

## Second and Third order Adjoint Methods for Sensitivity Analysis in Pollution Models

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Understanding the impact changes in pollutant emission from a foreign region on a target region of is a key factor for taking appropriate actions against the pollution and its drawbacks. This can be carried out by the sensitivity analysis of a response function with respect to the source of pollutant. The basic approach determines the sensitivity by carrying out multiple simulations with variation of source parameters. A systematic approach uses the first order adjoint formulation. Both approaches assume that the transport velocity and the initial distribution of the pollutant are known. However, they are given by the solution of a Data Assimilation problem whose ingredients include, but are not limited to, the pollutant source, the mathematical model and physical measurements. As a consequence, the sensitivity analysis should be carried out on the optimality system of the Data Assimilation problem. It leads to a non standard problem on a second order adjoint system whose the solution requires the third order adjoint.

We present the mathematical derivation of the third order adjoint method for the sensitivity analysis, along with some numerical experiments and a comparison with the first order adjoint approach.