Energy Extraction from Rotating Black Hole by Magnetic Reconnection

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Koide & Arai, ApJ 682, 1124-1133 (2008)

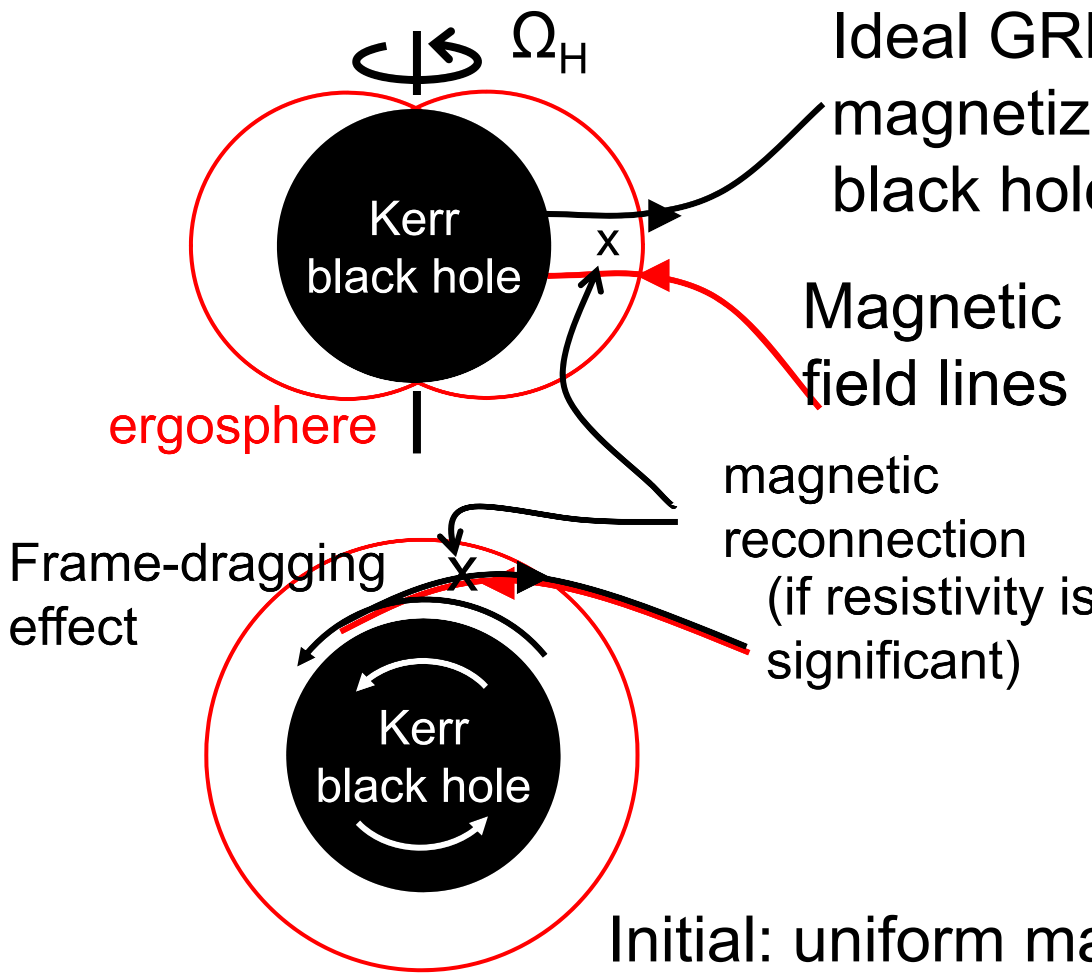
SCSAMM workshop 2009.8.28(Fri)

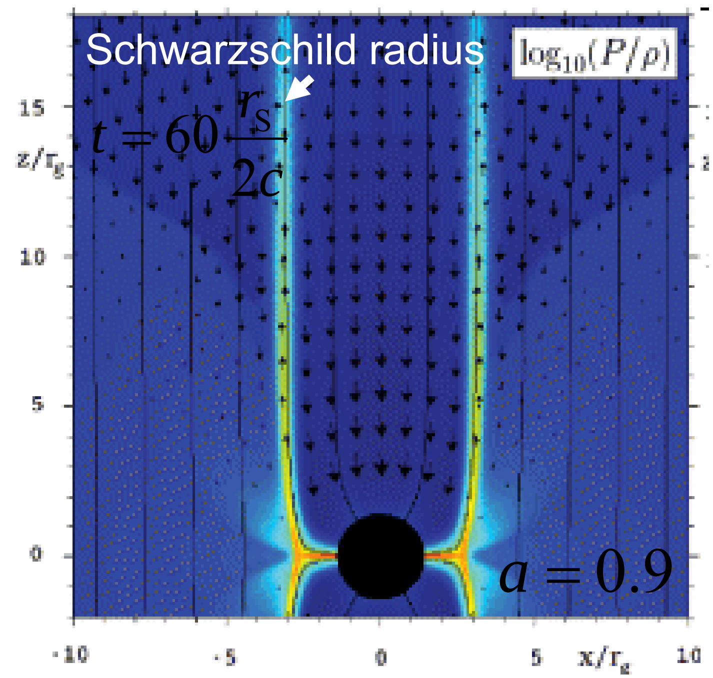
Outline

- Motivation
- Energy extraction mechanism from black hole: Penrose process
- A mechanism of energy extraction from black hole through magnetic reconnection
- Summary and future aspects

One of Motivation:
Magnetic reconnection in the ergosphere

Ideal GRMHD simulation of uniformly magnetized plasma around rotating black hole (Komissarov 2005)





Initial: uniform magnetic field (Wald solution)

⇒ sprit monopole magnetic field in ergosphere

⇒ magnetic reconnection in ergosphere

Notation of space-time and Ergosphere

Space-time around rotating black hole (Kerr metric)

$$\left(x^0, x^1, x^2, x^3\right) = \left(t, r, \theta, \phi\right) \qquad \text{metric} \quad ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Lapse function $\alpha = \sqrt{-g_{00} + \sum_{i=1}^{3} \frac{g_{i0}{}^2}{g_{ii}}}$ (gravitational red-shift, gravitational potential)

Angular velocity of frame-dragging $\omega_i = -\frac{g_{i0}}{g_{ii}}$

Shift vector $\beta^\phi = \frac{\sqrt{g_{\phi\phi}}\,\omega^\phi}{\alpha}$ (velocity of frame dragging)

ZAMO coordinates $\left(\hat{x}^0, \hat{x}^1, \hat{x}^2, \hat{x}^3\right)$ (local Minkowski frame)

(zero angular momentum observer)

Ergosphere: $\beta^\phi \geq 1$

Any matter, energy, and information never propagate against the rotation of black hole rotation in the ergosphere.

Unit system: $c = 1,\ \mu_0 = 1,\ \varepsilon_0 = 1$

Conservative quantity of a particle around rotating black hole

- Energy-at-infinity (total energy):

Rest mass energy + kinetic energy + gravitational potential

$$E^{\infty} = \alpha\hat{E} + \underbrace{\omega^{\phi} L^{\phi}}_{\text{"Frame-dragging potential"}} = \alpha\left(\hat{E} + \beta^{\phi} P^{\phi}\right)$$

Energy observed by ZAMO $\hat{E} = m\gamma$

Lorentz factor

- Angular momentum

$$L^{\phi} = \sqrt{g_{\phi\phi}}\,\hat{P}^{\phi}$$

Momentum $\hat{P}^{\phi} = m\gamma v^{\phi}$

When $L^{\phi} < -\hat{E} / \beta^{\phi}$ in ergosphere, the energy-at-infinity becomes negative, $E^{\infty} < 0$.

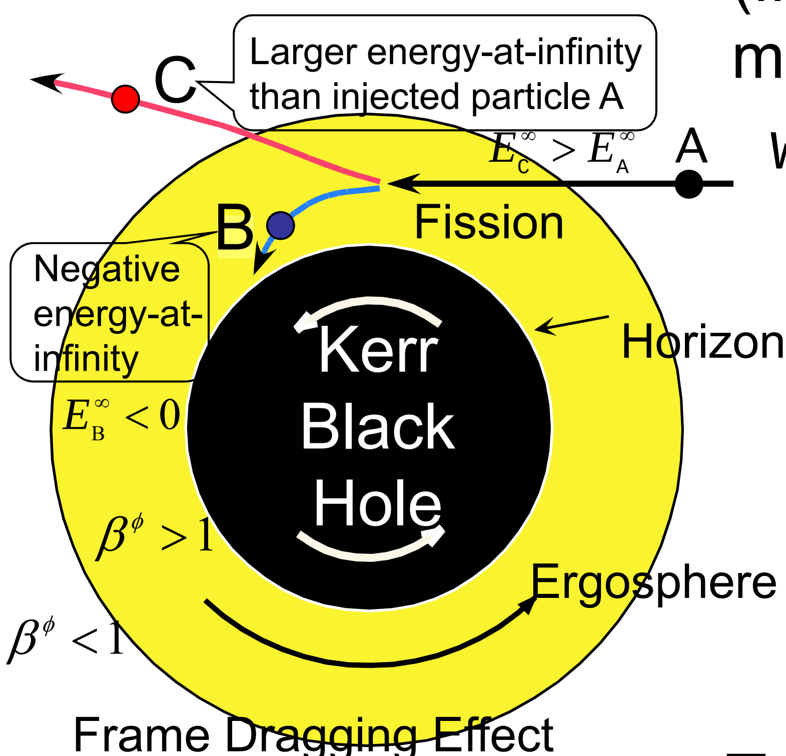
Penrose Process

Extraction of energy of rotating black hole through particle fission in ergosphere

Conservation of angular momentum $L_A^\infty = L_B^\infty + L_C^\infty$

Conservation of energy $E_A^\infty = E_B^\infty + E_C^\infty$

A→B+C



(fission ⇒ redistribution of angular momentum)

When $L_B^\phi < -\hat{E}_B / \beta^\phi$ in ergosphere, $E_B^\infty < 0$.

If particle B is swallowed by black hole, total mass of black hole decreases. The particle C obtains energy larger than that of injected particles: $E_C^\infty = E_A^\infty - E_B^\infty > E_A^\infty$

$$M'_{BH} = M_{BH} + E_B^\infty < M_{BH}$$

⇒ Energy extraction from black hole

Mechanism of extraction of black hole rotational energy

- Penrose process
- MHD Penrose process
- Blandford-Znajek mechanism

Through negative energy-at-infinity

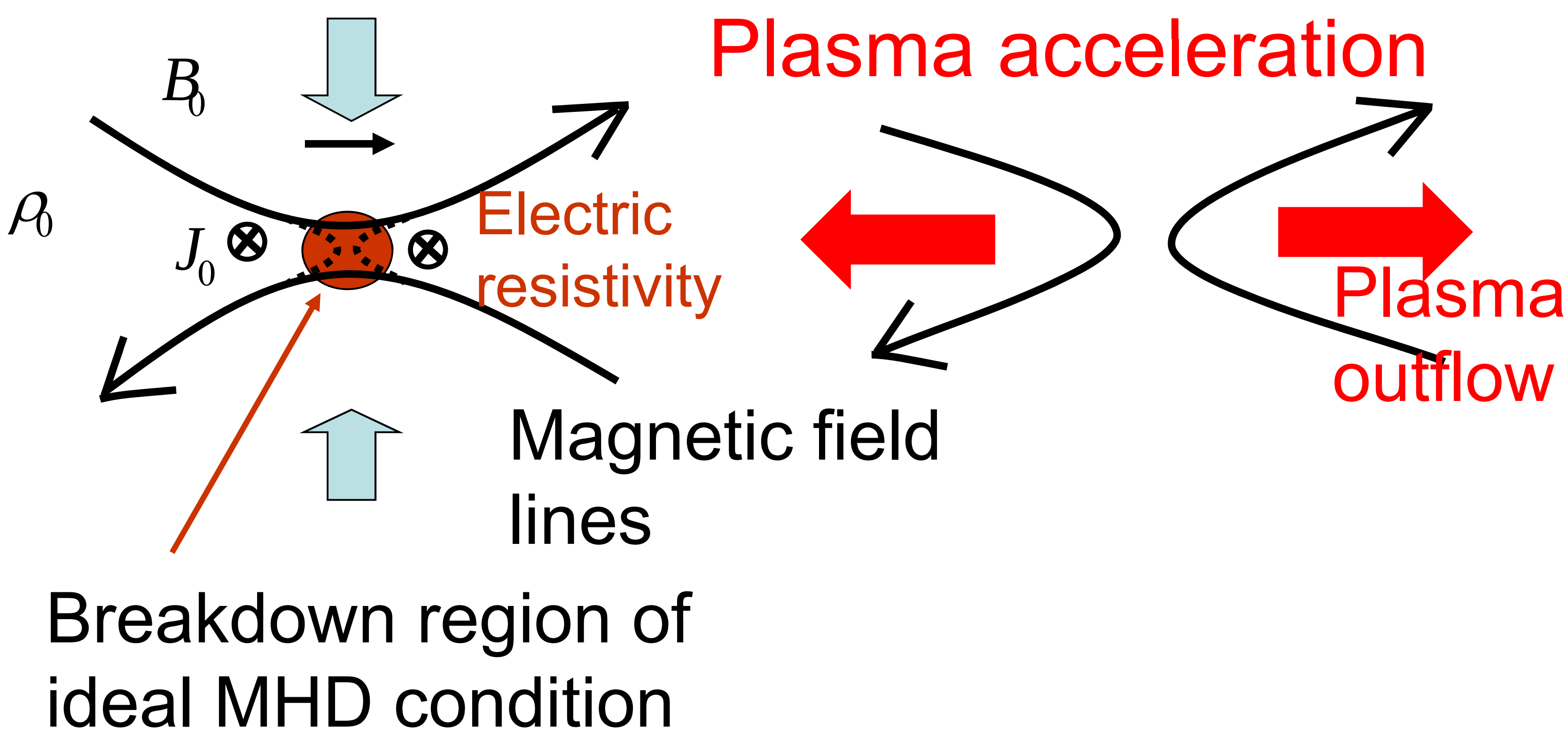
To realize negative energy-at-infinity, we require redistribution of angular momentum.

Mechanisms	Carrier of negative energy	Force for redistribution
Penrose	Particle	Particle fission
MHD Penrose	Plasma	Magnetic tension
Blandford-Znajek	Electromagnetic field	− Magnetic tension

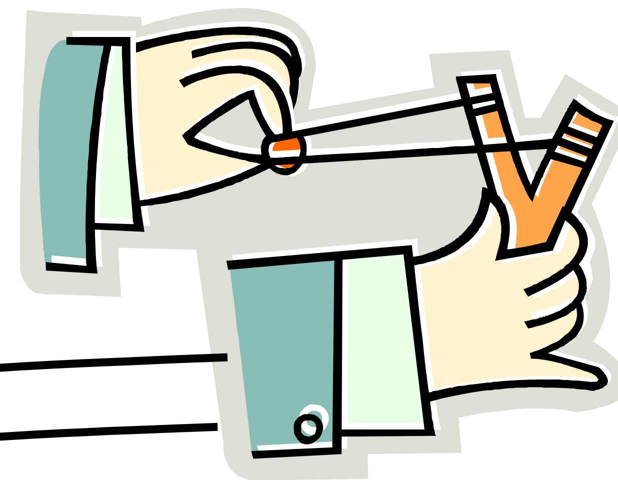
How about magnetic reconnection?

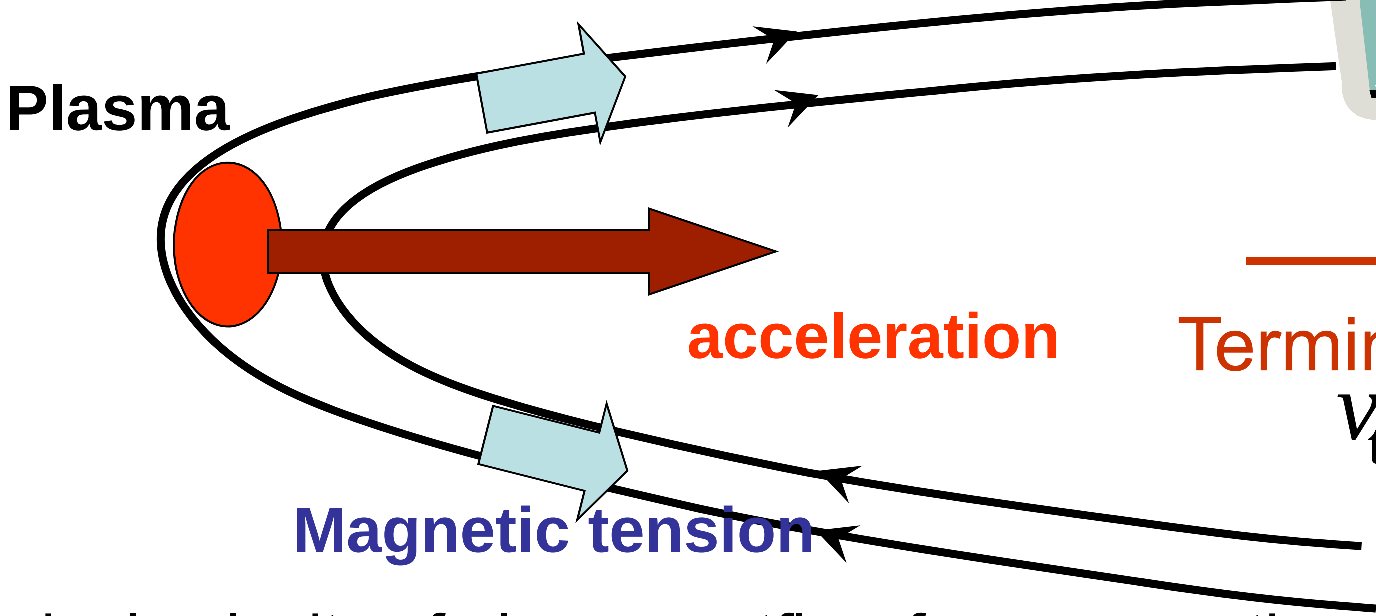
Magnetic reconnection： Plasma acceleration by reconnection of magnetic field lines

Ideal MHD condition
=frozen-in condition



Magnetic sling-shot





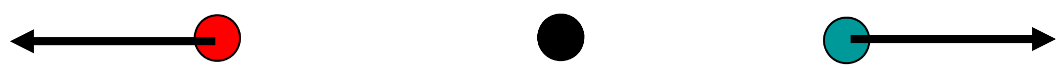
Terminal velocity of plasma outflow from magnetic reconnection when magnetic energy is converted to kinetic energy of plasma flow (Non-relativistic case)

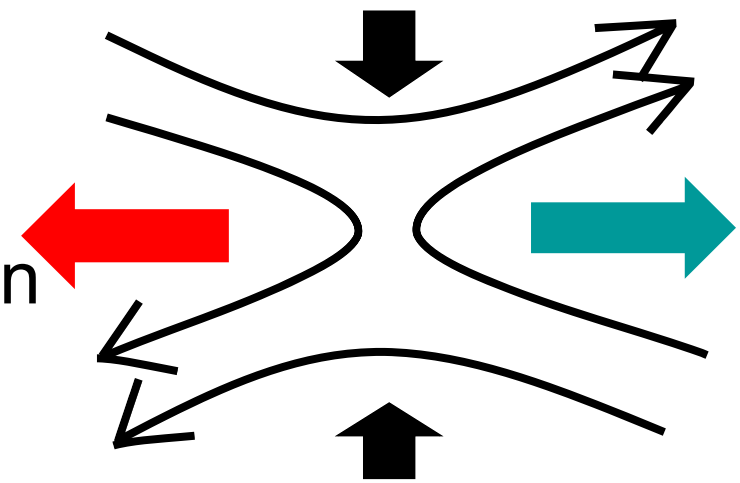
$$\frac{\rho_0}{2} v_{term}^{\;2} = \frac{B_0^2}{2} \quad \Rightarrow \quad v_{term} = \sqrt{\frac{B_0^2}{\rho_0}} = v_A$$ (Alfven velocity)

In relativistic case, we have to consider inertia of pressure.

Magnetic reconnection and particle fission

Penrose process

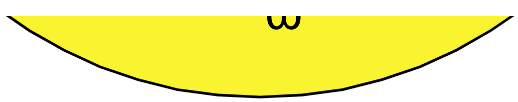




New mechanism of energy extraction from rotating BH?

ergosphere

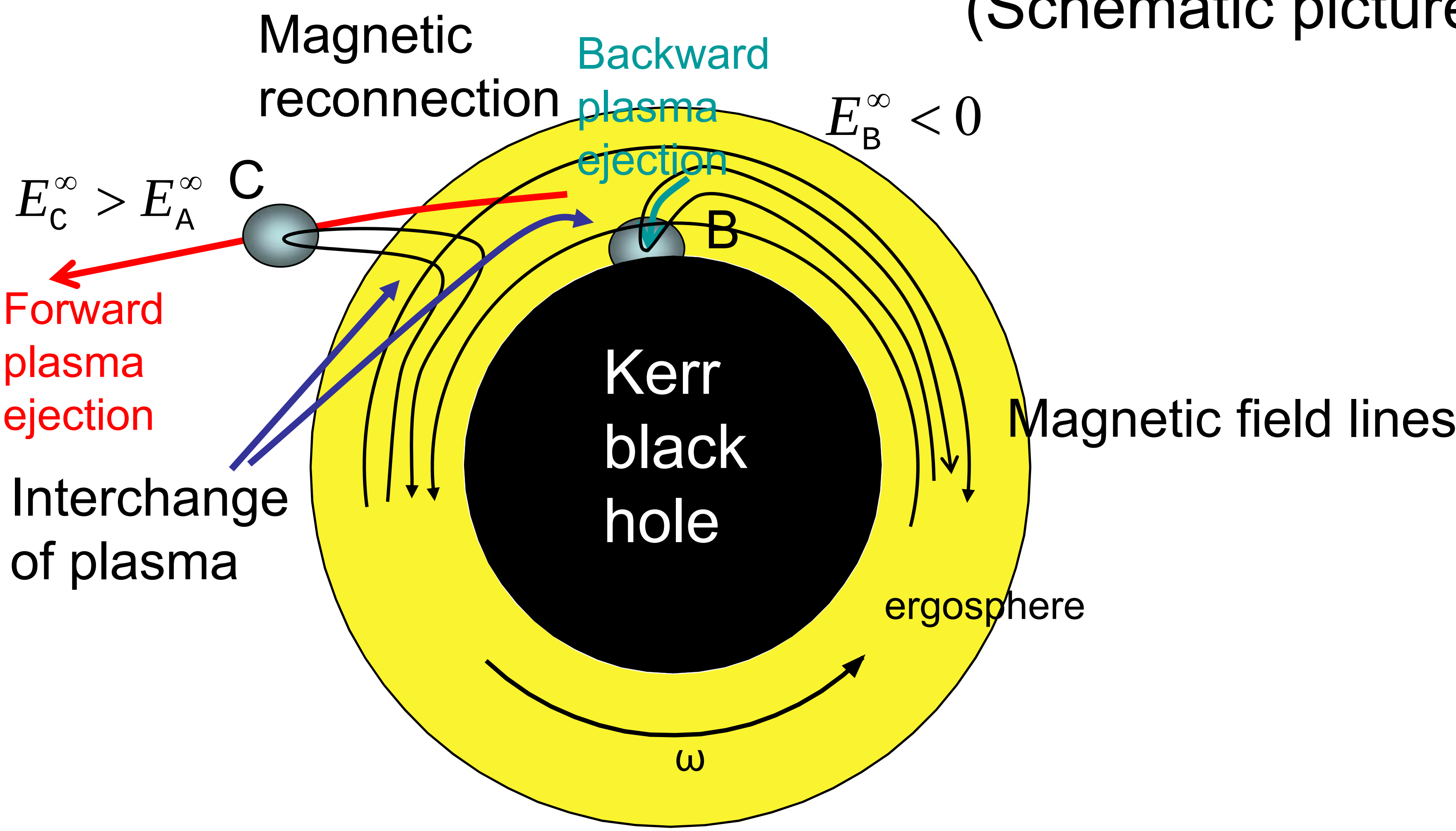
Frame-dragging
effect



Koide & Arai 2008

Escape and fall of a pair of plasma elements ejected from magnetic reconnection region

(Schematic picture)



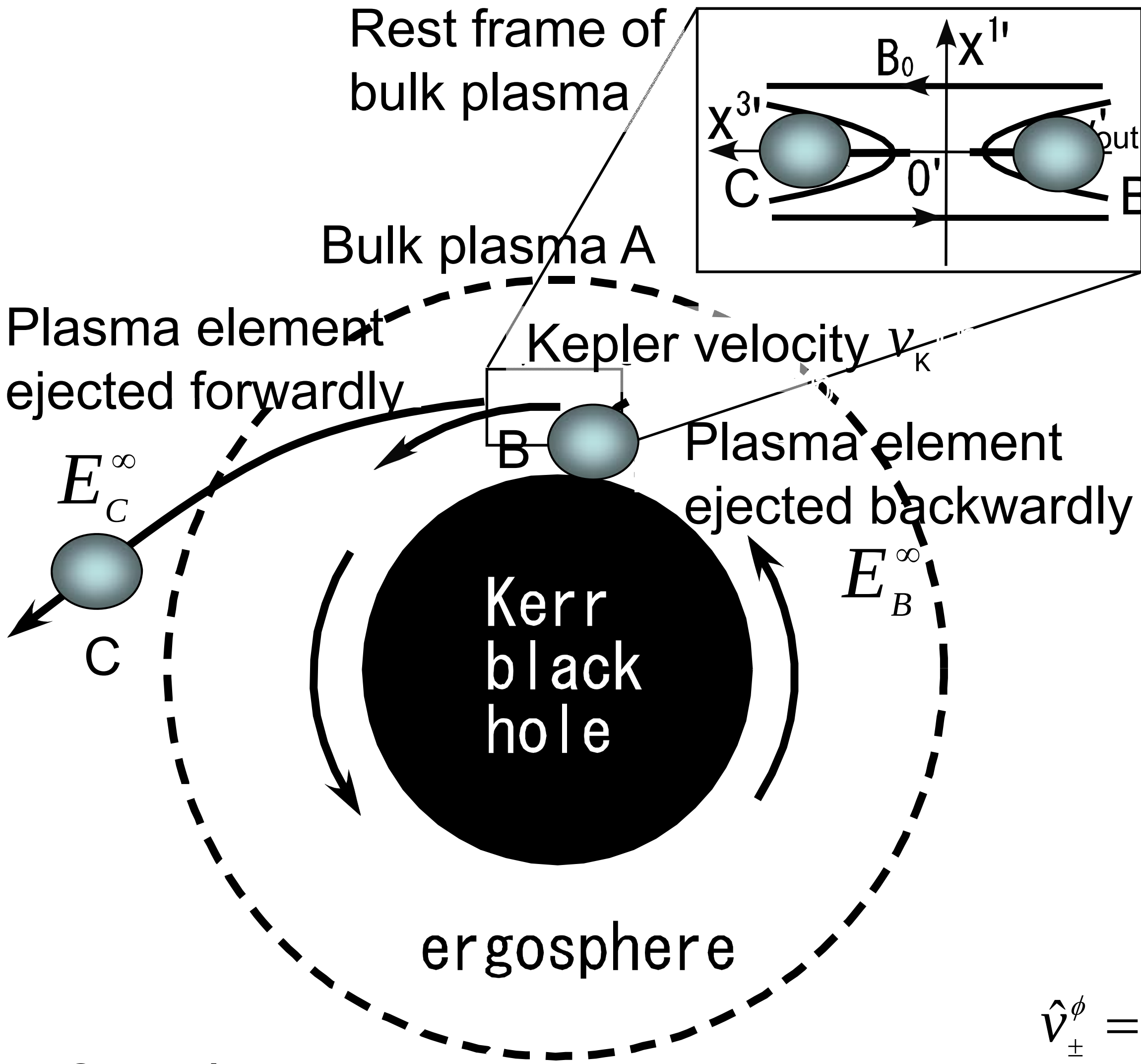
Very complex phenomena: numerical simulation with general relativistic MHD with resistivity (resistive GRMHD)

Simulation of magnetic reconnection in ergosphere

- We require numerical simulation to understand global mechanism of the magnetic reconnection in ergosphere.
- However, we have no numerical technique of resistive GRMHD. (This is reasonable because of standard formulation has causality problem.)
- To estimate the energy extraction of black hole due to magnetic reconnection, we use simplified model with combination of slab model of magnetic reconnection and general relativistic potential.

Analytic model of magnetic reconnection in ergosphere

(Local slab approximation)



θ=π/2

Energy-at-infinity of plasma element ejected forwardly/backwardly with the ejection velocity $v'_\phi = \pm v'_{out}$

$$E_\pm^\infty = \alpha\left(\hat{E}_\pm + \beta^\phi \hat{P}_\pm^\phi\right)$$

('+' = 'C', '—' = 'B')

$$\hat{E}_\pm = H\hat{\gamma}_\pm - \frac{\Gamma - 1}{\hat{\gamma}_\pm} U_{\mathrm{int}}$$

$$\hat{P}_\pm^\phi = H\hat{\gamma}_\pm \hat{v}_\pm^\phi$$

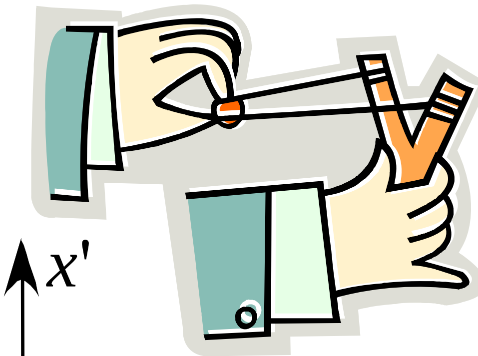
Enthalpy

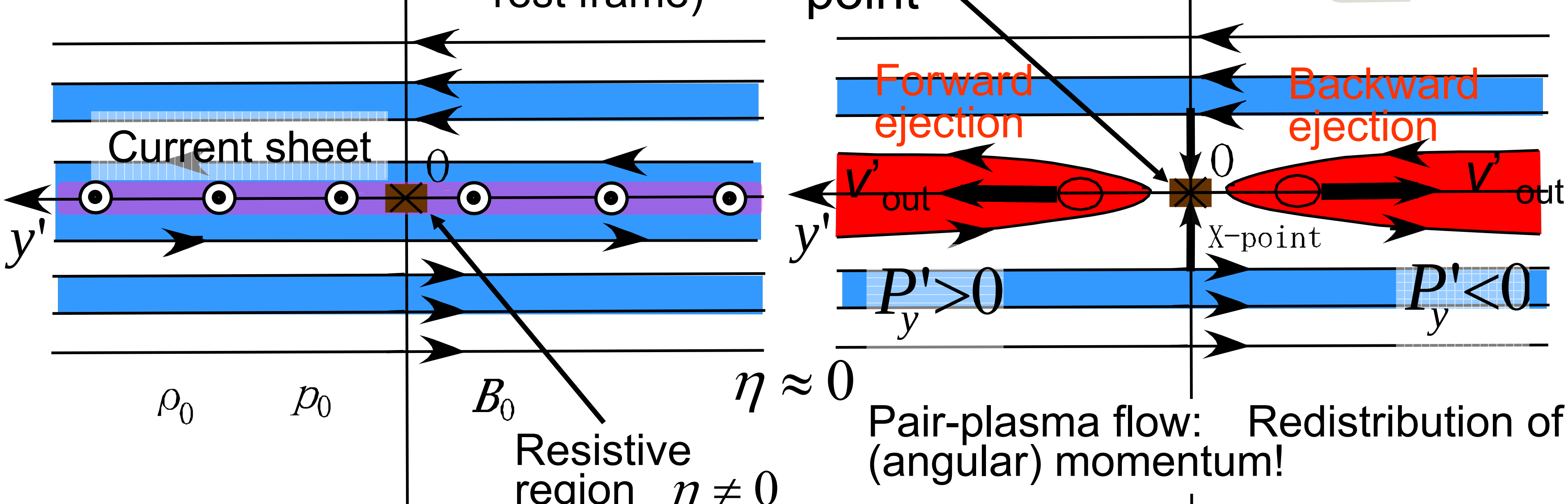
Thermal energy

$$\hat{v}_\pm^\phi = \frac{v_K \pm v'_{out}}{1 \mp v_{K0} v'_{out}}$$ Velocity of plasma element ejected observed by ZAMO

$$\hat{\gamma}_\pm = \left(1 - \hat{v}_\pm^{\phi\,2}\right)^{-1/2}$$ Lorentz factor

Slab model of relativistic magnetic reconnection





(Local approximation) We neglect

- Tidal force and Coriolis' force
- Global magnetic field reaction

Terminal velocity of plasma outflow from relativistic magnetic reconnection: inertia effect of pressure.

Complete energy conversion of magnetic energy to kinetic energy of plasma elements (assumption):

Particle energy after reconnection

$$\gamma'_{\mathrm{out}} H - \frac{(\Gamma-1)U}{\gamma'_{\mathrm{out}}} = \frac{B_0^2}{2n_0} + H - (\Gamma-1)U$$

Magnetic and thermal energy per particle before reconnection

Then, we get maximum of 4-velocity of the plasma element due to magnetic reconnection:

$$u'_{\mathrm{out}} = \gamma'_{\mathrm{out}} v'_{\mathrm{out}} = \left[\frac{1}{2}\left(1+\frac{u_{\mathrm{A}}{}^2}{2}-\varpi\right)^2 + (\varpi-1) + \frac{1}{2}\left(1+\frac{u_{\mathrm{A}}{}^2}{2}-\varpi\right)\sqrt{D}\right]^{1/2}$$

where

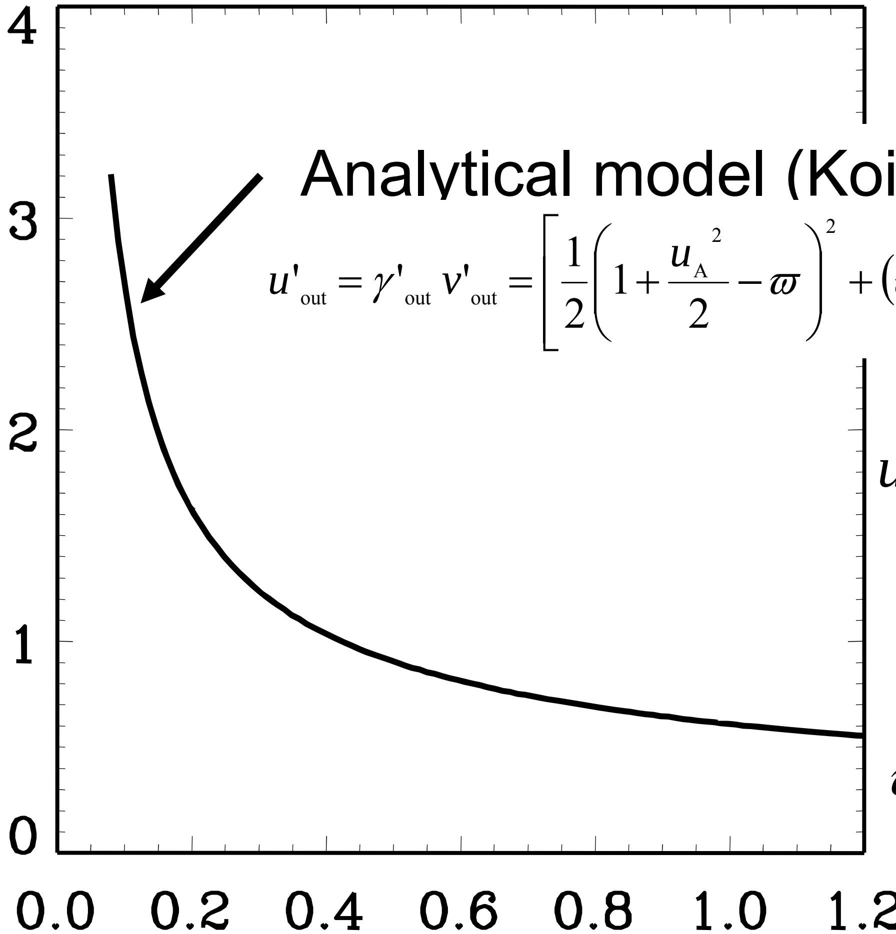
$$D = \left(1+\frac{u_{\mathrm{A}}{}^2}{2}-\varpi\right)^2 - 4\varpi$$

$$u_{\mathrm{A}} = \left[\frac{\rho_0}{B_0{}^2} + \frac{\Gamma}{(\Gamma-1)}\frac{p_0}{B_0^2}\right]^{-1/2}, \quad \varpi = \frac{p_0}{B_0^2}\left[\frac{\rho_0}{B_0{}^2} + \frac{\Gamma}{(\Gamma-1)}\frac{p_0}{B_0^2}\right]^{-1}$$

Relativistic magnetic reconnection

< Including inertial effect of (thermal) energy >

$$u_{\rm out}' = \gamma_{\rm out}' v_{\rm out}'$$



$$\beta_{\rm P} = \frac{p}{B_0^2/2}$$

Analytical model (Koide & Arai 2007)

$$u'_{\rm out} = \gamma'_{\rm out} v'_{\rm out} = \left[\frac{1}{2}\left(1 + \frac{u_{\rm A}{}^2}{2} - \varpi\right)^2 + (\varpi - 1) + \frac{1}{2}\left(1 + \frac{u_{\rm A}{}^2}{2} - \varpi\right)\sqrt{D} \right]^{1/2}$$

$$u_{\rm A} = \left[\frac{\rho_0}{B_0{}^2} + \frac{\Gamma}{(\Gamma - 1)} \frac{p_0}{B_0^2} \right]^{-1/2}$$

Alfven four-velocity, $\sqrt{\dfrac{B_0^2}{h_0}}$

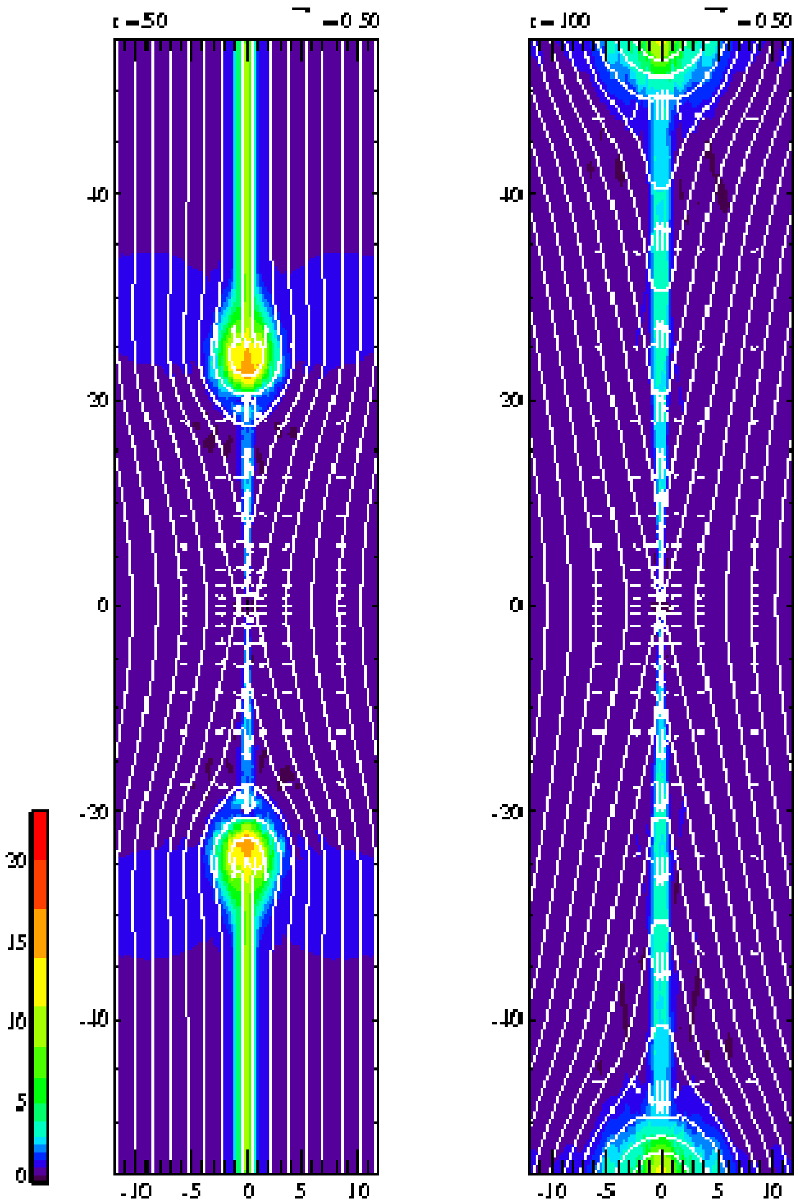
$$\varpi = \frac{p_0}{B_0^2} \left[\frac{\rho_0}{B_0{}^2} + \frac{\Gamma}{(\Gamma - 1)} \frac{p_0}{B_0^2} \right]^{-1}$$

Pressure per enthalpy

$\sqrt{p_0 / h_0}$

Γ: Polytoropic index

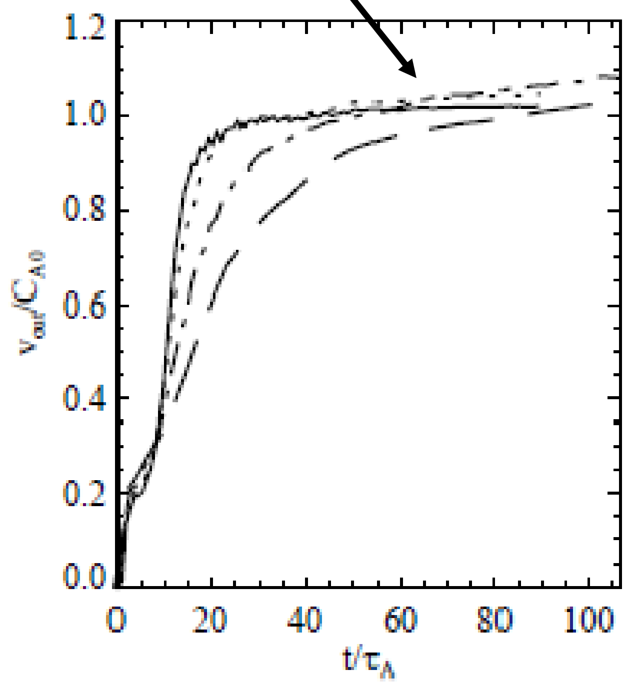
Simulation of Relativistic Magnetic Reconnection



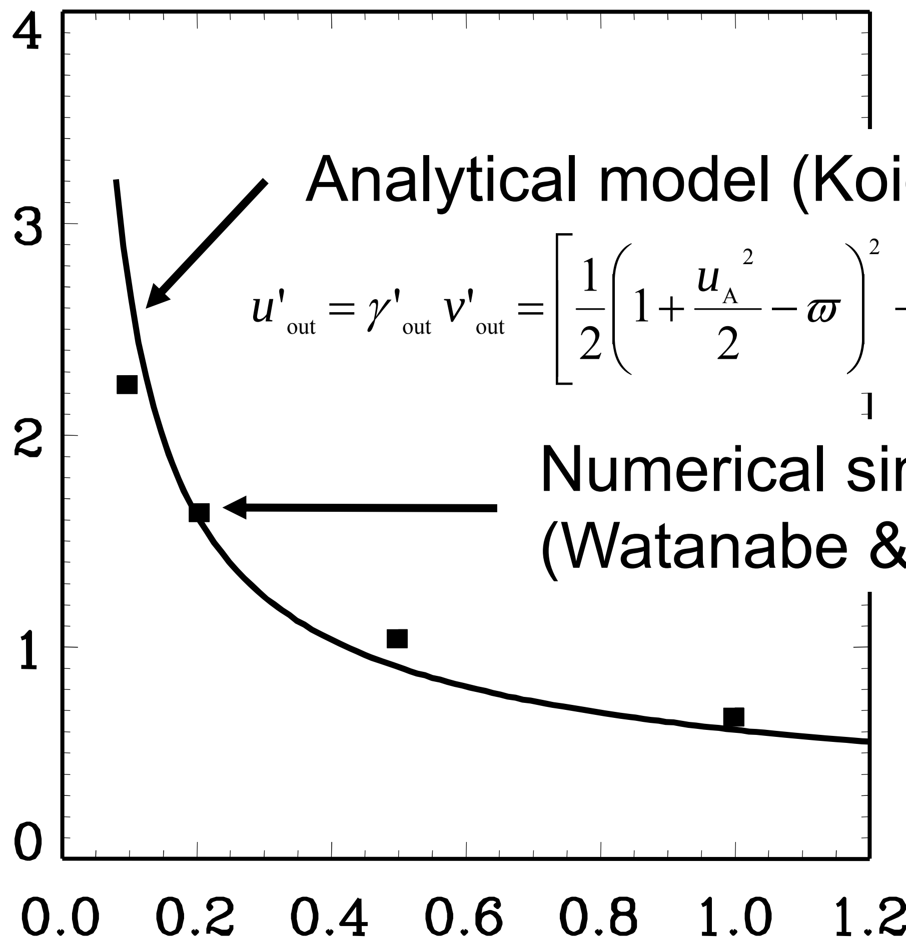
Watanabe & Yokoyama, ApJ. 2006

Momentum density: $\sim \rho c$ $\left(\dfrac{B^2}{\mu_0} \sim \rho c^2\right)$

Possibility of redistribution of significant momentum



Comparison between analytic and numerical solutions of relativistic magnetic reconnection



Analytical model (Koide & Arai 2007)

$$u'_{\mathrm{out}} = \gamma'_{\mathrm{out}} v'_{\mathrm{out}} = \left[\frac{1}{2}\left(1 + \frac{u_{\mathrm{A}}^{2}}{2} - \varpi\right)^{2} + (\varpi - 1) + \frac{1}{2}\left(1 + \frac{u_{\mathrm{A}}^{2}}{2} - \varpi\right)\sqrt{D} \right]^{1/2}$$

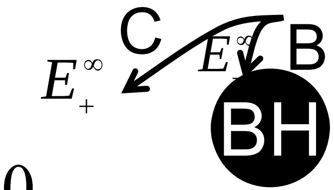
Numerical simulation (Watanabe & Yokoyama 2006)

$$u_{\mathrm{out}}' = \gamma_{\mathrm{out}}' v_{\mathrm{out}}'$$

$$\beta_{\mathrm{P}} = \frac{p}{B_0^2 / 2}$$

Conditions of energy extraction through magnetic reconnection

(1) Formation of plasma backward flow with negative energy-at-infinity：



$$\varepsilon_{-}^{\infty} = \frac{E_{-}^{\infty}}{H} < 0$$

(2) Escape of forward plasma flow to infinity:

Mass of plasma element

$$\Delta\varepsilon_{+}^{\infty} = \frac{E_{+}^{\infty} - M}{H} = \varepsilon_{+}^{\infty} - \left(1 - \frac{\Gamma}{\Gamma - 1}\varpi\right) > 0$$

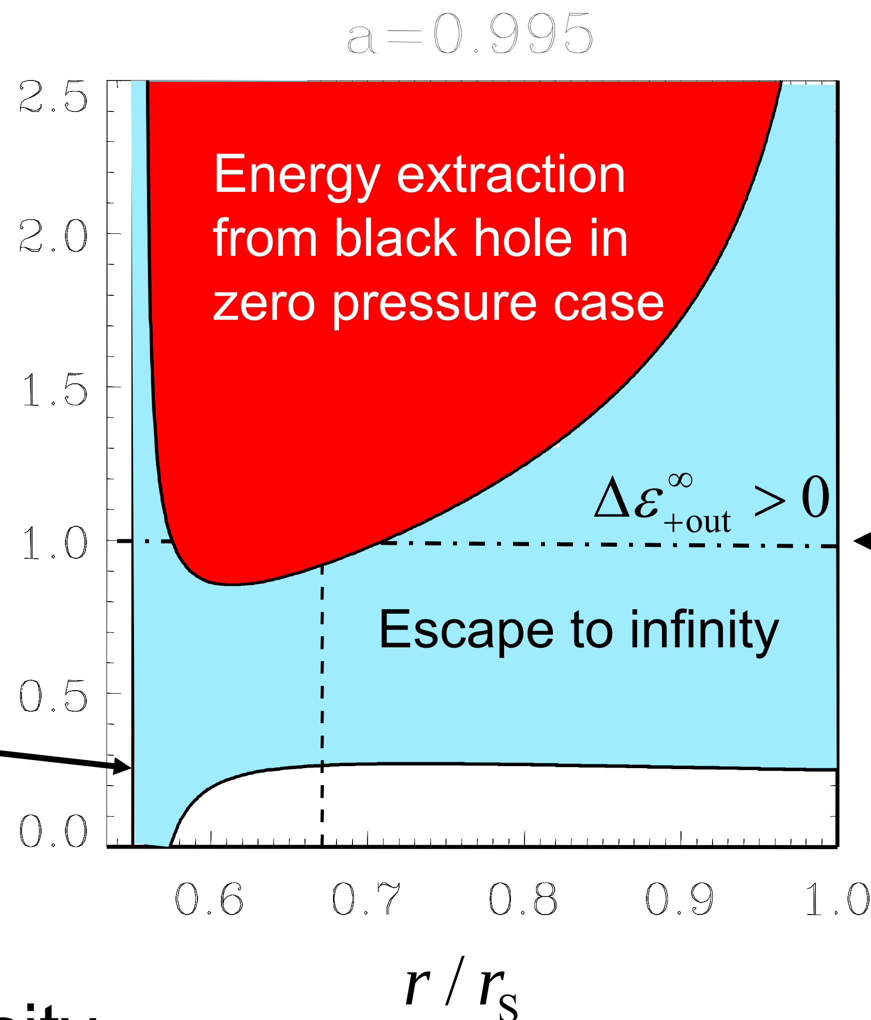
Specific energy-at-infinity of forward and backward plasma ejection estimated by slab model

$$\varepsilon_{\pm}^{\infty} = \frac{E_{\pm}^{\infty}}{H} = \alpha\hat{\gamma}_{\mathrm{K}}\left[\left(1+\beta_3\hat{v}_{\mathrm{K}}\right)\gamma'_{\mathrm{out}} \pm \left(\hat{v}_{\mathrm{K}}+\beta_3\right)\sqrt{\gamma'^{2}_{\mathrm{out}}-1} - \frac{\gamma'_{\mathrm{out}} \mp \hat{v}_{\mathrm{K}}\sqrt{\gamma'^{2}_{\mathrm{out}}-1}}{\gamma'^{2}_{\mathrm{out}}+\hat{\gamma}_{\mathrm{K}}^{2}\hat{v}_{\mathrm{K}}^{2}}\varpi\right]$$

Energy extraction condition through magnetic reconnection in ergosphere

Zero pressure case

$\theta=\pi/2$



$\hat{v}_0 = \hat{v}_{\mathrm{Kepler}}$

Bulk plasma rotates with Keplerian velocity

Relativistic reconnection

$\left(B_0^2 = h_0 > \rho c^2\right)$

$$\frac{B_0}{\sqrt{h_0}} = U_A$$

Alfven 4-velocity

Horizon

$$h_0 = \rho_0 c^2 + \frac{\Gamma}{\Gamma - 1} p_0$$

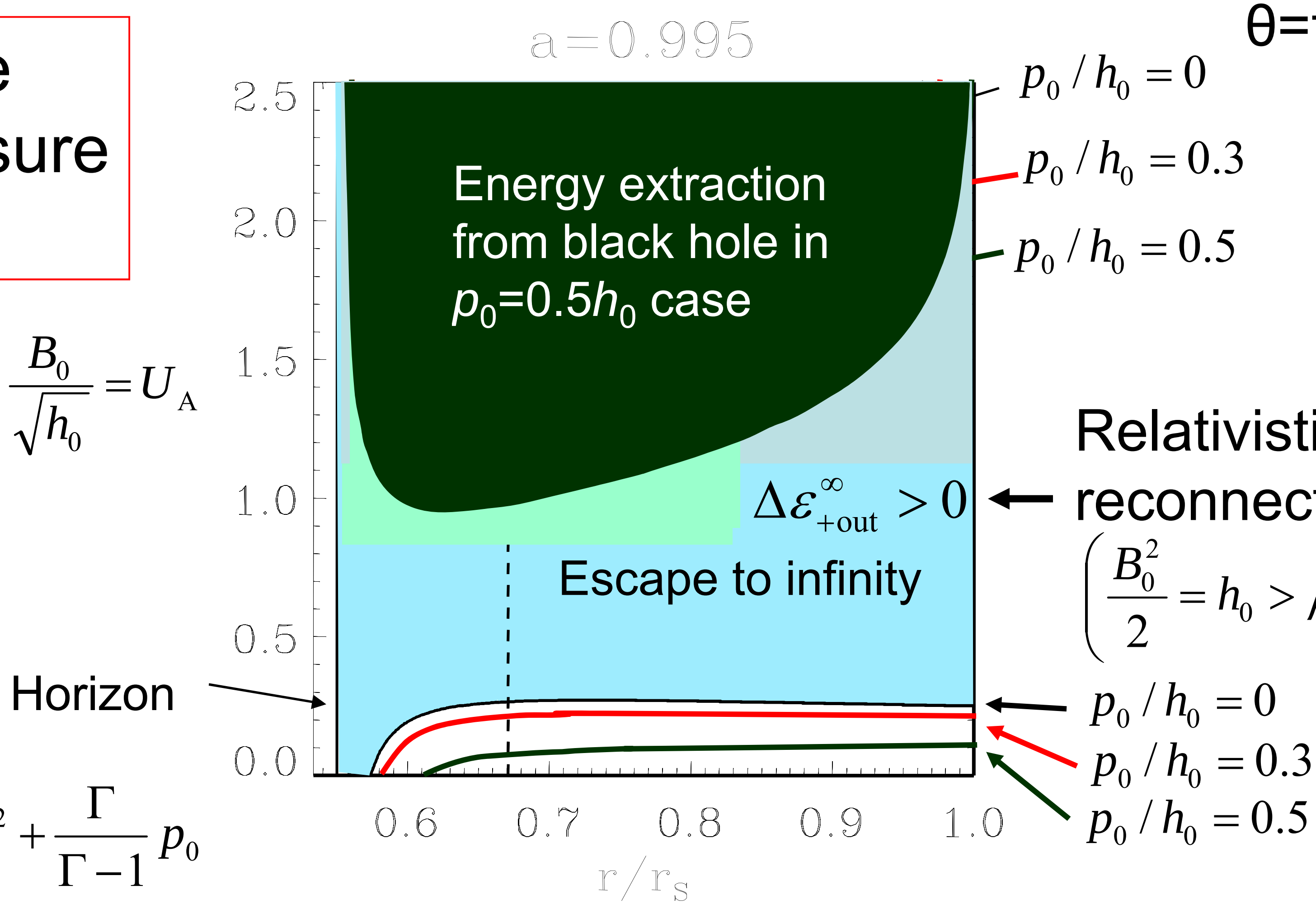
Enthalpy density

Energy extraction condition through magnetic reconnection in ergosphere

$\theta=\pi/2$

Finite pressure case

$$\frac{B_0}{\sqrt{h_0}} = U_A$$



Relativistic reconnection

$$\left(\frac{B_0^2}{2} = h_0 > \rho c^2\right)$$

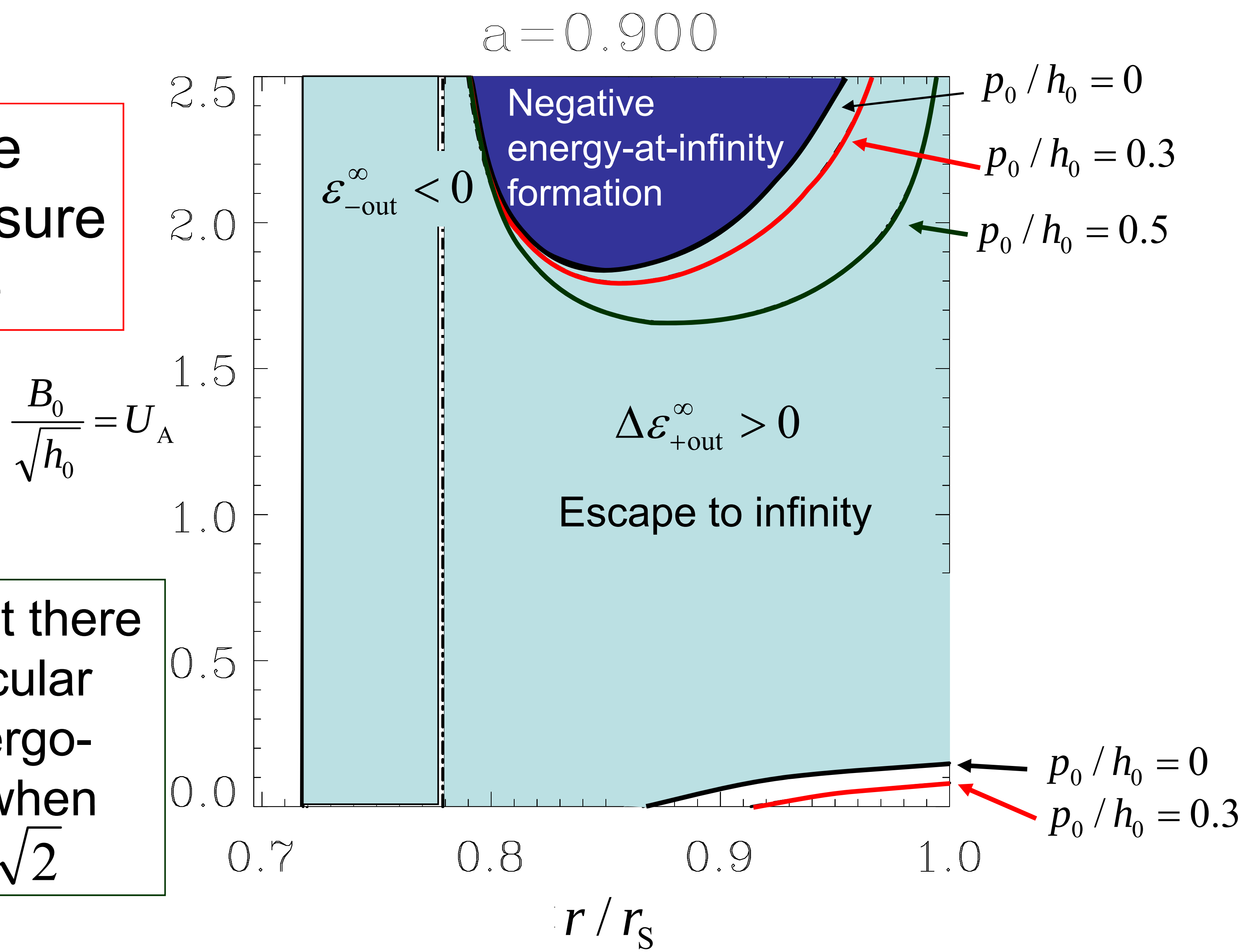
$$h_0 = \rho_0 c^2 + \frac{\Gamma}{\Gamma - 1} p_0$$

Enthalpy density

r/r_S

Slower rotating black hole case

Finite pressure case



Note that there is no circular orbit in ergo-sphere when $a < 1/\sqrt{2}$

Possibility of relativistic magnetic reconnection in individual objects

Relativistic magnetic reconnection is required to extract black hole rotational energy by magnetic reconnection:

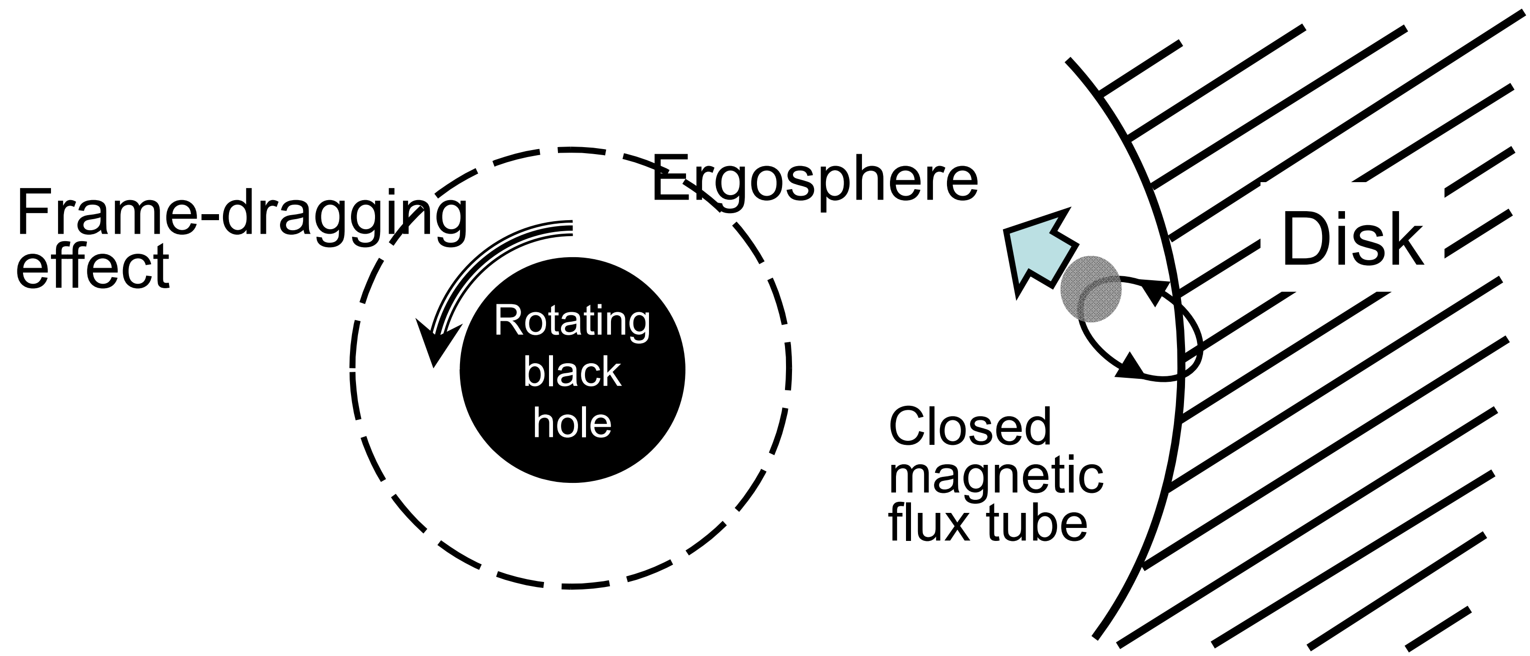
$$B_0 \geq B_{\mathrm{crit}} = \sqrt{\mu_0 \rho_0} c \quad \left(U_{\mathrm{A}} = 1,\ p_0 = 0\right)$$

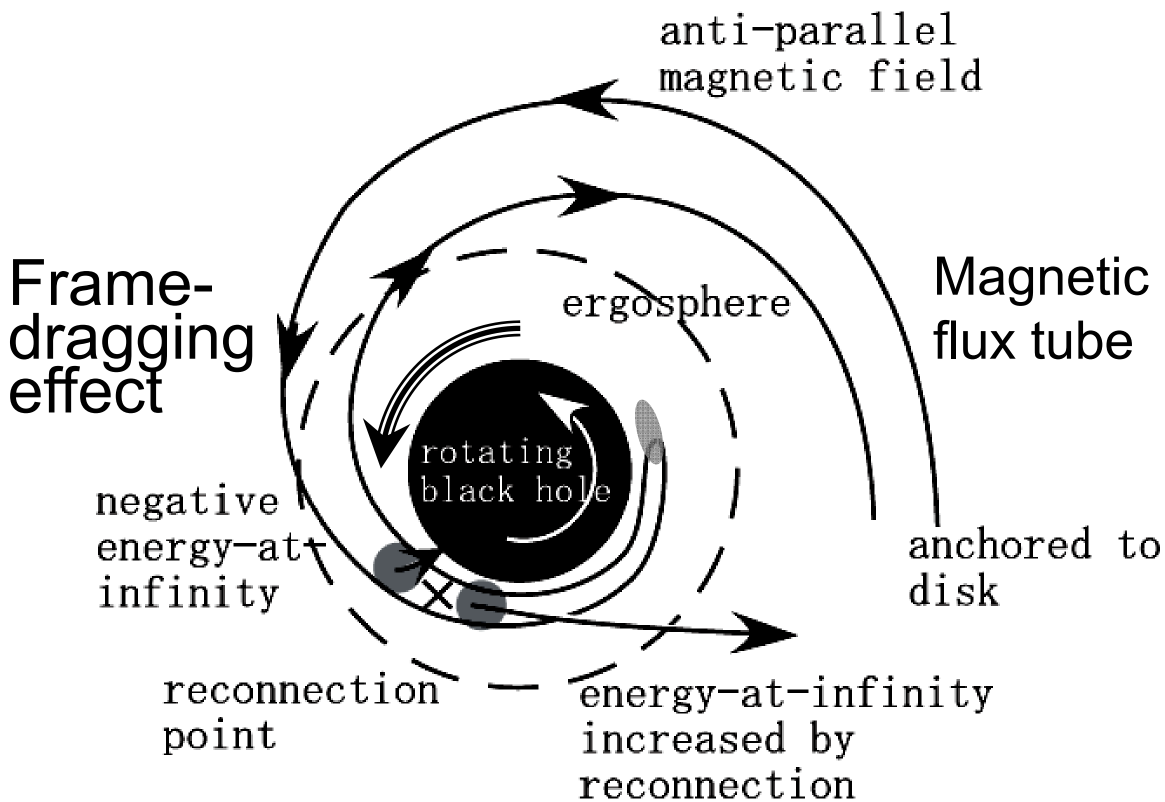
	individual	ρ_0 (g cm^{-3})	B_{crit} (G)	possibility
AGN	M87	2×10^{-17}	500	YES
GRB	(Collapsar model)	4×10^{11}	2×10^{15}	Marginal
Micro-QSO	GRS1915 +105	6×10^{-5}	8×10^{8}	NO

(SI unit system)

Typical density near the central black hole, which is assumed.

Energy extraction from black hole through magnetic reconnection in situation of magnetorotational instability





Koide & Arai, submitted

Numerical simulations with resistive GRMHD is demanded for further investigation.

Summary

We show one of a peculiar phenomena of magnetic reconnection around rotating black hole, extraction of black hole rotational energy through it. We clarify criterion condition of the mechanism in a rather simple situation.

- When we consider the magnetic reconnection in plasma rotating with Keplerian velocity around a rapidly rotating black hole (a=0.95), we need relativistic reconnection to extract the black hole energy.
- When the rotation of black hole becomes slower (a=0.9), the region of magnetic reconnection which extracts the black hole rotational energy becomes thinner and stronger magnetic reconnection is required. On the case of $a < 1/\sqrt{2}$ there is no circular orbit in the ergosphere and no possibility of the energy extraction in the simple situation.

Future Aspects

- To confirm the energy extraction mechanism of magnetic reconnection without artificial simplification, we have to perform global numerical simulation of generalized GRMHD with resistivity (Resistive GRMHD). Then we give consideration of the global magnetic field and complex plasma dynamics.
- However, standard resistive RMHD/GRMHD equations have causality problem, while ideal RMHD/GRMHD has no problem of causality because no electromagnetic wave can propagate in the ideal RMHD plasma.
- We have to reconsider the basic equations of resistive GRMHD/RMHD on the base of relativistic two-fluid equations (Khanna 1998, Koide 2008, 2009) or Boltzmann-Vlasov equations (Meier 2004, next talk) when we perform the resistive GRMHD simulations.

For example,

- Two-fluid model ⇒ generalized RMHD equations:

  (e.g. S. Koide, Phy. Rev. D 78, 125026 (2008),
  S. Koide, ApJ, 696, 2220-2233 (2009))

  When we consider inertia of current (electron) properly, we confirmed the group velocity is less than light speed in plasmas whose plasma parameter is much greater than unity.

- Numerical simulation with generalized GRMHD equations is required but it is difficult to perform because we have to take into account of displacement current term and inertia of electric charge and current. Especially, Ohm's law becomes very complex. This will be our next work.