If not GR, what?

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GR

$$S = \frac{-1}{16\pi G} \int d^4x \, \sqrt{-g}(R + 2\Lambda) + S_{\mathsf{matter}}[\mathsf{matter}, g_{ab}]$$

Why modified gravity?

Foil for testing GR

Puzzles of cosmology

(dark matter, inflaton origins and interactions, dark energy)

Hunt for residues of quantum gravity

(extra dimensions, LV from UV physics, non-locality…)

Singularities

Curiosity

How modified gravity?

Higher derivatives

$$f(R,\ \Box R,\ R_{ab}R^{ab},\ R_{abcd}R^{abcd},\ \ldots)$$

Non-local

$$f(\Box^{-1}R,\ \ldots)$$

Additional fields

scalar, vector, tensor

Et cetera

A priori theoretical Issues

New length scales in the modified action? Are they motivated?

Naturalness: are couplings allowed by the fields and symmetries included with their natural size?

Do the field equations admit initial value formulation?

Is the theory dynamically stable?

Is the energy of all excitations positive?

Phenomenology

Does the theory make well-defined predictions? (*cf. TeVeS and chiral gravity, e.g.*)

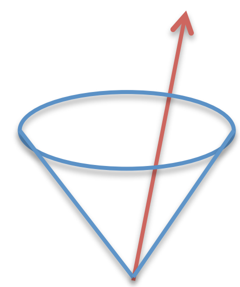
Do the predictions meet the many stringent tests: solar system dynamics, radiation damping, structure of compact bodies, cosmology (both homogeneous and perturbations), UHE cosmic rays, …?

Are there interesting predictions that might resolve puzzles like dark matter/energy, Inflaton field and dynamics, singularities, etc?

Are there other, new predictions that have not yet been tested?

“Einstein-aether theory”:

GR coupled to a unit timelike 4-vector



Will & Nordtvedt (1972)
Gasperini (1987)
TJ & Mattingly (2000)
Review: arXiv:0801.1547

General action with two derivatives:

$$S[g_{ab}, u^a, \lambda] = \frac{-1}{16\pi G}\int d^4x\,\sqrt{-g}[R + K_{ab}^{mn}\nabla_m u^a \nabla_n u^b + \lambda(g_{ab}u^a u^b - 1)]$$

$$K_{ab}^{mn} = c_1\, g_{ab}g^{mn} + c_2\,\delta_a^m\delta_b^n + c_3\,\delta_a^n\delta_b^m + c_4\, g_{ab}u^m u^n$$

Note: $\nabla_m u^n \sim \partial u + (\partial g)u$

Variations: more derivatives, functions of these scalars, field-dependent coefficients c_i

But working out consequences of this is hard enough… and I don’t want to introduce new length scale by hand…

Consequences:

Linearized wave modes

Newtonian limit and PPN parameters

Cosmology

Radiation

Equations of motion

Compact bodies and strong gravity

Stability
Cerenkov radiation
energy positivity

multiple speeds,
polarizations

$G_N = G/(1 - c_{14}/2)$

PPN same as GR except preferred frame parameters $\alpha_{1,2}$

$G_{cosmo} = G/(1 + (c_{13} + 3c_2)/2)$

Friedman eqn
Primordial fluctuations

Binary pulsars
Weak/strong self-field

preferred frame effects from strong fields

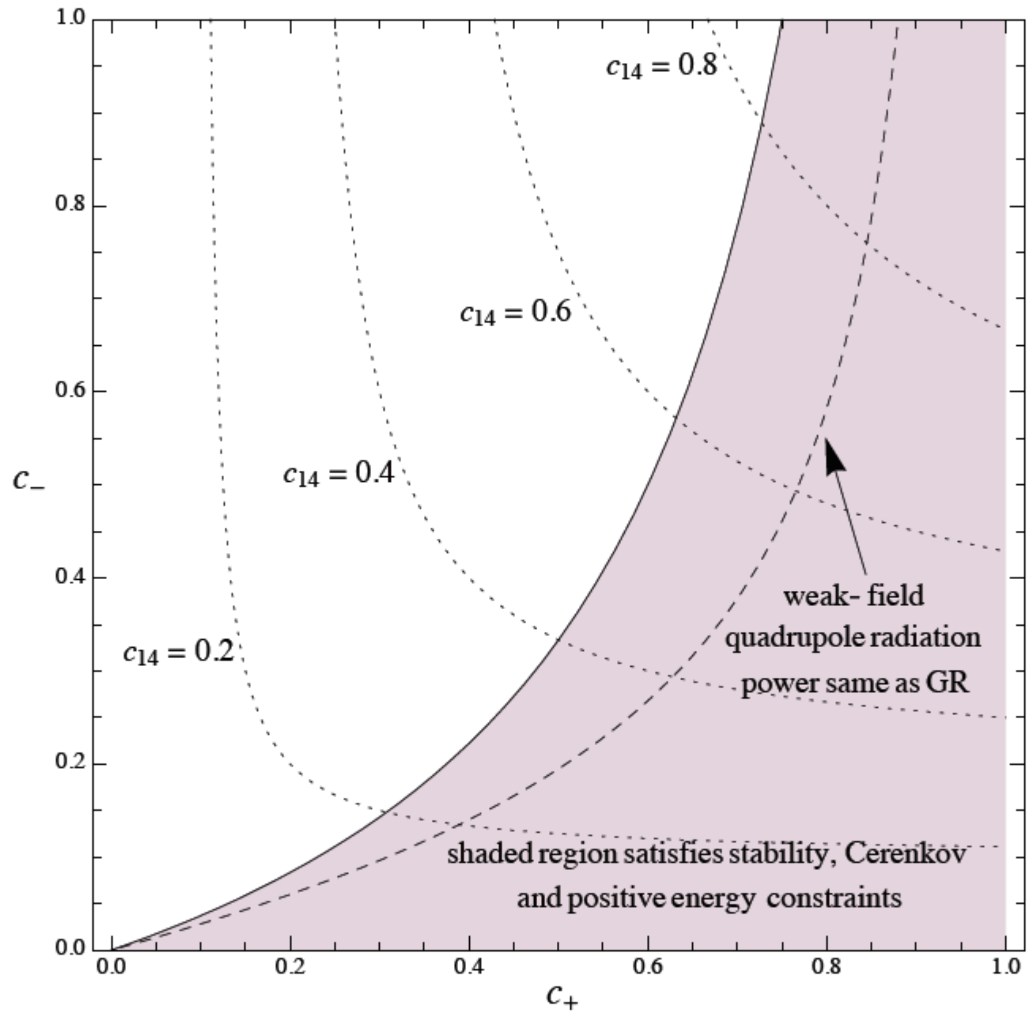
neutron stars
black holes

Preferred frame PPN parameters can be set to zero:

$$\alpha_1 = 0 \Rightarrow c_4 = -c_3^2 / c_1$$

$$\alpha_2 = 0 \Rightarrow c_2 = -(2c_1 + c_3 - c_3^2/c_1)/3 \qquad \text{(or } c_3 = c_4 = -c_1\text{)}$$

...leaves a (c_1, c_3) parameter space with all PPN parameters identical to those of GR!



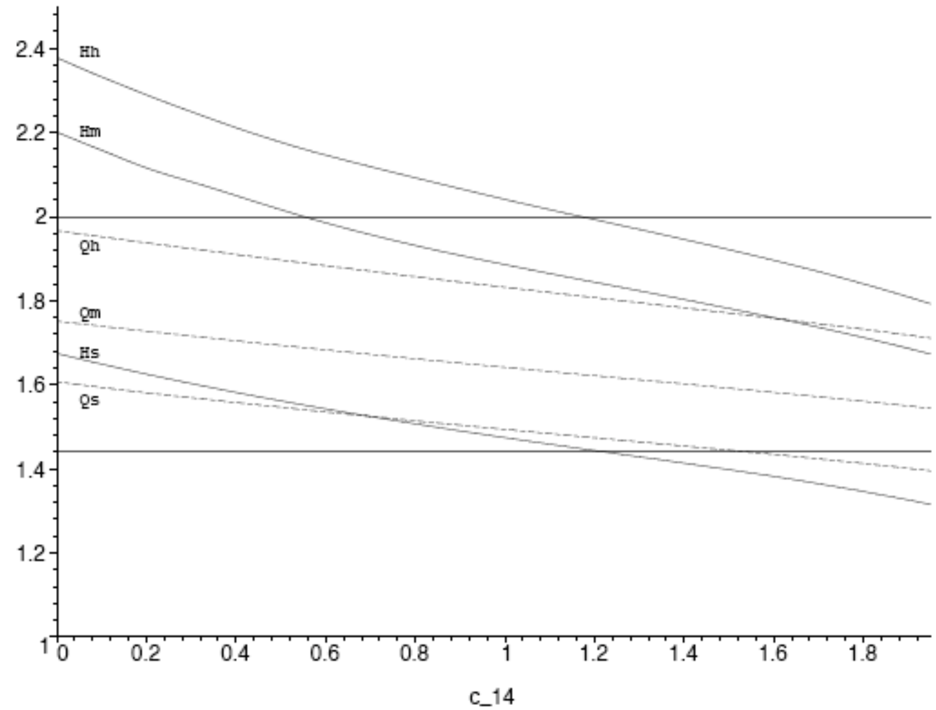
$$c_\pm = c_1 \pm c_3, \quad c_{2,4} \text{ chosen so that } \alpha_{1,2} = 0$$

Spherically symmetric static solutions

- Vacuum solution with Killing-parallel aether known analytically Eling & TJ (2006)
- Neutron star solutions found numerically for various equations of state Eling, TJ, Miller (2007)
- Stability established by analytic/numerical technique. Seifert (2007)
- Static black hole solutions found numerically. Eling & TJ (2006)
- Black holes formed by numerical collapse, stable. Garfinkle, Eling & TJ (2007)

Neutron stars

- ISCO (area) radius larger by (1 + 0.03 c_{14}), orbital frequency smaller by (1 - 0.04 c_{14})
- Surface redshifts as much as 10% larger for some EOS.
- Maximum mass ~ 6 – 15 % smaller, depending on equation of state, for c_{14} = 1



Black Holes

- Aether flows into black hole: different solution than outside star!
- Regularity at spin-0 horizon selects unique solution for each mass
- Solutions found both by radial integration of ODEs and by numerical collapse.
- Example with $c_3 = c_4 = 0$, and c_2 fixed so spin-0 mode speed is 1:
  - spin-0 horizon singular if $c_1 > 0.8$
  - ISCO radius is Schwarzschild times $(1 + 0.043\ c_1 + 0.061\ c_1^2 + \ldots)$
  - spacelike singularity at $r = 0$, oscillations as it is approached
- Examples for different values of c_i should be studied

Burning Questions

1. What is the strength of the spherical radiation emitted by a supernova?

2. What do non-rotating black hole solutions with other c_i look like?

3. What do the rotating black hole solutions look like?

4. What are the numerical values of the neutron star and black hole "velocity parameters" required to compute radiation damping and equation of motion corrections?

Radiation damping

(Foster, 2006,7)

Assume $\alpha_1 = \alpha_2 = 0$

• Monopole, dipole, and quadrupole radiation generally exists.

• *Weak self-gravity or* $c_i <\sim 0.01$: only quadrupole source significant, but radiation of spins 0,1,2. GR value implies one condition on c_1, c_3.

• *Strong self-gravity*: dipole radiation ~ (difference of "sensitivities")^2
Could be important for asymmetric binaries. Otherwise quadrupole and monopole dominate. Bounds not worked out accurately.

$$S = -m_0 \int d\tau \, [1 + \sigma(v^a u_a - 1) + \sigma'(v^a u_a - 1)^2 + \ldots]$$

NEED THE SENSITIVITY PARAMETERS!

Also needed for spherical supernova radiation and tests of Equations of motion (strong equivalence principle.)

$$E = m_0 + \tfrac{1}{2}(1+\sigma)m_0 v^2 + \tfrac{3}{8}(1+\sigma-\sigma')m_0 v^4 + \ldots .$$

$$\sigma = (\alpha_1 - \tfrac{2}{3}\alpha_2)(\Omega/m) + \mathscr{O}\Big(f[c_i](G_{\mathrm{N}} m/d)^2\Big)$$

Waves

Mattingly & TJ (2004)

5 "massless" modes	speed squared
spin-2: 2 gravitons	$\frac{1}{1-c_{13}}$
spin-1: 2 transverse aether-metric modes	$\frac{c_1+(c_3^2-c_1^2)/2}{c_{14}(1-c_{13})}$
spin-0: 1 longitudinal aether-metric mode	$\frac{c_{123}(2-c_{14})}{c_{14}(1-c_{13})(2+c_1+3c_2+c_3)}$

Polarization tensors…

Could be used to measure aether frame.

STABILITY* constraint: squared speeds > 0

CERENKOV constraint: squared speeds >1

Elliott, Moore & Stoica (2005)

*Carroll et al (2009) require stability for any mode normalized in any frame…

Wave Energy

Lim (2004), Eling (2005), Foster (2006)

Spin-2	Spin-1	Spin-0
+	$(2c_1 - c_1{}^2 + c_3{}^2)/(1-c_{13})$	$c_{14}(2- c_{14})$

Found using energy-momentum pseudotensors (Eling),
and using Noether current method (Foster).

POSITIVE ENERGY constraint: energy > 0

Polarizations

spin-2 $\quad h_{12}, \quad h_{11} = -h_{22}$

spin-1 $\quad v_I, \quad h_{3I} = \left[2c_{14}c_{13}^2/(2c_1 - c_1^2 + c_3^2)\right]^{1/2} v_I$

spin-0 $\quad v_0, \quad h_{00} = -2v_0, \quad h_{11} = h_{22} = -c_{14}v_0, \quad h_{33} = \left[2c_{14}(1+c_2)/c_{123}\right] v_0$

Given here in the gauge with metric perturbation h_0i=0 and
divergenceless vector perturbation v_i,i=0