conditions and forcing is considered. The consequence of inexact model covariance on optimal sequential Bayesian assimilation becomes the focus of the investigation. For the 2-mode system with uncertainty in initial conditions alone, an analytic solution to the governing dynamics can be found and used to solve Liouville's equation by the method of characteristics. Higher-mode systems are considered for exploration of covariance error due to both initial conditions and forcing. In the absence of an analytic solution for this case, Liouville's equation is solved by numerical methods.

[1] European Centre for Medium Range Weather Forecasts (ECMWF), Newsletter No. 134 – Winter 2012/13 .

[2] Platzman, G. W., 1964: An exact integral of complete spectral equations for unsteady one-dimensional flow. *Tellus*, **16**, 422 – 431.